CSE 2321: Homework 8 Solutions

1. Runtime of:

```c
find_max1(A, n)
/* A is an array of positive integers, n is the length of A */
max = 0;
for i ← 1 to n do
  if A[i] > max then
    max = A[i];
end
return max;
```

<table>
<thead>
<tr>
<th>Line</th>
<th>One-time Cost</th>
<th>Number of running times</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>c₂</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>c₃</td>
<td>n + 1</td>
</tr>
<tr>
<td>4</td>
<td>c₄</td>
<td>n</td>
</tr>
<tr>
<td>5</td>
<td>c₅</td>
<td>n (worst case)</td>
</tr>
<tr>
<td>8</td>
<td>c₈</td>
<td>1</td>
</tr>
</tbody>
</table>

Total running time is: $T(n) = c_2 + c_3(n + 1) + c_4n + c_5n + c_8$

2. Runtime of:

```c
x = 0;
for i = n² to n² + 5 do
  for j = 4 to n do
    x = x + i - j;
  end
end
return x
```

$$T(n) = \sum_{i=n^2}^{n^2+5} \sum_{j=4}^{n} c$$

$$= \sum_{i=n^2}^{n^2+5} (n - 4 + 1)c$$

$$= (n^2 + 5 - n^2 + 1)(n - 4 + 1)c$$

$$= 6c(n - 3)$$
3. Runtime of:

```plaintext
1  x = 0;
2  for i = 1 to n do
3      for j = 1 to 3i^3 do
4          x = x + i - j;
5      end
6  end
7  return x
```

\[
T(n) = \sum_{i=1}^{n} \sum_{j=1}^{3i^3} c
\]

\[
= \sum_{i=1}^{n} 3i^3 c
\]

\[
= 3c \sum_{i=1}^{n} i^3
\]

\[
\leq 3c \sum_{i=1}^{n} n^3
\]

\[
\leq 3cn^4 \rightarrow T(n) \in \Theta(n^4)
\]

4. Runtime of algorithm represented by the following summation:

\[
T(n) = \sum_{i=n}^{4n^3} \sum_{j=i}^{8n^3} c
\]

Upper Bound:

\[
T(n) = \sum_{i=n}^{4n^3} \sum_{j=i}^{8n^3} c
\]

\[
= \sum_{i=n}^{4n^3} (8n^3 - i)c
\]

\[
\leq \sum_{i=n}^{4n^3} (8n^3)c
\]

\[
\leq (4n^3 - n + 1)(8n^3)c
\]

\[
\leq (32n^6 - 8n^4 + 8n^3)c \rightarrow T(n) \in \Theta(n^6)
\]
Lower Bound:

\[ T(n) = \sum_{i=0}^{4n^3} \sum_{j=0}^{8n^3} c \]

\[ = \sum_{i=0}^{4n^3} (8n^3 - i)c \]

\[ \geq \sum_{i=0}^{4n^3} (8n^3 - 4n^3)c \]

\[ \geq (4n^3 - n + 1)(4n^3)c \]

\[ \geq (16n^6 - 4n^4 + 4n^3)c \rightarrow T(n) \in \Omega(n^6) \]

\[ \rightarrow T(n) \in \Theta(n^6) \]