Do Children Need Concrete Instantiations to Learn an Abstract Concept?

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Abstract

The effects of relevant concreteness on learning and transfer were investigated. Sixth grade students learned artificial instantiations of a simple mathematical concept. Some students were presented with instantiations that communicated concreteness relevant to the to-be-learned concept, while others learned generic instantiations involving abstract symbols. Results suggest that relevant concreteness may have some advantage over generic for learning. However, relevant concreteness hinders transfer of conceptual knowledge to novel isomorphic situations, while generic instantiations promote transfer.

Keywords: Cognitive Science; Psychology; Education; Learning; Transfer; Analogical reasoning.

Introduction

Concrete instantiations are popular tools for teaching abstract concepts in elementary and middle school (see Anderson, Reder, & Simon, 1996; Ball, 1992, for reviews). For example, children learn mathematical concepts such as place value with base ten blocks and fractions with representations of portions of pizza. However there is little empirical evidence of the effectiveness of such material for learning abstract concepts or for any advantage over generic, symbolic representations. Supporting evidence is often anecdotal or limited to demonstrations of knowledge in the learning domain. The goal of learning an abstract concept is not simply knowledge of one instantiation; it is the ability to transfer, or apply conceptual knowledge to a novel isomorphic situation.

Successful learning does not necessarily result in successful transfer (e.g. Gick & Holyoak, 1980, 1983). One factor that influences transfer of conceptual knowledge or problem solving strategies is the degree of similarity between the learned domain and the target domain. Superficial similarity between the domains such as storyline can facilitate the retrieval of an analogous, previously learned domain (Gentner, Rattermann, & Forbus, 1993). Also, elements that are similar across domains can promote transfer if they play analogous roles. However, when elements play different roles across domains, transfer typically goes awry (Ross, 1987, 1989).

Another factor that has been shown to affect transfer is the degree of concreteness of the learning domain (Goldstone & Sakamoto, 2003; Sloutsky, Kaminski, Heckler, 2005). Concrete instantiations communicate more information than their abstract counterparts. For example, consider the increase in information from an abstract stick figure to a detailed drawing, to a photograph. Sometimes this additional information may help communicate the to-be-learned concept and thus concreteness is "Relevant Concreteness". Other times it may be extraneous, creating "Irrelevant Concreteness".

A recent study examined the effects on learning and transfer of relevant and irrelevant concreteness (Kaminski, Sloutsky, & Heckler, 2005). College undergraduate students learned a simple mathematical concept that was instantiated through different artificial domains. The goal of the study was to investigate whether instantiating an abstract concept in a concrete manner would have benefits or costs for learning and transfer. The type of instantiation learned was a between-subjects factor. Participants learned instantiations that were generic, communicated relevant concreteness, communicated irrelevant concreteness, or communicated both relevant and irrelevant concreteness. For relevant concreteness, the storyline and symbols were designed to help communicate the relevant mathematical structure. Colorful, patterned symbols were used to add extraneous, perceptually engaging irrelevant concreteness. Relevant concreteness was shown to have an advantage for quick learning over irrelevantly concrete or generic instantiations. However with slightly lengthier training, the advantage of relevant concreteness over generic disappeared. Most importantly, both relevant and irrelevant concreteness hindered transfer, while generic instantiations promoted transfer (for the hindering effects of concreteness on transfer see also Goldstone & Sakamoto, 2003 & Sloutsky, Kaminski, Heckler, 2005).

The results of this study are striking because they contradict the intuition that facilitating learning will translate into facilitating transfer. However, the previous study involved only adult participants. Therefore, an important question remains unanswered. Is it possible that concreteness is helpful, but only for younger participants who cannot acquire an abstract concept otherwise? In particular, children may need a concrete instantiation to begin to grasp an abstract concept. This argument finds support in constructivist theories of development (e.g. Inhelder & Piaget, 1958) that posit that development proceeds from the concrete to the abstract and therefore learning should do the same. According to such theories, children under the age of twelve years are in the concrete operational stage of development in which thinking and problem solving are bound to the concrete. In addition, concrete instantiations may be more appealing to children than traditional generic symbols; and therefore children may be more engaged in learning (see Ball, 1992; Moyer, 2001).

On the other hand, there are several reasons to believe that concreteness will be at least as detrimental for children's transfer as it is for that of adults. First, successful transfer between a known base domain and a novel isomorphic target domain requires the recognition of common relational structure between domains; and there are several factors that affect this recognition. Superficial features of a representation may compete with relational structure for attention (Goldstone & Sakamoto, 2003). Therefore, the potential to be distracted from relational structure is greater for concrete instantiation than for generic instantiations. In addition, relational structure common to two situations is less likely to be noticed when the situations are represented in a more concrete, perceptually rich manner than in a more generic form (Gentner & Medina, 1998; Markman & Gentner, 1993). And finally, irrelevant information that may be communicated in a concrete instantiation can be misinterpreted as part of the relevant structure (Bassok & Olseth, 1995; Bassok, Wu, & Olseth, 1995).

Children may be more susceptible to the distractions of concreteness because they may be less able to control their attentional focus than adults (see Dempster & Corkill, 1999; Napolitano & Sloutsky, 2004). Children tend to notice object or attribute similarities between domains rather than relational similarities. When asked to match objects across two situations that share both object and relational commonalities, children tend to prefer object matches (Kotovsky & Gentner, 1996). Older children are better able than younger children to form matches bases on relations. Kotovsky and Gentner propose that this relational shift from attention to surface features to attention to relational structure is a result of knowledge of the given relation. For example, older children who are familiar with the concept of monotonic increase are able to form matches based on this relation, whereas younger children who are not familiar with this relation are less able to do so.

Another reason to believe that concrete instantiations may make transfer difficult for children is that for successful transfer, the elements of the learning domain may act as symbols for the elements of the transfer domain. It has been well documented that children have difficulty using concrete objects as symbols for other entities (DeLoache, 2000). While older children overcome this obstacle, increasing concreteness of entities can increase the difficulty of symbol use; and decreasing the concreteness can facilitate symbol use for younger children. For children to use an object as a symbol for something else, they must achieve representational insight in which they recognize that an object can have a *dual representation*, both as a concrete object itself and as an abstract referent to something else (see DeLoache, 2000; Uttal, Liu, & DeLoache, 1999). Dual representation is not achieved in a stage-like, all or nothing, manner. Instead, it depends on the particular stimuli and situation.

Uttal, Liu, and DeLoache (1999) argue that the often ineffective use of manipulatives for teaching mathematical concepts can be attributed, at least in part, to children's difficulty achieving dual representation. Children often fail to recognize the manipulatives as an instantiation of the to-be-learned concept (see also Ball, 1992).

Therefore, there are sufficient reasons to believe that concreteness may hinder children's ability to transfer conceptual knowledge. The purpose of the present research was to investigate the effects of relevant concreteness on children's ability to learn and transfer conceptual knowledge. In particular, children learned either a generic instantiation or a relevantly concrete instantiation of the same mathematical concept used in our earlier studies. Then they were presented with a novel isomorphic transfer domain and asked to answer questions about it.

Experiment

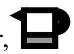
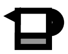
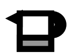
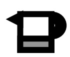
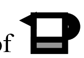
Method

Participants Nineteen sixth-grade students (seven female, twelve male, mean age = 11.8 years) from two middle schools in suburbs of Columbus, Ohio participated in the experiment. Children were randomly assigned to one of two conditions that specified the type of instantiation (i.e., generic or concrete) they learned.

Materials and Design The experiment consisted of two phases. In phase 1, participants learned a mathematical concept, using either a generic or relevantly concrete instantiation, with the type of instantiation varying between subjects. In phase 2, participants were tested on an isomorphic transfer domain.

The to-be learned concept was the same concept used in our previous research (Kaminski, Sloutsky, & Heckler, 2005; Sloutsky, Kaminski, & Heckler, 2005). This was a commutative group of order three. In other words the rules were isomorphic to addition modulo three. The idea of modular arithmetic is that only a finite number of elements (or equivalent classes) are used. Addition modulo 3 considers only the numbers 0, 1, and 2. Zero is the identity element of the group and is added as in regular addition: $0 + 0 = 0$, $0 + 1$

= 1, and 0 + 2 = 2. Furthermore, 1 + 1 = 2. However, a sum greater than or equal to 3 is never obtained. Instead, one would cycle back to 0. So, 1 + 2 = 0, 2 + 2 = 1, etc. To understand such a system with arbitrary symbols (not integers as above) would involve learning the rules presented in Table 1. However, a context can be created in which prior knowledge and familiarity may assist learning. In this type of situation the additional information is relevant to the concept.

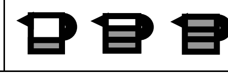
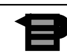
To construct a condition that communicates relevant concreteness, a scenario was given for which students could draw upon their everyday knowledge to determine answers to test problems. The symbols were three images of measuring cups containing varying levels of liquid (see Table 1). Participants were told they need to determine a remaining amount when different measuring cups of liquid are combined. In particular,  and  will fill a container. So for example, combining  and  would have  remaining. Additionally, participants were told that they should always report a remainder. Therefore they should report that the combination of  and  would have remainder . In this domain,  behaves like 0 under addition (the group identity element).  acts like 1; and  acts like 2. For example, the combination of  and  does not fill a container and so  remains. This is analogous to 1 + 1 = 2 under addition modulo 3. Furthermore, the perceptual information communicated by the symbols themselves can act as reminders of the structural rules. In this case, the storyline and symbols may facilitate learning.

In the Generic condition, the concept was described as a symbolic language in which three types of symbols combine to yield a resulting symbol (see Table 1). Combinations are expressed as written statements.

Training and testing in both conditions were isomorphic and presented via computer. Training consisted of an introduction and explicit presentation of the rules through examples. For instance, participants in the relevantly concrete conditions were told that combining  and  has a remainder of . Analogously, in the not relevantly concrete conditions where students were told that symbols combine to yield a resulting symbol the analogue to the above rule was presented as ◆, ◆ → ●. Questions with feedback were given and complex examples were shown.

After training, the participants were given a 16-question multiple choice test designed to measure the ability to apply the learned rules to novel problems. Many questions required

Table 1: Stimuli and rules across domains.

	Relevant Concreteness		No Relevant Concreteness	
Elements			● ◆ 	
Rules of Commutative Group:				
Associative	For any elements x, y, z: $((x + y) + z) = (x + (y + z))$			
Commutative	For any elements x, y: $x + y = y + x$			
Identity	There is an element, **I**, such that for any element, x: $x + \mathbf{I} = x$			
Inverses	For any element, x, there exists another element, y, such that $x + y = \mathbf{I}$			
Specific Rules:	 is the identity		 is the identity	
	These combine	Remainder	Operands	Result
			● ●	◆
			◆ ◆	●
			● ◆	

application of multiple rules. The following are examples of test questions in the not relevantly concrete conditions.

(1) What can go in the blanks to make a correct statement?

___ , ◆ , ___ , ● → ● ?

(2) Find the resulting symbol:

◆ , ● ,  , ◆ → _____.

Participants in the Relevant Concreteness condition saw the analogues of these questions.

After training and testing in the base domain a novel transfer domain was presented. The same transfer domain was used for both conditions and was described as a children's game involving three objects (see Table 2). Children sequentially point to objects and a child who is "the winner" points to a final object. The correct final object is specified by the rules of the game (rules of an algebraic group). Participants received no explicit training in the target domain. They were not explicitly taught the rules of the system. Instead they were told that the game rules were like the rules of the system they just learned and they need to figure them out by using their newly acquired knowledge (i.e. transfer). After being asked to study a series of examples from which the rules could be deduced, 16-question multiple-choice test was given. The test was isomorphic to the test given in phase 1. Questions were presented individually on the computer screen along with four key examples at the bottom of the screen. The same four examples were shown with all test questions. Following the multiple-choice questions, participants were asked to match each element of the transfer domain to its analogous element in the base domain and then to rate the similarity of the learning and transfer domains.

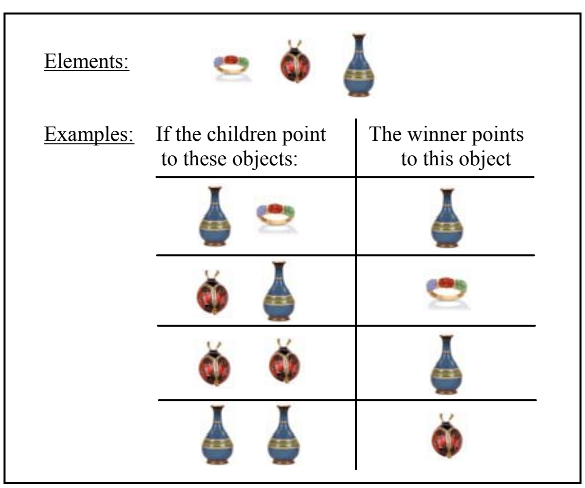
In previous experiments involving adults (Kaminski, Sloutsky, & Heckler, 2005), no participant was able to score above change on a test of the target domain without first learning an isomorphic domain. It is unlikely that children would score above chance in this domain without having previously learned an isomorph. Therefore, in the present experiment, target scores that are above chance suggest successful transfer of conceptual knowledge. Also, note that in a separate experiment, adults who read descriptions of the learning and transfer domains, but received no explicit training of the rules, found both the generic and concrete domains equally similar to the target domain. Thus, any differences in transfer performance across conditions cannot be attributed to differential similarity of learning and transfer domains.

Procedure Training and testing were presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded. A female experimenter was present while students completed the activity.

Results and Discussion

Two participants (one Generic, one Relevant Concreteness) were eliminated from the analysis because their scores in the learning domain (Phase 1) were two standard deviations below the mean score of their respective conditions.

Table 2: Stimuli for transfer domain.



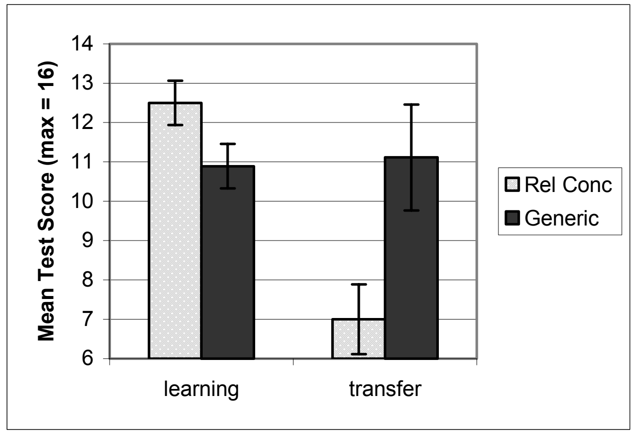


Figure 1: Mean Test Scores.
Note: Error bars represent standard error of mean.

As shown in Figure 1, in both conditions participants successfully learned in the base domain, with mean learning scores being significantly above chance score of 6, one sample t-tests, *ts* > 8.67, *ps* < .001 (see Figure 1). At the same time, there was a clear advantage of generic instantiations for transfer. These findings were supported by a two by two mixed ANOVA revealing a significant interaction, F (1, 15) = 13.9, $p < .003$. While there was a marginal advantage of Relevant Concreteness for learning, independent samples t-test t (15) = 2.00, $p = .063$, there was a marked advantage of Generic instantiation for transfer, independent samples t-test t (15) = 2.49, $p < .03$. Furthermore, in the Relevant Concreteness condition, transfer scores were no different than a chance score of 6, one sample t-test t (7) = 1.128, $p = .296$.

Additional analyses focused on the ability to match corresponding elements across domains, which differed markedly between the Generic condition and the Relevant Concreteness condition. Only one of eight participants in the Relevant Concreteness condition correctly matched elements. While seven of nine participants in the Generic condition made the correct correspondences, χ^2 (*df*=1, N=17) = 7.2, $p <$ 0.008. Furthermore, there was a very high correlation between matching ability and test score, point biserial correlation, $r_{pb} = .83$. The mean transfer score for those who made the correct matching was 12.5 (*SD* = 3.02), while the mean score for those who did not make the correct matching was 6.22 (*SD* = 1.39). This difference was clearly significant, independent samples t-test, t (15) = 5.61, $p < .001$.

Similarity ratings also differed as a function of ability to match elements. Participants who correctly matched elements rated the domains as highly similar, mean = 4.5 (*SD* = .756) on a scale from 1 (completely different) to 5 (almost identical). At the same time, participants who did not match elements correctly gave a mean similarity rating of 2.7 (*SD* = .866). Again this was a significant difference, independent samples t-test, t (15) = 4.62, $p < .001$. Taken together, these findings suggest that that those participants who aligned the two domains exhibited a greater ability to match elements between the domains, perceived the domains as more similar, and exhibited greater transfer. Furthermore, the likelihood of alignment was greater with generic than with relevantly concrete instantiations.

General Discussion

The results of this study demonstrate that children do not need a concrete instantiation to acquire an abstract concept. Some concreteness, relevant concreteness, can help to communicate the relevant structure in the context of learning; relevant concreteness was shown to have a slight advantage over generic instantiations of the same concept for initial learning. However, generic instantiations can also be learned well by children. Most importantly knowledge acquired through a generic instantiation can be transferred to a novel isomorph, while knowledge of a relevantly concrete instantiation does not transfer spontaneously. For relevantly concrete instantiations, the structural knowledge appears to be bound to the learning domain so that it cannot be easily recognized elsewhere.

These findings suggest that transfer could be construed as a process of analogical reasoning. Analogy involves several subprocesses: (1) representation of the target domain, (2) retrieval of prior domain, (3) alignment of elements and mapping of structure across domains, and (4) implementation of the analogy (see Rattermann, 1997 for review). Of crucial importance is alignment and mapping of structure (see Gentner, 1983). In the present study, participants were explicitly told that their knowledge of the first domain could be applied to the second. Therefore, failure for participants to transfer was not due retrieval failure. Failure was also not due poor learning, as the Relevant Concreteness participants actually had higher learning scores than the Generic participants. Transfer failure appears to be due to inability to align and map structure. Relevant Concreteness participants were not able to recognize structure and match elements across domains.

The fact that participants who were able to match elements scored highly on the transfer test and those who were not able to match scored poorly supports the notion that structural alignment is a necessary step in transfer across isomorphs. Also, in agreement with structure mapping theories (Markman & Gentner, 1993), participants who were able to align found the learning and transfer domains to be highly similar, while those who were not able to align did not.

The results of this study have important implications for teaching. If indeed the goal of teaching abstract concepts, such as mathematical and scientific concepts, is transfer, then elaborate teaching of concrete instantiations is not likely to help attain that goal. Moreover, generic external representations such as traditional symbolic notation can be well learned by children and will increase the likelihood of transfer.

In conclusion, while the ease of learning can make relevantly concrete instantiations appealing for teaching, these instantiation are unlikely to promote transfer. Generic instantiations, on the other hand, can be learned by children and once learned they can give children the power to gain knowledge of novel analogues.

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