Abstract
This study investigated the nature of internal representations constructed from learning concrete or generic instantiations of an abstract concept. Undergraduate students learned instantiations of a mathematical group that were generic, communicated concreteness relevant to the concept, or communicated concreteness irrelevant to the concept. Students who learned relevantly concrete instantiations were unable to recognize relational structure in the context of novel elements, while students who learned with no relevant concreteness were able to recognize the structure. This differential ability suggests participants constructed different representations which may in turn be responsible for differential transfer, with generic instantiations promoting transfer and relevantly concrete hindering. When given the alignment of elements across domains, all subjects transferred equally well, suggesting that alignment of elements helps disembed relational structure.

Keywords: Cognitive Science; Psychology; Education; Learning; Transfer; Analogical reasoning, Knowledge Representation.

Introduction
Transfer, or the ability to use prior knowledge in a novel situation, is a major goal of education, yet it is often difficult to achieve. Researchers have documented both transfer failures and transfer successes (e.g., Gick & Holyoak, 1980, 1983). Successful transfer across isomorphs has been construed as a process of analogical reasoning from a learned base domain to a novel target transfer domain which requires success on each of four subprocesses: (1) representation of the transfer domain, (2) retrieval of prior domain, (3) alignment of elements and mapping of structure across domains, and (4) implementation of the analogy (see Rattermann, 1997 for review). Of crucial importance is alignment and mapping of structure (see Gentner, 1983) which requires first that relational structure is recognized in the transfer domain.

One factor affecting both retrieval and alignment is the degree of similarity of the base and transfer domains. Superficial similarity between the base and transfer domains such as storyline can facilitate the retrieval of an analogous, previously learned domain (Gentner, Rattermann, & Forbus, 1993; Ross, 1987, 1989). In addition, elements that are similar across domains can promote transfer if they play analogous roles. On the other hand, if similar elements play different roles across domains, the likelihood of successful transfer diminishes significantly (Ross, 1987, 1989).

In the absence of glaring similarities across domains, what other characteristics of the learning domain might affect transfer to a novel isomorph? Concreteness of the learning domain has been shown to hinder transfer (Goldstone & Sakamoto, 2003; Sloutsky, Kaminski, & Heckler, 2005). However, not all concreteness is the same. The concreteness involved in the earlier studies also hindered learning (Sloutsky et al, 2005) or did not significantly facilitate it (Goldstone & Sakamoto, 2003). Another possibility with some intuitive appeal is that a domain that promotes quick learning would also promote transfer. However, the results of a previous study demonstrated that quick learning does not necessarily translate into successful transfer (Kaminski, Sloutsky, & Heckler, 2005). College undergraduates learned a simple mathematical concept that was instantiated through different artificial domains. The goal of the study was to investigate whether instantiating an abstract concept in a concrete manner would have benefits or costs for learning and transfer. Of particular interest was the impact of a type of concreteness that might give rise to the “ah-ha” effect by helping to communicate the relevant concept. This “Relevant Concreteness” underlies many instructional materials such as base ten blocks and portions of pizzas that are used to teach arithmetic. For relevant concreteness, the storyline and symbols were designed to help communicate the relevant mathematical structure. Colorful, patterned symbols were used to add extraneous, perceptually engaging “Irrelevant Concreteness”. Therefore, subjects learned one of four domains: (1) Generic, in which arbitrary black symbols were used and storyline offered no insight to the relevant concept, (2) Irrelevant Concreteness, same storyline as the Generic, but perceptually rich symbols, (3) Relevant Concreteness, in which the storyline and symbols encouraged participants to draw upon their everyday knowledge to learn the structure, and (4) Relevant &
Irrelevant Concreteness, same storyline as Relevant Concreteness with perceptually rich symbols (see Table 1).

The results of the study found, as expected, an advantage of relevant concreteness for learning. With minimal training, students who learned the relevantly concrete domain performed significantly higher when tested than students who learned the domain with no relevant concreteness. However, this benefit was limited only to learning with minimal training. With protracted training, students who learned the generic domain scored as highly as those who learned with relevant concreteness. Only subjects who learned the irrelevently concrete domain scored lower that the relevantly concrete group. Most interesting was transfer performance. When presented with a novel isomorphic domain, subjects who learned the generic instantiation ably transferred conceptual knowledge, while subjects who learned with relevant or irrelevant concreteness did not. Interestingly, both irrelevant and relevant concreteness hindered transfer, but a comparison of both learning and transfer suggests that they did so for different reasons. Irrelevant concreteness hindered learning and thus subsequently hindered transfer, while the relevant concreteness appeared to hinder transfer by possibly obfuscating the analogy between the learning and transfer domains.

Additional (albeit inconclusive) evidence that relevant concreteness obfuscated the analogy between the learning and transfer domains comes from similarity ratings. When asked to rank similarity between each learning domain and the transfer domain prior to learning, participants ranked all learning domains as equally similar to the transfer domain. However, after training, similarity between the generic learning domain and the transfer domain increased, whereas similarity between concrete learning domain and the transfer domain remained low. One possibility is that it is easier to align a generic learning domain with the transfer domain than a concrete learning domain. Given that alignable structures are considered more similar than non-alignable (Markman & Gentner, 1993), it is possible that differential alignability could underlie both differential transfer and differential similarity.

Another possibility is that failure to transfer is due to an inability to recognize the relational structure in the transfer domain. Learning a generic instantiation allowed recognition of structure while learning with a relevantly concrete instantiation did not. This suggests that categorically different internal representation were constructed depending on what type of instantiation was learned. Internal representations of a concept might contain the following types of information: (1) relational structure that defines the concept, (2) elements that instantiate the concept, and (3) other extraneous information such as storyline. To possess structural knowledge of a particular instantiation implies that the internal representation contains both elements and relations. The fact that participants in all conditions performed well on the learning test indicates that elements and relations were represented. But perhaps for Relevant Concreteness, the element information and relational information are so tightly bound that the relational information cannot be recognized elsewhere, while for Generic and Irrelevant Concreteness, the relational information can be disassociated from the learning elements.

The goal of experiment 1 was to test the hypothesis that learning a relevantly concrete instantiation leads to a representation in which relational information is bound to element information, while learning with a generic or irrelevently concrete instantiation does not. In particular, after learning, can a participant recognize truths and violations of relational structure when expressed with either familiar or novel elements?

**Experiment 1**

**Method**

**Participants** Fifty three undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students were randomly assigned to one of three conditions that specified the type of instantiation they learned.

**Materials and Design** The experiment consisted of two phases. In phase 1, all participants learned an instantiation of a mathematical concept. The type of instantiation learned was a between-subjects factor: Relevant Concreteness, Irrelevant Concreteness, and Generic. In phase 2, participants were presented with expressions involving either familiar elements from phase 1 or novel elements and were asked whether the rules are the same as those of phase 1.

Phase 1 used the same to-be-learned concept that was used in our previous research (Kaminski et al., 2005; Sloutsky et al., 2005). This was a commutative group of order three. In other words the rules were isomorphic to addition modulo three. The idea of modular arithmetic is that only a finite number of elements (or equivalent classes) are used. Addition modulo 3 considers only the numbers 0, 1, and 2. Zero is the identity element of the group and is added as in regular addition: \(0 + 0 = 0, 0 + 1 = 1,\) and \(0 + 2 = 2.\) Furthermore, \(1 + 1 = 2.\) However, a sum greater than or equal to 3 is never obtained. Instead, one would cycle back to 0. So, \(1 + 2 = 0, 2 + 2 = 1,\) etc. To understand such a system with arbitrary symbols (not integers as above) would involve learning the rules presented in Table 1. However, a context can be created in which prior knowledge and familiarity may assist learning. In this type of situation the additional information is relevant to the concept.

In the Relevant Concreteness condition, the symbols were three images of measuring cups containing varying levels of liquid (see Table 1). Participants were told they need to determine a remaining amount when different measuring cups of liquid are combined. In particular, \(\) and \(\) will
Table 1: Stimuli and rules across domains.

<table>
<thead>
<tr>
<th>Elements</th>
<th>Relevant Concreteness</th>
<th>Generic</th>
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<tbody>
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<td>![Generic symbols]</td>
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**Rules of Commutative Group:**

- **Associative**
  - For any elements \(x, y, z\):
  \(((x + y) + z) = (x + (y + z))\)

- **Commutative**
  - For any elements \(x, y\): \(x + y = y + x\)

- **Identity**
  - There is an element, \(I\), such that for any element, \(x\):
  \(x \cdot I = x\)

- **Inverses**
  - For any element, \(x\), there exists another element, \(y\), such that \(x \cdot y = I\)

**Specific Rules:**

- **is the identity**
- **is the identity**

<table>
<thead>
<tr>
<th>These combine</th>
<th>Remainder</th>
<th>Operands</th>
<th>Result</th>
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<tr>
<td>![Symbols]</td>
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fill a container. So for example, combining \(\text{ }\) and \(\text{ }\) would have \(\text{ }\) remaining. Additionally, participants were told that they should always report a remainder. Therefore they should report that the combination of \(\text{ }\) and \(\text{ }\) would have remainder \(\text{ }\). In this domain, \(\text{ }\) behaves like 0 under addition (the group identity element). \(\text{ }\) acts like 1; and \(\text{ }\) acts like 2. For example, the combination of \(\text{ }\) and \(\text{ }\) does not fill a container and so \(\text{ }\) remains. This is analogous to \(1 + 1 = 2\) under addition modulo 3. Furthermore, the perceptual information communicated by the symbols themselves can act as reminders of the structural rules. In this case, the storyline and symbols may facilitate learning.

The conditions with no relevant concreteness were presented to the participants as a symbolic language in which three types of symbols combine to yield a resulting symbol (see Table 1). Combinations are expressed as written statements. In the Generic condition, the symbols were black; and in the Irrelevant Concreteness condition, the symbols were colorful and patterned.

Training and testing in all conditions were isomorphic and presented via computer. Training consisted of an introduction and explicit presentation of the rules through examples. Questions with feedback and examples with complex combinations were given.

After training, the participants were given a 24-question multiple-choice test designed to measure the ability to apply the learned rules to novel problems. Many questions required the application of multiple rules. The following are examples of test questions in the Generic condition.

1. What can go in the blanks to make a correct statement?
   \(____, \Diamond, ____\), \(\otimes \rightarrow \bullet ?\)

2. Find the resulting symbol:
   \(\Diamond, \bullet, \bullet, \bullet \rightarrow ____\).

Participants in the Relevant Concrete condition saw the analogues of these questions.

Phase 2 of the experiment consisted of 26 test trials. On each trial, participants were presented with a set of three expressions. They were told that each set is from a new system and asked whether the new system follows the same type of rules as the system they had previously learned. Four types of trials were used. Table 2 shows examples of each type of trial, as expressed for the Relevant Concrete condition. For the Generic and Irrelevant Concreteness conditions, the analogous statements were expressed with the generic black symbols or their colorful counterparts respectively. Six trials involved the same elements as the learning phase and the same relational structure (E+/R+). Six trials involved the familiar elements, but different relational structure (E+/R-). Six trials involved novel elements and the familiar relational structure (E-/R+). Another six trials involved both novel elements and novel relations (E-/R-). In addition two questions were posed in which familiar elements were cross-mapped to play different roles in the same relational structure.

For example, in the statement \(\text{ }\) is playing the role that \(\text{ }\) held in the learning domain.

**Procedure**
All training and testing was presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.

**Results and Discussion**
Three participants (one Perceptually Rich, two Generic) were removed from the analysis for failing to learn; their
learning test scores were less than 11 and no different than chance score of 9. In all conditions, participants successfully learned the concept. The mean test scores of 19.6 ($SD = 4.1$) for the Relevant Concreteness group, 17.3 ($SD = 4.0$) for the Perceptually Rich group, and 18.2 ($SD = 2.8$) for the Generic group were above a chance score of 9, one sample $t$-tests, $t > 8.54$, $p < .0005$. The differences between groups was not significant, one-way ANOVA, $F(2, 47) = 1.67$, $p = .199$, $\eta^2_p = .066$. While there were no differences in learning across condition, there were considerable differences in ability to discriminate familiar and novel relational structure in phase 2 depending on the presence of familiar or novel elements. To measure discriminability in the context of familiar elements, the number of “yes – same structure” responses for E+/R+ trials minus the number of erroneous “yes – same structure” responses for E+/R- trials was calculated (see Figure 1) and submitted to an ANCOVA with condition as a covariate. The results found no difference in discriminability across condition, $F(2, 42) = .289$, $p > .75$, $\eta^2_p = .01$; and a significant effect of learning $F(1, 46) = 11.67$, $p < .0005$, $\eta^2_p = .20$. This discriminability accuracy supports the proposition that successful learning results in representations that contain both elements and relations.

However, when trials involved novel elements, there were striking differences in discriminability, where participants in the Relevantly Concrete condition were unable to recognize the familiar structure while participants in both the Generic and the Perceptually Rich conditions were able. Scores of the number of “yes – same structure” responses for E-/R+ trials minus the number of erroneous “yes – same structure” responses for E-/R- trials were calculated (see Figure 1) and submitted to an ANCOVA with condition as a between-subjects factor and learning test score as a covariate. Results revealed a significant effect of condition, $F(2, 46) = 12.22$, $p < .0005$, $\eta^2_p = .35$, with a lesser effect of learning $F(1, 46) = 7.62$, $p < .009$, $\eta^2_p = .14$. Therefore, in the context of novel elements, the ability to recognize learned structure does not depend as much on how well the initial instantiation was learned, but rather on what type of initial instantiation was learned.

Responses to cross-mapped trials provide additional evidence that when learning a relevantly concrete instantiation, structure is tightly bound to the elements as presented during learning. These trials presented the learned structure, but switched the roles of familiar elements. None of the participants in the Relevant Concreteness condition were able to recognize structure when elements were crossed mapped, while 25% of participants in the Generic condition and 29% in the Perceptually Rich condition correctly recognized familiar relational structure with cross mapped elements. Scores for these questions were submitted to an ANCOVA with condition as a factor and learning test score as a covariate. There was a main effect of condition, $F(2, 46) = 3.68$, $p < .04$, $\eta^2_p = .14$, and no significant effect of learning scores, $F(1, 46) = 1.49$, $p > .22$, $\eta^2_p = .03$.

Therefore, the type of instantiation from which the concept was learned significantly affected the learner’s ability to recognize the same relational structure in the context of novel elements. Learning an instantiation that communicated no relevant concreteness, whether generic or perceptually rich, allowed participants to recognize relational structure elsewhere, while learning a relevantly concrete instantiation did not. For relevant concreteness, relational structure is embedded in the learning context creating an inability to recognize structure in an isomorph that results in an obstacle for successful transfer. What might help the learner overcome this obstacle? One possibility is explicitly stating the correspondence between elements of the learning and transfer domains. Giving the alignment of elements should help the learner recognize common structure in the transfer domain and subsequently align the two domains and successfully transfer. The purpose of Experiment 2 was to investigate whether giving participants the correspondence between elements would facilitate transfer.

![Figure 1](Figure 1: Discriminability: “Same Structure” Responses on R+ trials minus “Same Structure” Responses on R- trials presented as a percentage. Error bars represent standard error of the mean.)
Experiment 2

Method

Participants Eighty-three undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students were randomly assigned to one of five conditions that specified the domain they learned in the first phase of the experiment and whether or not they were given the correspondence of elements.

Materials and Design Material were similar to those used in the previously discussed transfer study. The experiment included two phases: (1) training and testing in a learning domain and (2) testing of the transfer domain. Two types of learning instantiations were considered: Relevant Concreteness and Generic. Training and testing of these domains was identical to that used in Experiment 1. The same transfer domain was used for all conditions and was isomorphic to the Relevant Concreteness and Generic domains. The experiment has a 2 (Learning Domain: Relevantly Concrete vs. Generic) by 2 (Alignment: Mapping vs. No Mapping) between-subjects design. Therefore, there were four conditions, Relevant Concreteness Map, Relevant Concreteness No Map, Generic Map, Generic No Map, and a Baseline. A fifth learning domain was constructed as a baseline for spontaneous performance in the transfer domain. This domain involved unrelated arithmetic and matching questions, thus training in the learning domain should not facilitate performance in the transfer domain in this condition. For Generic and Relevant Concreteness, half of the subjects were given the correspondence (or mapping) of elements across domains and half of the subjects were not.

In the four experimental conditions, the learning domain tests were the same 24-question tests used in Experiment 1. The transfer domain test was isomorphic to these tests. Training in the learning domain across the four conditions was isomorphic and was identical to that of Experiment 1. The transfer domain was described as a children’s game involving three objects: . Children sequentially point to objects and a child who is “the winner” points to a final object. The correct final object is specified by the rules of the game (rules of a mathematical group). Participants were not explicitly taught these rules. Instead they were told that the game rules were like the rules of the system they just learned and they need to figure them out by using their prior knowledge (i.e. transfer). In addition to this suggestion to transfer, participants in the Map conditions were also given the correspondence between elements of the learning and transfer domains. For example, in the Generic Map condition, they were told that is like , is like , and is like . Participants in the Relevant Concreteness Map condition were shown the analogous correspondences. In the Generic No Map and Relevant Concreteness No Map, the correspondences were not given. Then participants were asked to study a series of examples from which the rules could be deduced, afterward the multiple-choice test was given. Questions were presented individually on the computer screen along with four key examples at the bottom of the screen. The same four examples were shown with all test questions. Following the multiple-choice questions, participants in the four experimental conditions were asked to indicate a level of similarity between the learning and transfer domains.

Procedure As in experiments 1, training and testing were presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.

Results and Discussion

Three participants (one Relevant Concreteness Map, one No Relevant Concreteness Map, one No Relevant Concreteness No Map) were eliminated from the data because their learning or transfer scores were more than two standard deviations below the mean of their respective conditions. Participants in all conditions successfully learned, mean learning test score = 20.4 (SD = 2.35) for Relevant Concreteness and mean = 19.0 (SD = 4.32) for Generic. Mean scores were significantly above chance score of 9, one sample t-tests, t (31) > 13.08, ps < .001. There was no significant difference in learning between Concrete and Generic conditions, independent samples t-test, t (62) = 1.65, p > .10.

There were clear differences in transfer across conditions (see Figure 2). Transfer scores were submitted to an ANCOVA with learning domain condition (Generic or Relevant Concreteness) and alignment (Map or No Map) as factors and learning score as a covariate. The results revealed significant effects of both condition, $F(1, 57) = 10.12, p < .003, \eta^2 = .15$; and alignment, $F(1, 57) = 9.04, p < .005, \eta^2 = .14$, and a significant interaction between the two $F(1, 57) = 15.59, p < .0005, \eta^2 = .22$. Learning score was also a contributing factor, $F(1, 57) = 30.32, p < .0005, \eta^2 = .35$. In other words, participants who learned the relevantly concrete instantiation successfully transferred only when given the correspondence of elements. However, giving the correspondence offered no additional benefit for those who learned the generic instantiation; they were able to transfer with or without being given the correspondence of elements.

In addition, similarity ratings followed the same pattern as transfer. Participants in the Generic conditions and the Relevant Concreteness Map condition rated the similarity of the learning and transfer domain as highly similar on a scale from 1 (completely dissimilar) to 5 (structurally identical), mean = 4.6 (SD = .71) with no differences between conditions, ANOVA $F(3, 60) = 9.37$, post-hoc Tukey ps > .303. A mean rating of 3.2 (SD = 1.4) for the Relevant
Concreteness group was lower than that of the other conditions, post-hoc Tukey $p < .02$.

In sum, giving participants in the Relevant Concreteness condition the correspondence of elements allowed them to transfer as well as participants in the Generic conditions.

**General Discussion**

Previous research has demonstrated that relevantly concrete instantiations of an abstract concept can promote quick learning, but dramatically hinder transfer. At the same time generic instantiations can be learned as well and in addition can facilitate transfer. The current research elucidates what underlies this differential transfer ability.

Successful transfer requires that the relational structure of an isomorphic domain is recognized to the extent that the learned and transfer domains can be aligned and the analogy subsequently implemented. Experiment 1 demonstrated that learning with relevant concreteness hinders the ability to recognize relational structure in a novel isomorph, while learning with generic or irrelevantly concrete instantiations does not. The inability to recognize structure creates an obstacle to transfer. However, Experiment 2 demonstrated that if given the correspondence of elements across domains, learners can overcome this obstacle and successfully transfer. This suggests that aligning the elements helps students disembed the relational structure from the learning context.

Successful transfer depends on more than simply the similarity between domains. In fact, perceived similarity may be a product of alignment and transfer. Students who learned the relevantly concrete instantiation and were given the correspondence of elements not only successfully transferred conceptual knowledge, but also rated the learning and transfer domains as highly similar, while students who were not given the correspondence failed to transfer well and did not rate the domains as highly similar.

The appeal of relevant concreteness in teaching is certainly understandable. Presenting an abstract concept to students through a familiar instantiation can make learning quicker and easier than teaching with bland, generic symbols. However, for abstract concepts, the goal of learning is not simply knowledge of one instantiation; it is the ability to recognize novel instantiations. The power of abstract concepts lies in their ability to provide insight and understanding of the new through transfer. Giving the correspondence of elements can promote transfer, but certainly this correspondence is not always available. This research provides additional support for the argument that the benefits of relevant concreteness for learning come at the cost of transfer.

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**References**


