What are fallacies good for? Representational speed-up in propositional reasoning

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Abstract

Two experiments examine speed-up in argument pairs of various propositional forms. In the first experiment, participants were presented with pairs of conditional arguments. Some of these pairs had a form of a valid Modus Ponens (MP) inference, whereas other pairs had a form of a fallacy of Affirming the Consequent (AC). In both argument pairs, presentation of the prime led to a significant speed-up in the probe argument. In the second experiment, in addition to AC-AC and MP-MP pairs, AC-MP and MP-AC pairs were also included. Results indicated that AC primes led to a speed-up of MP probes, and MP primes led to a speed-up in AC probes. The results are discussed in relation to theories of propositional reasoning.

Introduction

The ability to reason deriving conclusions from available information is an integral aspect of human cognition. A large component of this ability is propositional reasoning, or reasoning with logical connectives AND, OR, IF…THEN, and NOT. There are two major theoretical approaches to propositional reasoning, the syntactic approach and the semantics approach. According to the former, reasoners extract the syntactic form of the argument and apply certain formal rules of inference, or inferential schemata, to the extracted form (Braine & O'Brien, 1991; Rips, 1994). For example, reasoners easily conclude that B is the case, using the modus ponens (MP) schema, when presented with the following premises:

$A \rightarrow B$ (If A then B)
A.

The syntactic approach thus hinges on assumptions that reasoners (a) veridically represent information in the premises and (b) apply inferential schemata to these representations. However, both assumptions are not uncontroversial. For example, according to the semantic approach, the untrained mind is not equipped with formal rules of inference. Furthermore, reasoning, to a large extent, is a function of representations of information in the premises. In turn, these representations are not veridical but are often incomplete or defective (Johnson-Laird & Byrne, 1991; Evans & Over, 1996; Sloutsky & Goladvarg, 1999; Sloutsky, Rader, & Morris, 1998).

One of the semantic theories of propositional reasoning, the Mental Model Theory (Johnson-Laird & Byrne, 1991) suggests that inferences, such as considered above, occur in the following manner. First, the reasoner construes the initial representation of the premises:

First premise	Second premise
$A \quad B$	A
…	

The first line in the leftmost column makes explicit the possibility in which both A and B co-occur, and the second model (ellipses) corresponds to those possibilities in which the antecedent of the conditional is false. The theory accordingly assumes that individuals do not normally make these possibilities explicit (Johnson-Laird & Byrne, 1991). The line in the rightmost column represents the second premise. Combining the two models together leads to the inference that B.

There is a plethora of empirical studies contrasting predictions stemming from the two approaches. One major result of these comparisons is that the Mental Model Theory is capable of accounting for a variety of systematic errors observed in reasoning (Johnson-Laird & Savary, 1999; Newsome & Johnson-Laird, 1996; Sloutsky & Johnson-Laird, 1999; Yang & Johnson-Laird, in press; see also Johnson-Laird, 1999; Johnson-Laird & Byrne, 1991, for reviews). One of these errors accounted for by the Mental Model Theory is the fallacy of Affirming the Consequent (AC). AC has the following form:

$A \rightarrow B$
B.
Therefore A.

The inference is a fallacy because there is nothing in the argument suggesting that B could not occur without A. The mental model explanation of this fallacy is that initial representations of MP arguments and AC arguments are identical. As a result, people tend to draw conclusions, both when presented with valid MP arguments and invalid AC arguments.

This paper offers a further examination this issue. If inferences in Modus Ponens arguments occur due to the MP schema, as specified by the syntactic approach, then the use of the schema should lead to a speed-up in subsequent applications of the schema (see Smith, Langston, & Nisbett, 1992). At the same time, inferring conclusions from AC arguments should not lead to a speed-up in MP arguments, because there is no schema for AC. However, if people reason from mental representations, as according to the semantic approach, then arguments that have identical mental representations should speed up each other. We therefore, predicted that (1) AC arguments should speed-up AC arguments and (2) MP arguments should speed-up MP arguments. We further predicted that (3) AC arguments should speed-up MP arguments and (4) MP arguments

should speed-up AC arguments. The first two hypotheses were tested in Experiment 1, whereas the last two were tested in Experiment 2.

There was also a critical point added to Experiment 1. According to the syntactic theory of mental logic (Braine & O'Brien, 1998), conjunctive arguments (CONJ) of the form *A & B* could be simplified using conjunction elimination schema of the form:

A & B

Therefore *A*.

On the other hand, the semantic theory of mental models suggests that conjunctions have similar (although not identical) representations as conditionals. Therefore, an important question is whether or not conjunctive arguments can also be speeded up by subsequent use. There is evidence that during text comprehension, conjunctions do not result in automatic, on-line inferences, whereas conditionals do (Gernsbacher, 1997; Millis, Golding, & Barker, 1995). The importance of this question is that, if hypotheses are confirmed, the examination of conjunctive arguments will allow us to assess the generality of findings: whether all forms that have similar representations prime each other, or if priming is limited to If…Then forms only.

Table 1: Sample stimuli by argument type and prime type.

Prime Type	Argument Type		
	MP	CONJ	AC
Related Prime (select a conclusion)	If there is an Ace then there is a Jack. There is an Ace. • No conclusion follows • There is a Jack • There is no Jack • There is a Two	There is an Ace and there is a Jack. There is an Ace. • No conclusion follows • There is a Jack • There is no Jack • There is a Two	If there is an Ace then there is a Jack. There is a Jack. • No conclusion follows • There is an Ace • There is no Ace • There is a Two
Unrelated Prime (select a conclusion)	There is a Three or there is a Seven, but not both. There is a Three • No conclusion follows • There is a Seven • There is no Seven • There is a Jack	There is a Three or there is a Seven, but not both. There is a Three • No conclusion follows • There is a Seven • There is no Seven • There is a Jack	There is a Three or there is a Seven, but not both. There is a Three • No conclusion follows • There is a Seven • There is no Seven • There is a Jack
Probe (answer Yes or No)	If there is a Queen then there is a Six. There is a Queen. • There is a Six	If there is a Queen then there is a Six. There is a Six. • There is a Queen	There is a Queen and there is a Six. There is a Queen. • There is a Six

Experiment 1

The first goal of this experiment was to test hypotheses 1 and 2, suggesting that there is AC-AC and MP-MP priming. The second goal was to examine whether or not there is CONJ-CONJ priming.

Method

Participants A total of 86 participants from Ohio State University took part in the experiment for an introductory psychology course credit. These participants represented three groups, with each group receiving a particular argument type. There were 31 participants in the Modus Ponens (MP) group, 29 participants in the Conjunction group (CONJ) and 26 participants in the Affirmation of Consequent (AC) group. All participants were fluent English speakers.

Materials In each of the three groups, stimuli considered of 60 critical items and 120 filler items. Critical items consisted of 30 prime-probe pairs. Half of primes had the same argument form as probes, whereas another half of the primes had a different argument form. Examples of stimuli for each of the group are presented in Table 1. Filler items consisted of primes and probes that were similar to those in the Table, except that they had a different logical form. Primes had a form of inclusive OR (e.g., There is a Joker or an Ace, or both), whereas probes had a form of exclusive or (There is a Joker or an Ace, but not both). Participants were randomly assigned to one of the three groups.

Design and Procedure The experiment had a 3 Argument Type (AC, MP, CONJ) by 2 Prime Type (Related, Unrelated) mixed design with Prime Type as a repeated measure. Stimuli were presented on a PC screen and controlled by Superlab Pro for Windows (Cedrus Corporation, 1997). Participants were tested individually. Participants were told that they would read arguments on the computer. They were further told that sometimes they would need to select a conclusion from a set of conclusions, and sometimes to determine whether or not a conclusion follows logically from the premises by answering either Yes (follows) or No (does not follow). Participants were asked to respond as quickly and accurately as possible. Then they were given examples of conclusions that do and do not follow logically from premises accompanied with explanations. Finally, they were presented with four practice trials, two of which included selecting a conclusion from a

list, and another two included a Yes/No response. These practice trials were accompanied by feedback, such that participants were told whether or not their inference was warranted and why. After finishing the practice trials, participants were presented with experimental trials. Each participant received 30 experimental items (60 arguments) and 60 filler items (120 arguments) with a total of 180 arguments. Participants read each argument in self-paced fashion. Once the argument that served as a prime was answered, a probe argument appeared on the screen. Argument pairs were separated by 300 ms interstimuli intervals. The experiment took approximately 40 minutes.

Results and Discussion

In all reported analyses, degrees of freedom are based on subjects * item variability. Accuracy by argument and prime type are presented in Table 2 and response times are presented in Figure 1. For AC arguments accuracy was below chance $t(389) < -7$, $p < .0001$, two-tailed. For MP and CONJ arguments, accuracy was above chance ts (389) > 7, $p < .0001$, two-tailed. Because comparisons of response times across between-subject conditions could be misleading, we perform only comparisons across within-subject conditions. For CONJ condition, there were no significant differences in responses to the probe questions between related and unrelated primes. In fact, unrelated primes resulted in slightly (but not significantly) faster responses than related primes. At the same time, in the AC condition, $t(353) = 3.5$, $p < .0001$ and MP condition, $t(425) = 3.8$, $p < .0001$, related primes resulted in a significant speed-up of responses to the probe questions. These data indicate that while there was no speed-up in the CONJ condition, both AC and MP arguments were speeded-up by more 500 ms when preceded by a related prime. Having established that responses to both AC and MP arguments could be speeded up by a related prime, we deemed it necessary to answer another question: what constitutes a related prime? According to syntactic theories of reasoning, related prime would the one that is based on the same inference rule. However, syntactic theories do not posit a rule for AC inferences. Recall that we hypothesized that the prime is related whenever it has a identical mental representation with the probe. For example, according to the Mental Model Theory, AC and MP have identical mental representations. In this case, both an AC prime should speed up both AC and MP probes, and MP prime should speed up AC and MP probes. This prediction was tested in Experiment 2. Note, that there was not priming in CONJ-CONJ pairs; this issue will be addressed in the General Discussion section.

Table 2: Percent of accurate responses by argument type and prime type.

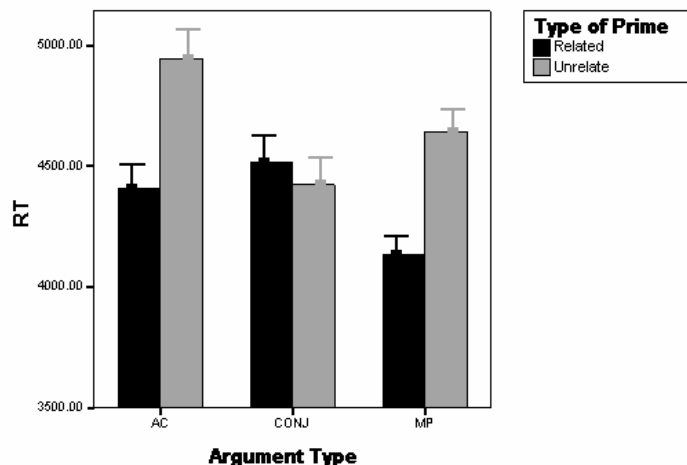| | Prime Type | |
Argument Type	Related	Unrelated
AC	32.80	31.28
MP	95.90	92.20
CONJ	68.50	68.00

Figure 1: Response times by argument type and prime type. Error bars represent standard errors of the mean

Experiment 2

Experiment 2 differs from Experiment 1 in two respects. First, in Experiment 2, both types of argument and types of prime varied within subjects. Each participant received two types of arguments (AC and MP) and three types of prime (AC, MP, and XOR, which was considered unrelated). Types of argument were fully crossed with types of prime. Second, because there was no CONJ-CONJ speed-up, conjunctive arguments were eliminated.

Method

Participants A total of 26 participants from Ohio State University took part in the experiment for an introductory psychology course credit. All participants were fluent English speakers.

Materials & Procedure The experiment had a 3 Argument Type (AC, MP) by 3 Prime Type (AC, MP, XOR) within-subject design. The experimental procedure was identical to that of the first experiment, except that the total number of items in the current experiment was 240. The experiment took approximately 55 minutes.

Results and Discussion

As in the previous experiment, degrees of freedom are based on subjects * item variability. Accuracy rates by argument and prime type are presented in Table 3. These rates were subjected to one-sample t-tests. The analyses indicate that for AC arguments accuracy was below chance, $t < -7$, $p < .0001$, whereas for MP argument accuracy was above chance ts > 7, ps $< .0001$. Figure 2 presents relative speed-up for prime-probe pairs; estimates for relative speed-up were derived as $RT_{unrelated prime} - RT_{related prime}$. Speed-up rates presented in Figure 2 were subjected to one-sample t-tests. Recall that it was predicted that for both types of arguments, MP and AC primes should lead to a speed-up above XOR primes that were considered unrelated. As depicted on Figure 2, all "related" primes resulted in a speed-up, all ts > 2, ps $< .05$. Speed-up effects presented in

Figure 2 were also subjected to a repeated measures ANOVA. Although overall effects of the prime-probe pair were significant, $F(3, 320) = 3.4$, $p < .02$, due to large variance, neither of the individual contrasts reached significance. These findings are consistent with predictions that speed-up occurs due to a common mental representation.

Table 3: Percent of accurate responses by argument type and prime type

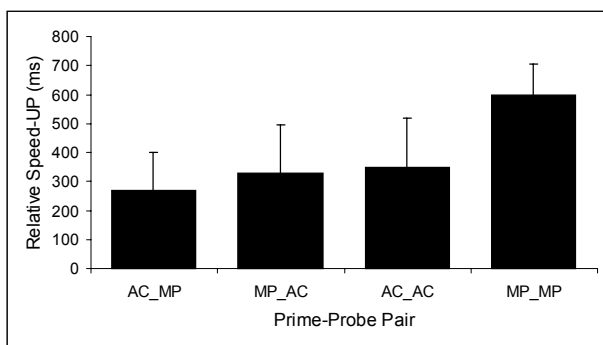| Prime Type	Argument Type	
	AC	MP
AC	39.74	95.64
MP	39.75	98.21
XOR	39.49	97.69

Figure 2: Relative speed-up by prime-probe pairs. Error bars represent standard errors of the mean.

General Discussion

The results of the two reported experiments indicate that both MP and AC arguments speed-up each other. These findings support predictions that priming could be due to a common mental representation rather than due to a common syntactic rule. Indeed, what do AC and MP arguments have in common? First, they have the common linguistic form "If…then," and second, they have a similar mental representation. It seems more likely that the observed speed-up is due to the similarity of mental representation rather than due to the similarity of linguistic form. This suggestion is based on indirect evidence (e.g., Lea, 1995; Rader & Sloutsky, 2000) that when inference in the priming argument is blocked (e.g., *If there is an Ace then there is a King. I really need an Ace.*), priming does not occur.

It also seems important that there was no speed-up in CONJ-CONJ pairs, even though these arguments have identical linguistic form and identical representation. One important difference of Conjunctive arguments is that, unlike conditionals, conjunctive arguments do not lead to an automatic, on-line inference (Gernsbacher, 1997; Millis, Golding, & Barker, 1995). Taken together, these findings suggest that the identical mental representation is not sufficient for priming: only those forms exhibit speed-up that (a) have the identical mental representation and (b) lead to an automatic inference. Furthermore, priming effects occur in both valid (MP) and invalid (AC) conditional

arguments.

Taken together, results of the two experiments suggest that the presence of MP arguments may lead participants to consider AC-AC pairs as sets of invalid arguments (after all the participants are college undergraduates who may be familiar with basic principles of logic). This consideration did not lead to an increase in accuracy, but could have slowed down their responses. This consideration will be tested in future research. In particular, AC-AC and MP-MP arguments could be presented in separate blocks within subjects, in which case there should be priming effect when AC-AC pairs are presented prior to MP-MP pairs. Another prediction is that strengthening of the associative link between the antecedent and the consequent in the AC argument (*If it flies then it is a bird. It is a bird.*) should result in an increase in the speed-up in AC-AC pairs. This is because when the antecedent and the consequent are highly associated, people are less likely to notice that the inference is invalid (Markovits, 1993; Markovits, Fleury, Quinn, & Venet, 1998). Alternatively, weakening of the associative link between the antecedent and the consequent in the AC argument (*If you throw a watermelon to the window, the window breaks. The window is broken.*) should result in an a decrease in the speed-up in AC-AC pairs.

While these possibilities will be tested in further experiments, results of the current experiments seem to indicate that independently of the validity, MP and AC conditional arguments tend to speed-up each other. This finding seems to support the idea that those arguments that share mental representation and lead to an automatic, on-line inference are likely to get primed by each other independently of their validity.

Acknowledgements

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