Question 1. [4 points] Show that the following implication is a tautology without using a truth table.

$$[(P \lor Q) \land (P \rightarrow R) \land (Q \rightarrow R)] \rightarrow R.$$ 

Solution.
Question 2. [4 points] Goldbach’s conjecture in Number Theory states that: “Every even integer greater than 2 can be expressed as the sum of two primes”. No one has yet proved or disproved this conjecture.

(a) Symbolize Goldbach’s conjecture using quantifiers.

(b) State the negation of Goldbach’s conjecture (do not write “It is not true that …”) and symbolize it using quantifiers.

Solution.
Question 3. [4 points] Use the Principle of Mathematical Induction to verify that, for \( n \) any positive integer, \( 2^{2^n} - 1 \) is divisible by 3.

Solution.
Question 4. [4 points] Construct a truth table for $P \leftrightarrow (\neg Q \lor R)$ and write its corresponding disjunctive normal form (DNF) expression.

Solution.
Question 5. [4 points] Write the the negation, converse, inverse, and contrapositive of the conditional statement:

“If Baha is my advisor, then I can finish my dissertation and I can enjoy my life in Columbus.”

Solution.

Negation:

Converse:

Inverse:

Contrapositive: