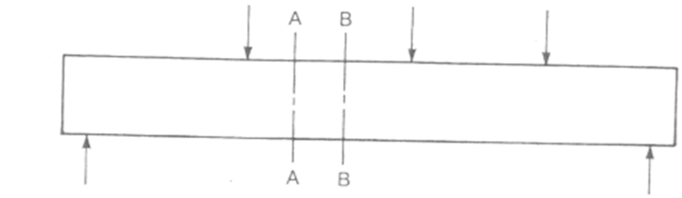
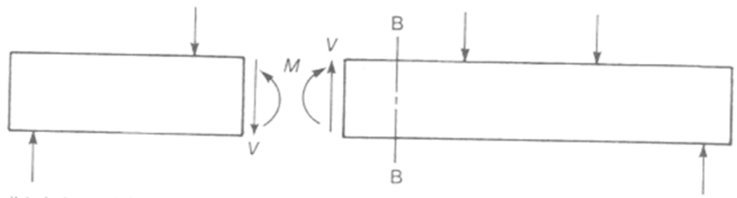
SHEAR DESIGN

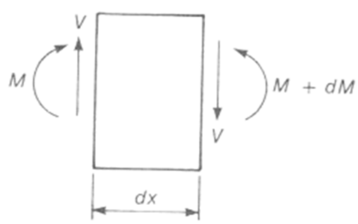
Internal Forces and Stresses due to Bending and Shear



(a) Beam



(b) Internal forces on section A–A.

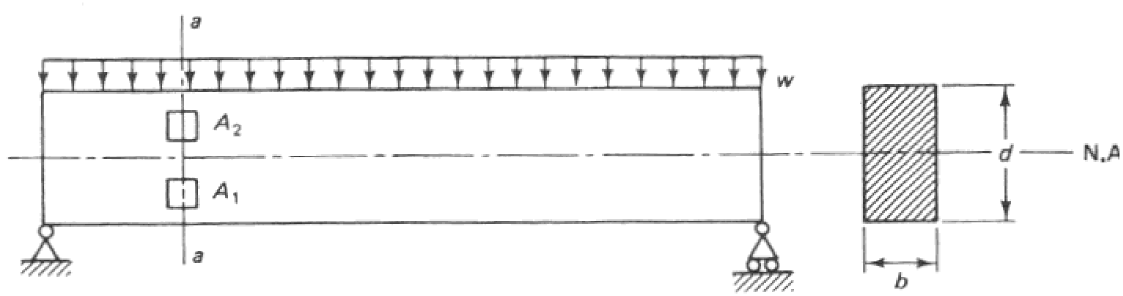


(c) Internal forces on portion between sections A–A and B–B.

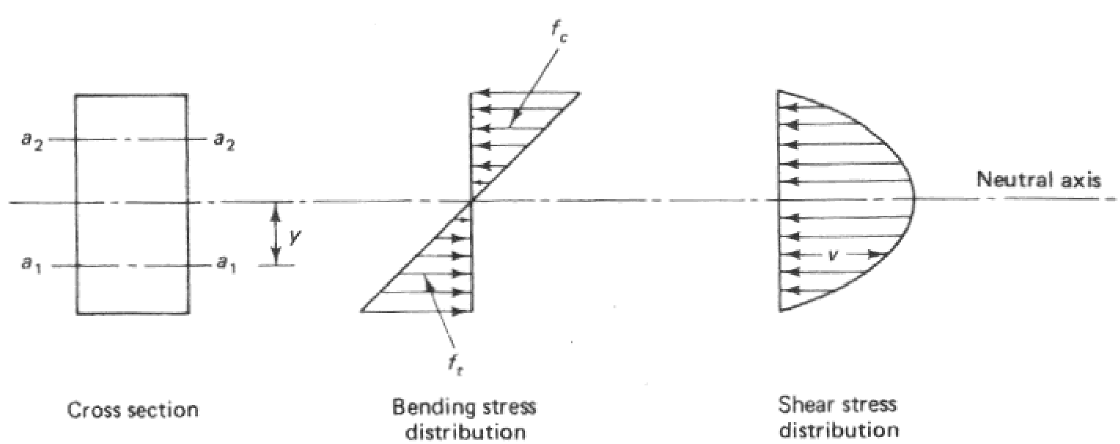
$$Vdx = dM$$

$$V = \frac{dM}{dx}$$

Slope of moment = shear



(a)



(b)

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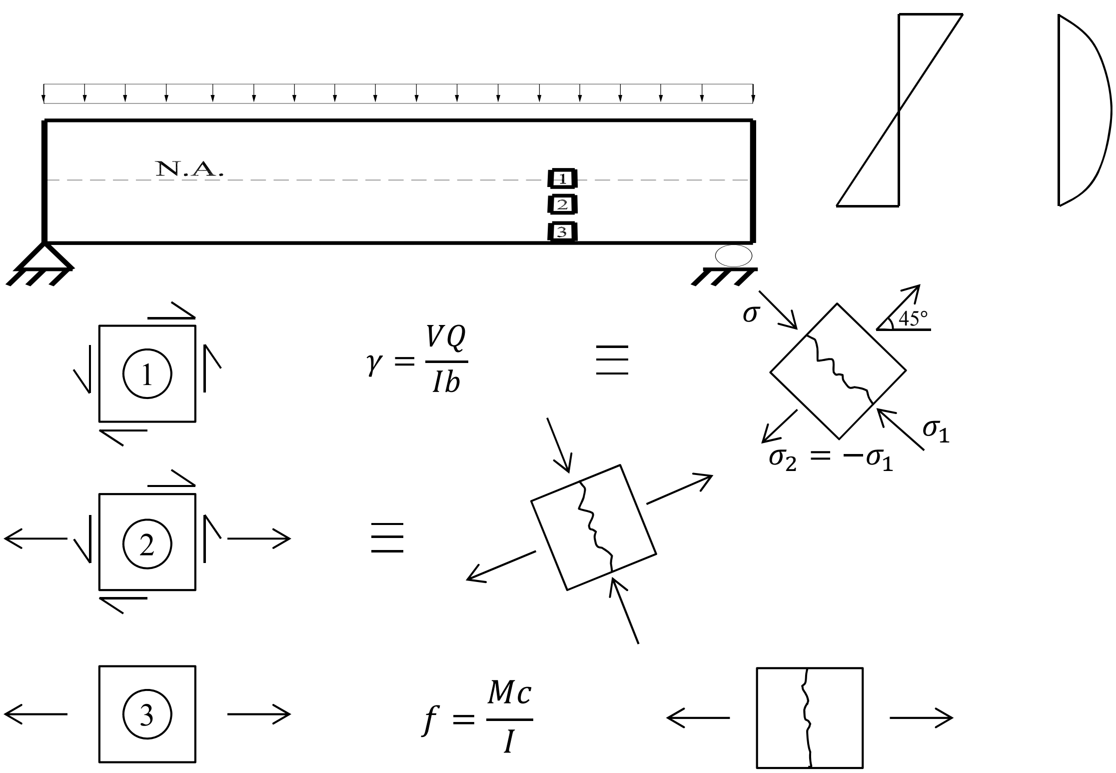
Shear in Reinforced Concrete Beams

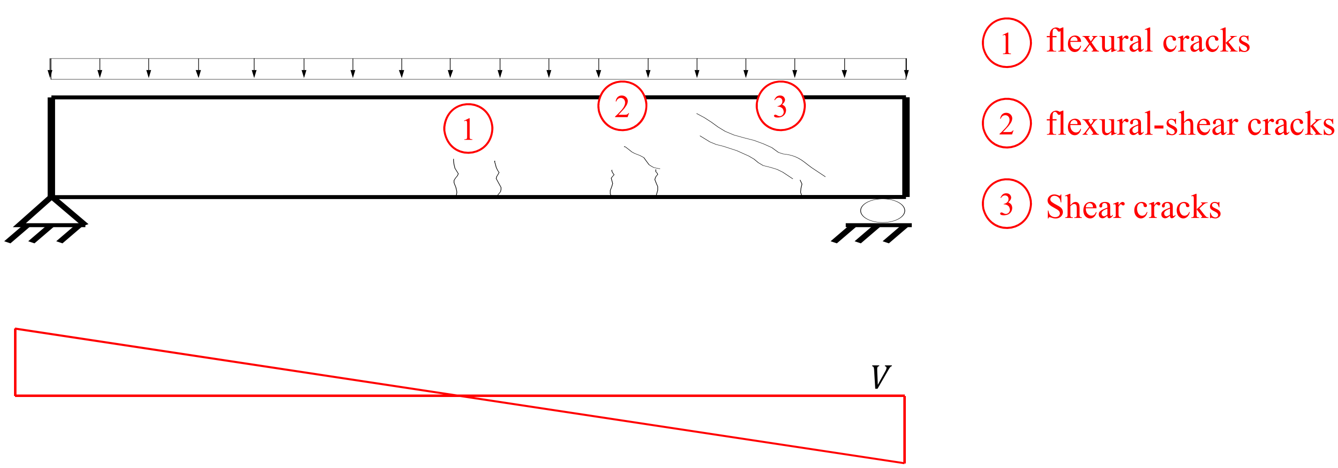
Generally, we try to design beams, so they will not fail except in extreme loading. If they fail, they should fail in flexure. Shear failure takes place suddenly (fairly *brittle)* and without warning.
Check shear and add shear reinforcement to ensure flexure failure, which is preceded by cracking and large deflections.

Shear in Reinforced Concrete Beams without Stirrups

Consider beam to be homogeneous, isotropic, and elastic to gain some insight.



Crack progression in rectangular beams



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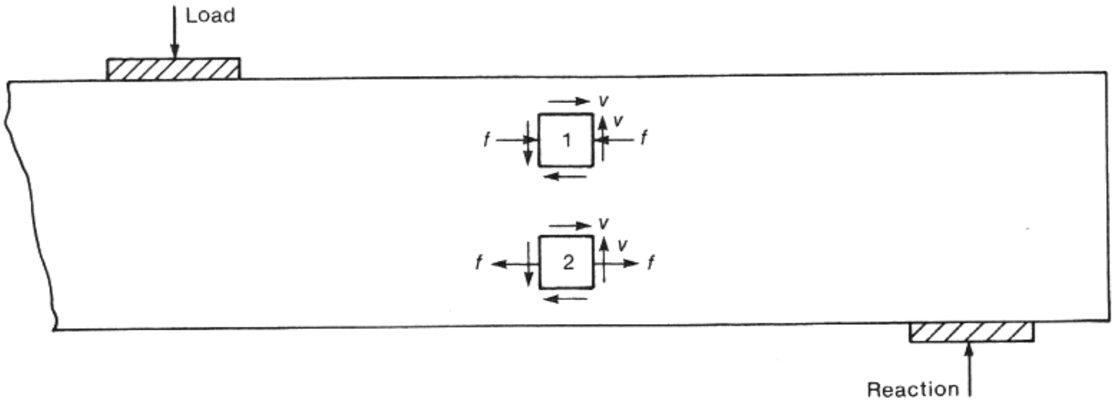
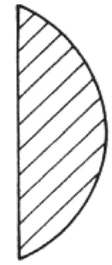
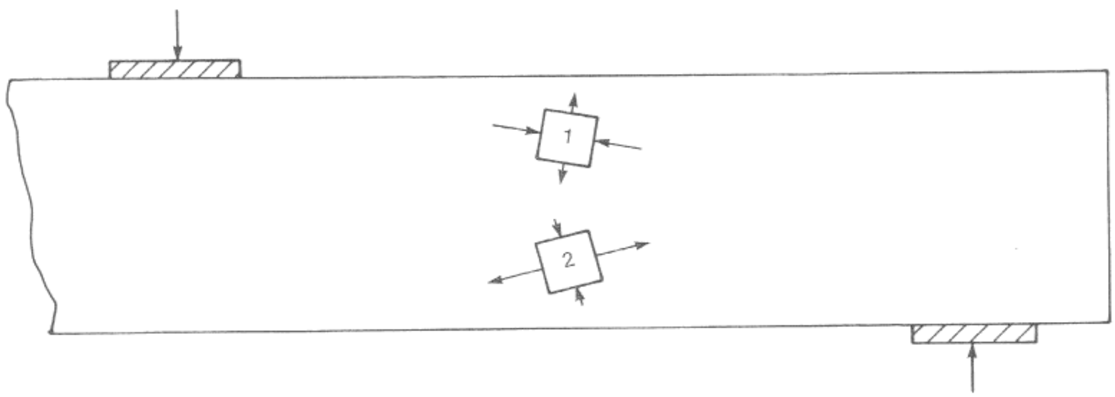


(a) Flexural and shear stresses acting on elements in the shear span.

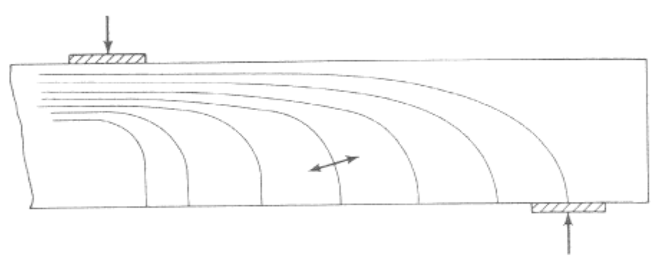


(b) Distribution of shear stresses.

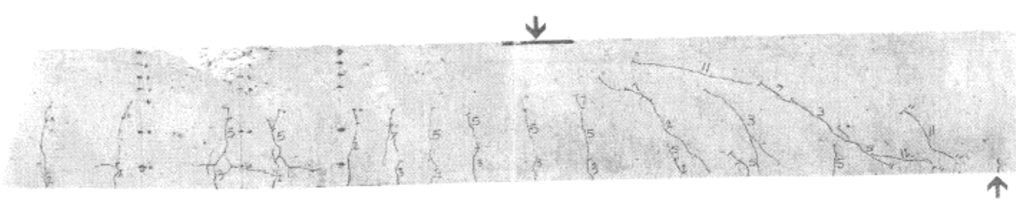


(c) Principal stresses on elements in shear span.

Normal, shear, and principal stresses in homogeneous uncracked beam.



(a) Principal compressive stress trajectories in an uncracked beam.



(b) Photograph of half of a cracked reinforced concrete beam.

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Principal compressive stress trajectories and inclined cracks. (Photograph courtesy of J. G. MacGregor.)

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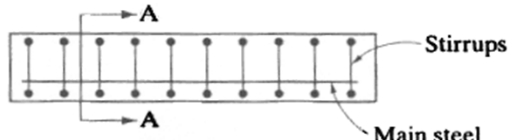
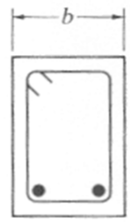
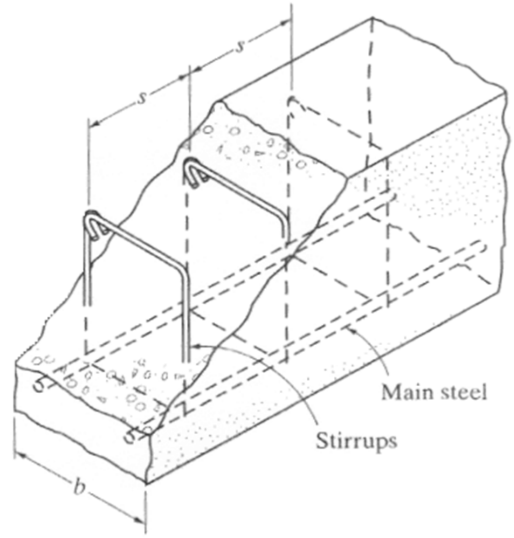
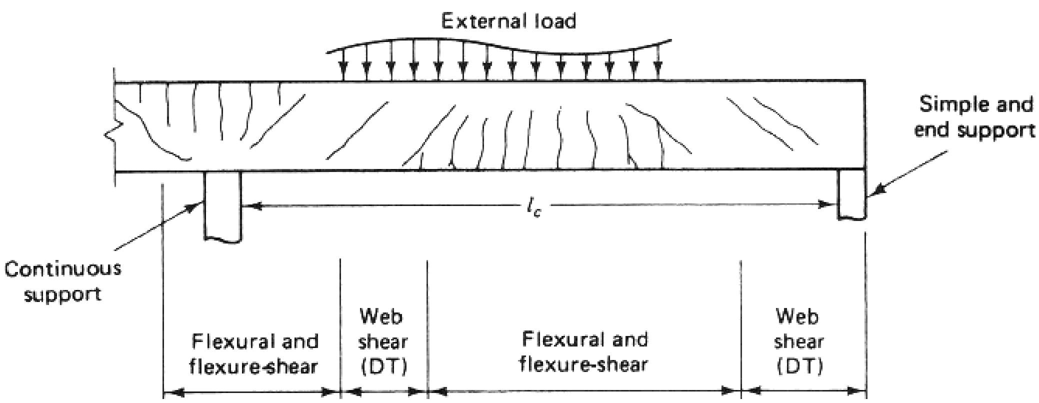


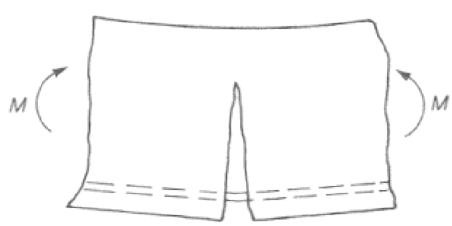
Elevation



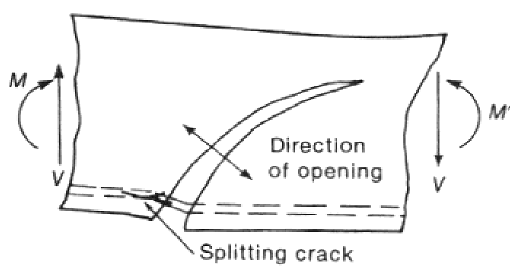
Section A–A



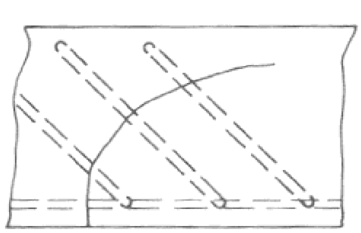


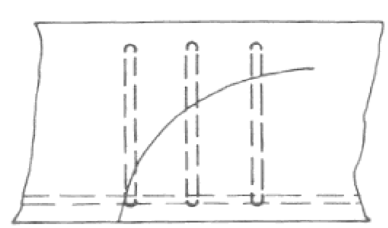


(a) Flexural crack.



(b) Inclined crack.


(c) Inclined shear reinforcement.


(d) Vertical shear reinforcement.

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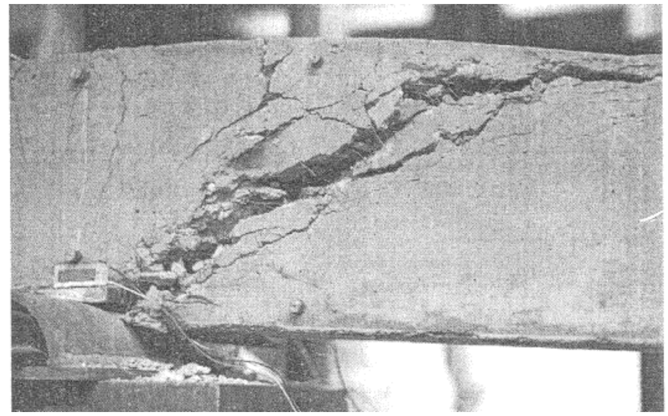
Total Shear Strength, V_n

Nominal shear strength, V_n is the summation of concrete and steel components.

$$V_n = V_c + V_s$$



Strength Design:

$$\phi V_n \geq V_u \qquad \phi = 0.75$$

Concrete shear strength, V_c

Strength of a beam without stirrups is related empirically to the quality of concrete. For design:

$$V_c = 2\sqrt{f_c'}\, b_w d \ \text{(psi)}$$

Stirrups/hoops in beams

Functions:

To provide tensile resistance across shear cracks

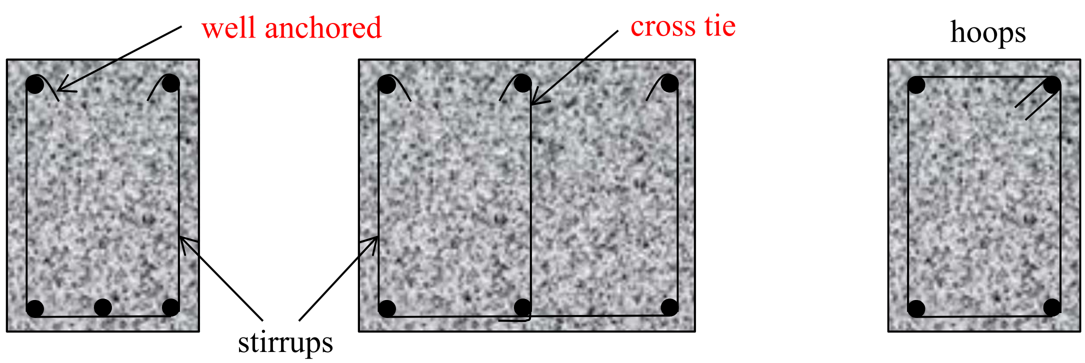
To carry a portion of the shear force

To hold longitudinal reinforcement in place (during casting)

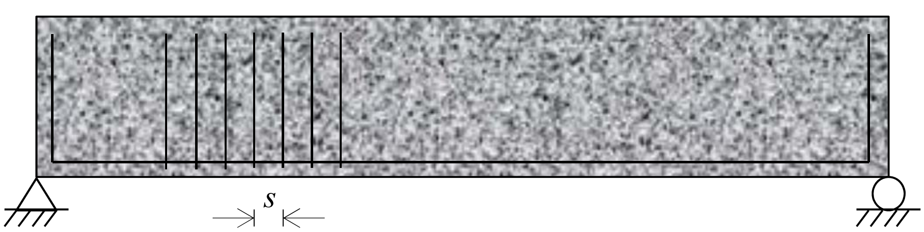
To provide confinement (especially in columns, and 135° hooks)

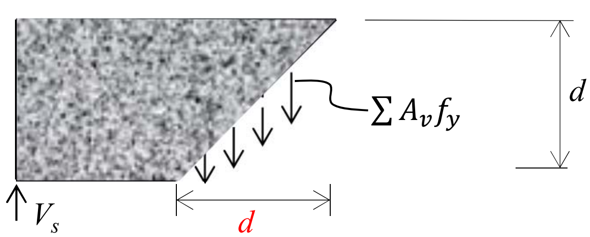
Hoops (or closed stirrups) are required where torsion or cyclic load reversals are expected.

Shapes:



ACI 318 Design Model for Shear: Assume 45° shear cracks





$$A_v = (\text{\# of legs crossing the crack})A_s$$

$$V_s = \sum A_v f_y = nA_v f_y = \frac{A_v f_y d}{s}$$

n=d/s

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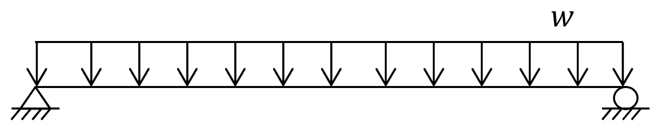
Shear Design Procedure:

a) *Estimate the required shear strength (Draw shear demand, V_u diagram using factored loads)*

$$\phi V_n \geq V_u \quad (V_n = V_c + V_s)$$

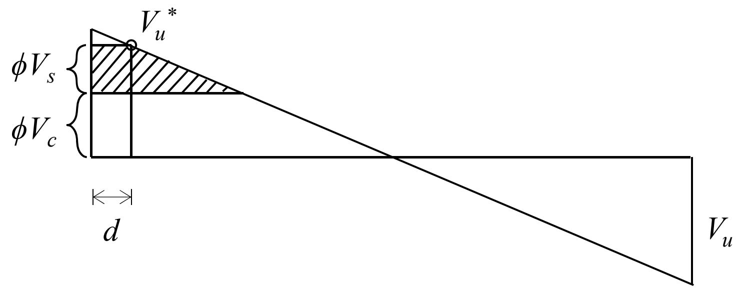


b) *Compute concrete component, V_c*

$$V_c = 2\sqrt{f_c'}\,b_w d$$

c) *Check to see if stirrups required:*

(Determine the region where stirrups required)



- No stirrups needed, if $V_u < \frac{\phi V_n}{2}$
- Minimum stirrups, if $V_u \geq \frac{\phi V_n}{2}$
- $\phi V_s = V_u - \phi V_c$

(Under distributed loading, the critical section is at a distance, *d* away from the support)

d) *Maximum stirrups:*

$$V_s = \frac{V_u - \phi V_c}{\phi} \leq 8\sqrt{f_c'}\,b_w d$$

If V_u requires more V_s than this, make beam bigger!

e) *Design stirrups:*

$$A_{v,min} = \frac{50 b_w s}{f_y},\ s_{min} \approx 4 \text{ in. (sometimes less)}$$

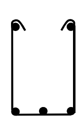
Either select A_v and find *s*; OR select *s* and find A_v $\qquad V_s = \frac{A_v f_y d}{s}$

f) *Spacing limits:*

- If $V_s \leq 4\sqrt{f_c'}\,b_w d$, $\qquad s \leq d/2$ *or 24* in. (whichever is smaller)
- If $4\sqrt{f_c'}\,b_w d \leq V_s \leq 8\sqrt{f_c'}\,b_w d$ $\qquad s \leq d/4$ *or 12* in.

g) *Practical aspects and other limitations*

- #3 bars are smallest size stirrups (#3 or #4 common)
- Typical spacing increment is 1 in. (maximum 2-3 different spacings)
- 1st stirrups at *s*/2 from support face (commonly, start @2 in from support face)
- Anchor stirrups at ends.



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Shear Analysis Example

What is the maximum factored shear force, V_u allowed on this member?

$f_c' = 4000$ psi, $f_y = 40{,}000$ psi

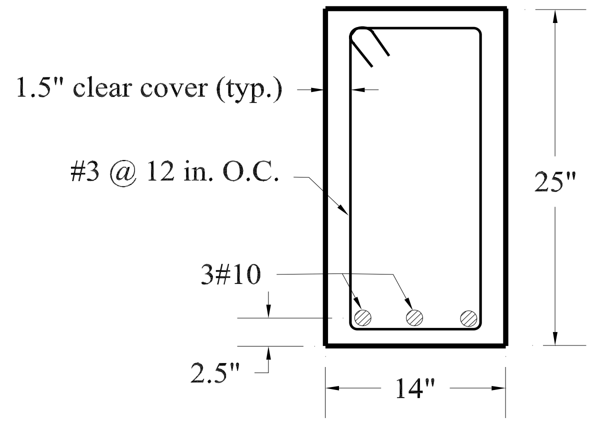
$$V_c = 2\sqrt{f_c'}b_w d = \frac{2\sqrt{4000}(14)(22.5)}{1000}$$

$$V_c = 39.8 \text{ kips}$$

$$V_s = \frac{A_v f_y d}{s} = \frac{2(0.11)(40)(22.5)}{12} = 16.5 \text{ kips}$$

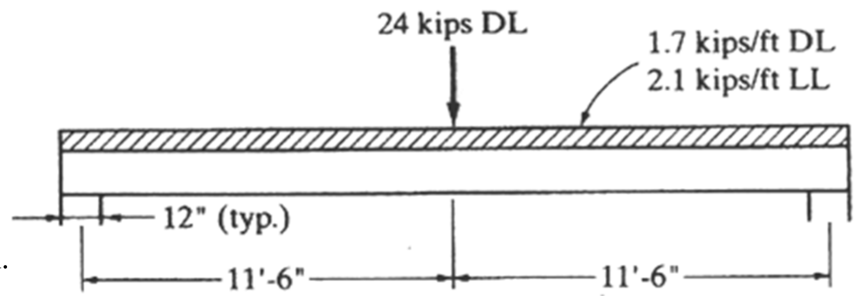
Maximum $V_u = \phi V_c + \phi V_s = 0.75(39.8 + 16.5)$

$V_u = 42.2$ kips



Shear Design Example

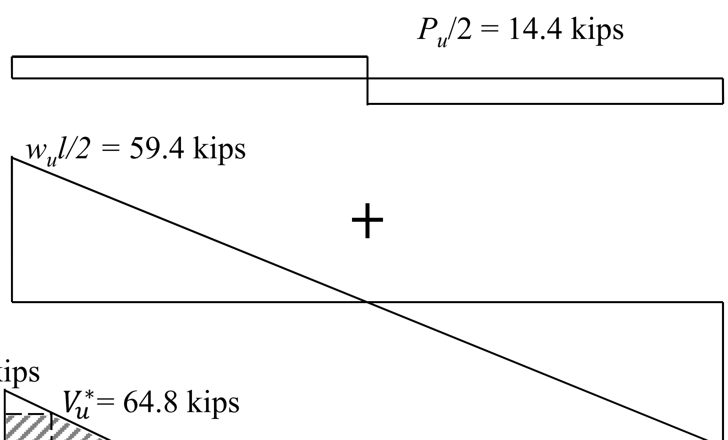
The distributed dead load shown includes weight of the beam. Beam width, b =16 in., d =20 in., f_c' = 4000 psi, f_y = 60,000 psi. Design stirrups and sketch the stirrup pattern.



1) Draw V_u diagram

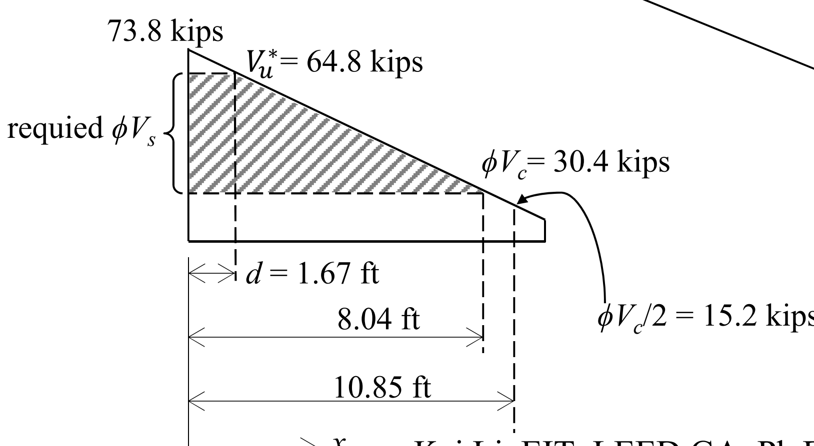
$w_u = 1.2(1.7)+1.6(2.1) = 5.4$ k/ft

$P_u = 1.2(24) = 28.8$ kips



$V_{u,max} = 11(5.4) + 28.8/2$
$= 59.4+14.4$
$= 73.8$ kips

$$V_u^* = 73.8 - \frac{20}{12}(5.4\frac{\text{k}}{\text{ft}})$$
$= 64.8$ kips



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2) Are stirrups required?

$\phi V_c = \phi 2\sqrt{f_c'} b_w d = 0.75(2)\sqrt{4000}(16)(20)$

$\phi V_c = 30.4$ kips $\rightarrow \dfrac{\phi V_c}{2} = \dfrac{30.4}{2} = 15.2 \text{ kips} < 64.8 \text{ kips}$

So, stirrups are required

3) Stirrups required to (within the x length from support face):

$$\frac{73.8 - 15.2}{5.4} = 10.85 \text{ ft from support face}$$

Location where $V_u = \phi V_c$

$$\frac{73.8 - 30.4}{5.4} = 8.04 \text{ ft from support face}$$

4) Within the length between *d* distance from the support face and *x* (i.e., for 1.67 ft $\leq$ x $\leq$ 8.04 ft), the required shear strength from stirrups is:

$\phi V_s = V_{u,max} - \phi V_c - mx$

$\quad = 73.8 - 30.4 - 5.4x = 43.4 - 5.4x$

5) Assume No. 3 stirrups with two legs

$A_v = 0.11 \text{ in.}^2 (2) = 0.22 \text{ in.}^2$

$$required\ s^* = \frac{\phi A_v f_y d}{(req'd \phi V_s^*)} = \frac{0.75(0.22)(60)(20)}{64.8 - 30.4} = 5.76 \text{ in.}$$

Use 5 in.

6) Maximum spacing according to ACI 318 code:

$4\sqrt{f_c'} b_w d = 4\sqrt{4000}(16)(20) = 81 \text{ kips}$

$$V_s^* = \frac{\phi V_s^*}{\phi} = \frac{req'd \phi V_s}{\phi} = \frac{64.8 - 30.4}{0.75} = 45.9 \text{ kips} < 81 \text{ kips}$$

s_{max} =d/2 or 24 in., d/2 = 10 in. (controls)

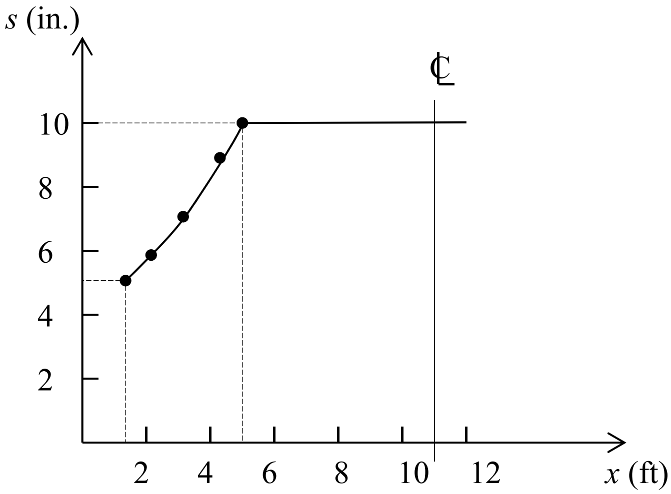
Also, $s_{max} = \dfrac{A_v f_y}{50 b_w} = \dfrac{0.22(60{,}000)}{50(16)} = 16.5 \text{ in.}$

Summary:

@ x = 1.67 ft, s = 5 in.

@ x = 8.04 ft to midspan, s = 10 in (max spacing)

Between x = 1.67 ft and 8.04 ft, → function of x



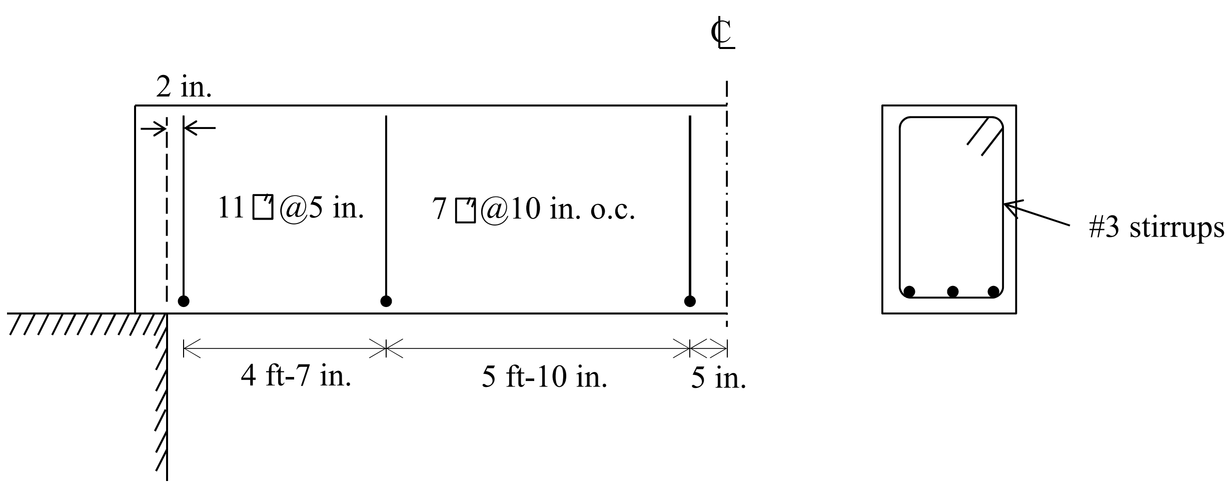
7) Required s as a function of x

$$\text{Req'd } s = \frac{\phi A_v f_y d}{(\text{req'd } \phi V_s)} = \frac{0.75(0.22)(60)(20)}{43.4 - 5.4x} = \frac{198}{43.4 - 5.4x}$$

8) Find x (distance from support face) for 10 in max spacing

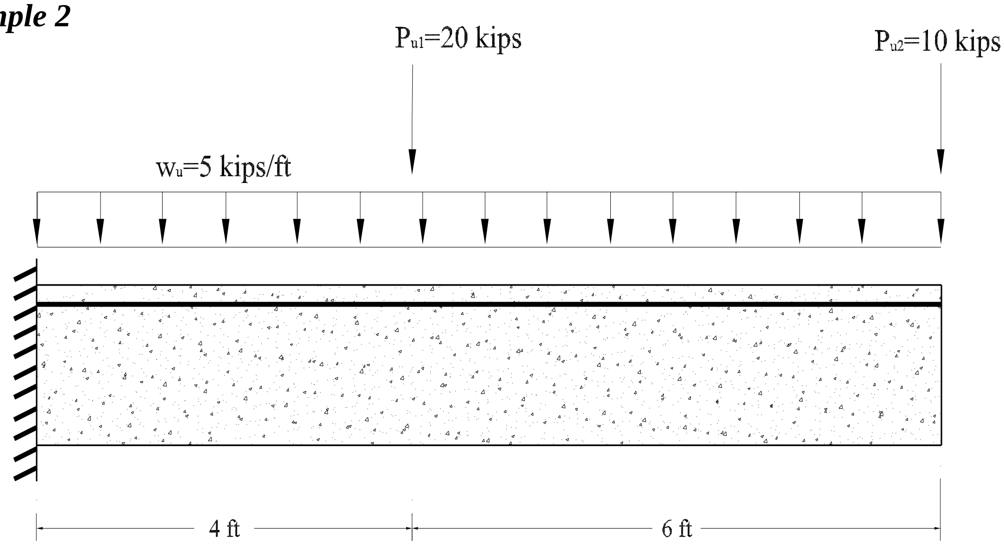
$10(43.4 - 5.4x) = 198 \rightarrow x = 4.37$ ft

$x > 4.37$ ft → s = 10 in.
$x < 4.37$ ft → s = 5 in.

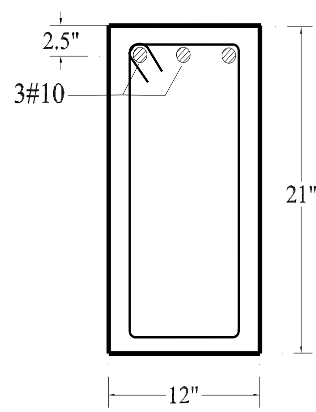


Shear Design Example 2

$f_c' = 4000$ psi
$f_y = 60{,}000$ psi



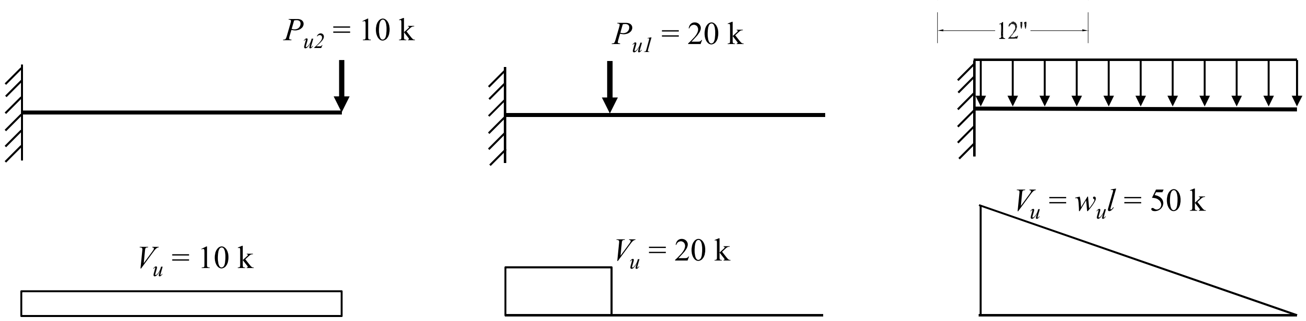
The 10-ft-span cantilever beam carries two ultimate (factored) point loads and factored distributed load of 5 kips/ft (including beam's own weight).

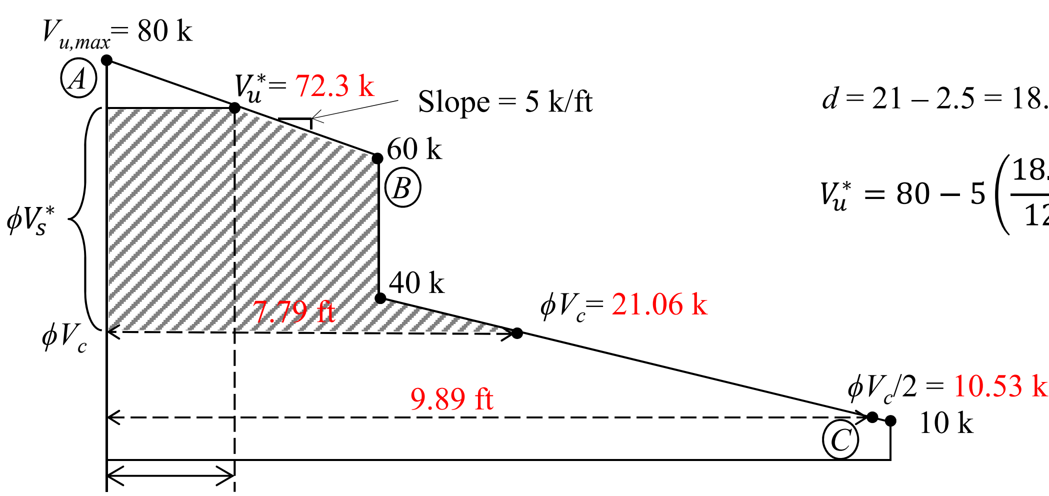


Design the shear reinforcement along the length of the beam. Sketch the stirrup pattern.

SOLUTION:

1) Draw V_u diagram





$d = 21 - 2.5 = 18.5$ in.

$$V_u^* = 80 - 5\left(\frac{18.5}{12}\right) = 72.3 \text{ k}$$

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2) Are stirrups required?

$\phi V_c = \phi 2\sqrt{f_c'}b_w d = 0.75(2)\sqrt{4000}(12)(18.5) = 21.06$ kips

$\frac{\phi V_c}{2} = 10.53 \text{ kips} < V_u^* \rightarrow \text{Therefore, stirrups are required.}$

$$\begin{cases} \text{For } V_u < \frac{\phi V_c}{2} \rightarrow \text{No stirrups required,} \\ \text{For } \frac{\phi V_c}{2} \leq V_u < \phi V_c \rightarrow \text{Minimum stirrups required} \\ \text{For } V_u > \phi V_c \rightarrow \text{shear reinforcement is required} \end{cases}$$

3) Stirrups required to (between support face and $\phi V_c/2$):

$$x = 4 + \frac{40 - 10.53}{5} = 9.89 \text{ ft}$$

Location where $V_u = \phi V_c$:

$$y = 4 + \frac{40 - 21.06}{5} = 7.79 \text{ ft}$$

4) At *d* distance away from the support face, required shear strength provided by stirrups:

Required $\phi V_s^* = V_u^* - \phi V_c = 72.3 - 21.06 = 51.24$ kips

5) Assume No. 3 stirrups with two legs
$0.11 \text{ in.}^2 (2) = 0.22 \text{ in.}^2 = A_v$

$$\phi V_s = \frac{\phi A_v f_y d}{s} \rightarrow s^* = \frac{\phi A_v f_y d}{\phi V_s^*} = \frac{0.75(0.22)(60)(18.5)}{51.24}$$

$s^* = 3.57$ in.   Use $s^* = 3.5$ in.   if $s^* < s_{max}$

6) Maximum spacing according to ACI 318 code:

$$V_s^* = \frac{\phi V_s^*}{\phi} = \frac{51.24}{0.75} = 68.32 \text{ kips} \qquad \begin{aligned} &> 4\sqrt{f_c'}bd = 56.16 \text{ k} \\ &< 8\sqrt{f_c'}bd = 112.32 \text{ k} \end{aligned}$$

$$\rightarrow s_{max} = \min\left(\frac{d}{4} \; or \; 12 \text{ in.}\right) \rightarrow s_{max} = \frac{d}{4} = \frac{18.5}{4} = 4.625 \text{ in.}$$

$$Also, s_{max} = \frac{A_v f_y}{50b} = \frac{0.22(60{,}000)}{50(12)} = 22 \text{ in.}$$

At critical section use s* = 3.5 in.   #3 stirrups @3.5 in. o.c.

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4') At point B:

Design shear, V_u = 60 kips > ϕV_c

Required $\phi V_s = V_u - \phi V_c$ = 60 – 21.06 = 38.94 kips

$$V_s = \frac{38.94}{0.75} = 51.92 \text{ kips}$$

5')

$$s = \frac{A_v f_y d}{V_s} = \frac{0.22(60)18.5}{51.92} = 4.70 \text{ in.}$$

$V_s < 4\sqrt{f_c'}bd = 56.16$ k $\rightarrow s_{max} = d/2$ = 9.25 in.

Can use s = 4.5 in. at B

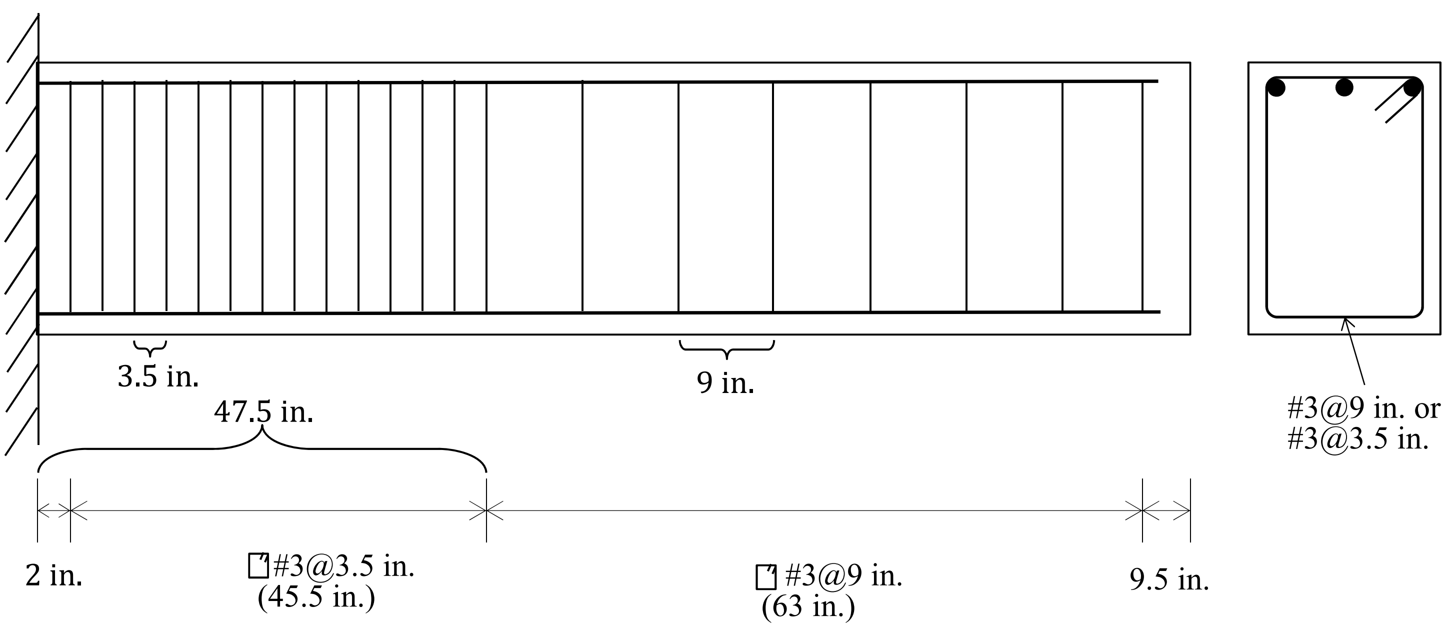
4'') Part BC:

V_u = 40 kips > ϕV_c

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{40 - 21.06}{0.75} = 25.25 \text{ kips} < 4\sqrt{f_c'}bd \ \ (s_{max} = 9.25 \text{ in. controls})$$

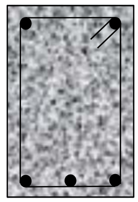
$$s = \frac{A_v f_y d}{V_s} = \frac{0.22(60)18.5}{25.25} = 9.67 \text{ in.}$$

For BC, use #3 @9 in.



TORSION

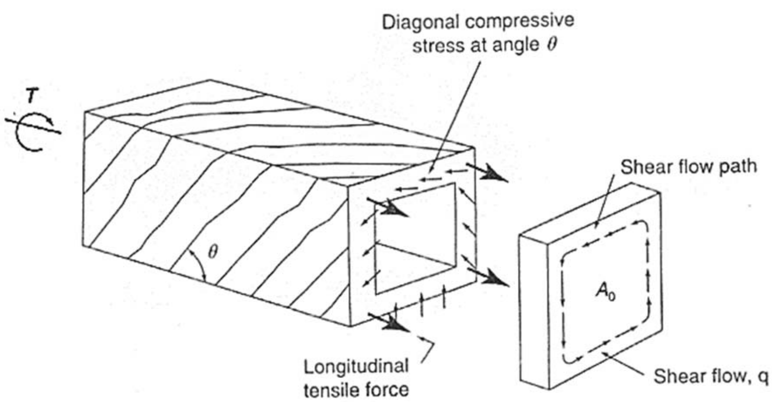
$$T_n \geq T_u$$



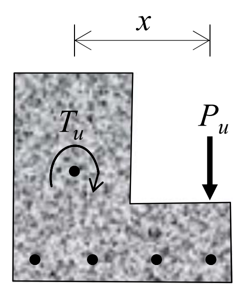
T_u = factored torsion

After cracking, torsion is resisted by stirrups, longitudinal bars, and concrete compression diagonals

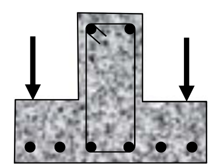
Figure 5.40 Torsional failure of web-reinforced beam after spalling of cove (Collins and Mitchell, Ref. 5.11).

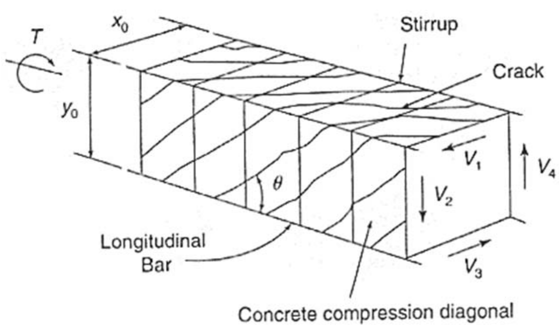


(a) Thin-walled tube analogy



$T_u = P_u \cdot$

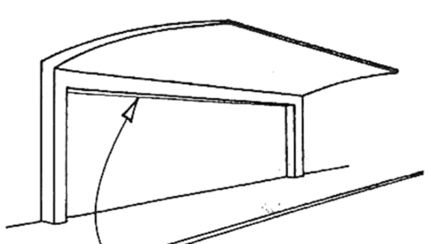




(b) Space truss analogy

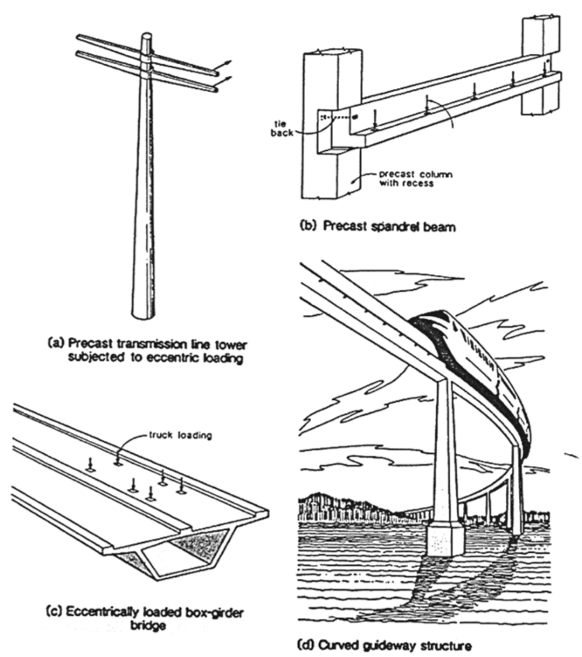
ACI STANDARD/COMMITTEE REPORT

COMMENTARY



From Collins & Mitchell

Design for torsion



(a) Precast transmission line tower subjected to eccentric loading

(b) Precast spandrel beam

(c) Eccentrically loaded box-girder bridge

(d) Curved guideway structure

Figure 8-1 Examples of prestressed concrete structures with significant torsion.

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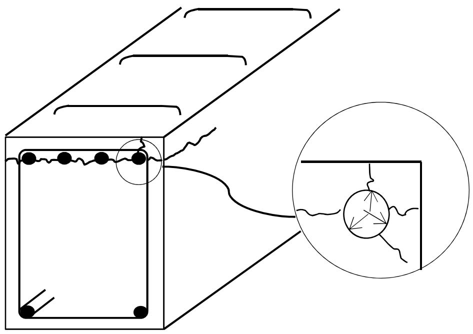
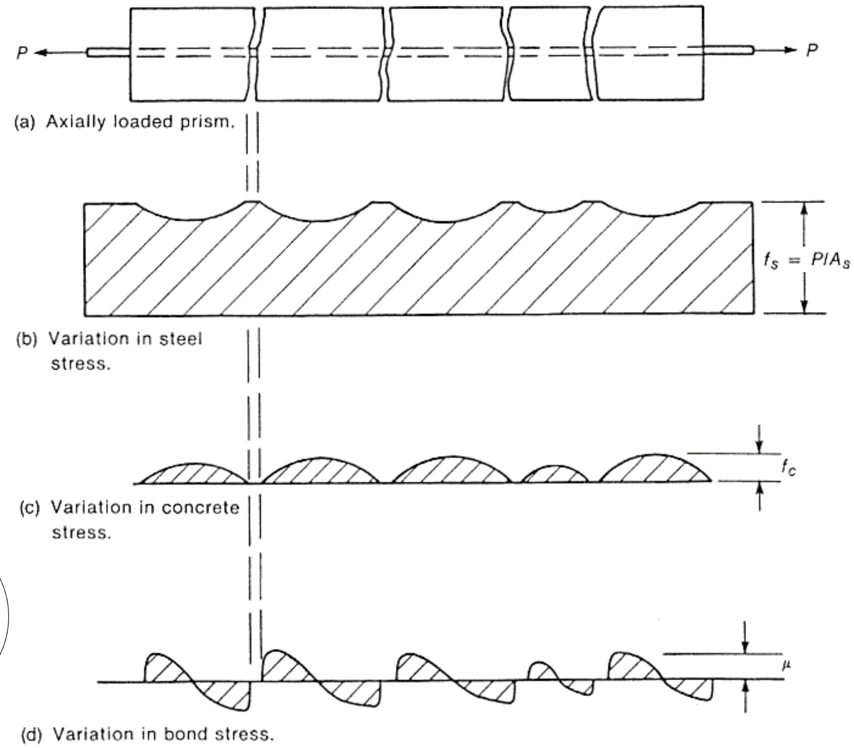
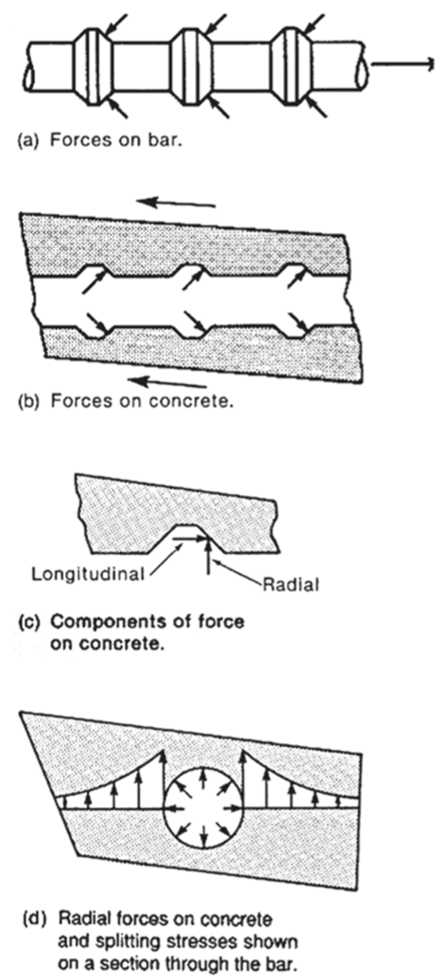
Development length and Splices

- We have assumed plane sections remain plane: $\varepsilon_s = \varepsilon_c$
- This requires bond between steel and concrete
- Design must ensure that adequate bond exists
- Provide required development length







8–2 Mechanism of Bond Transfer

Bond Mechanics

* Chemical adhesion*: weak*
* Friction*: weak*
* Bearing against deformation*: strong*

Bearing against deformation creates a radially outward force on concrete → concrete splitting

Strength depends on:

1. Cover, and spacing
2. Concrete strength, $f_c{}'$, top/bottom, LWC
3. Length
4. Bar size
5. Transverse steel
6. Epoxy, coating

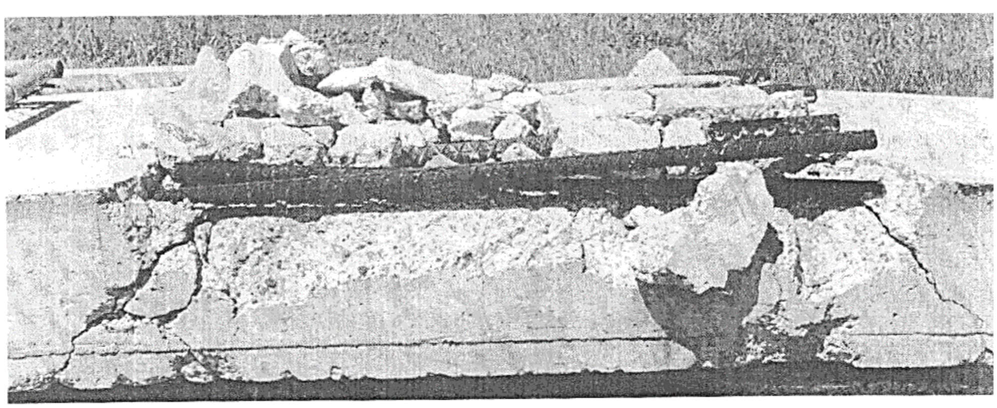
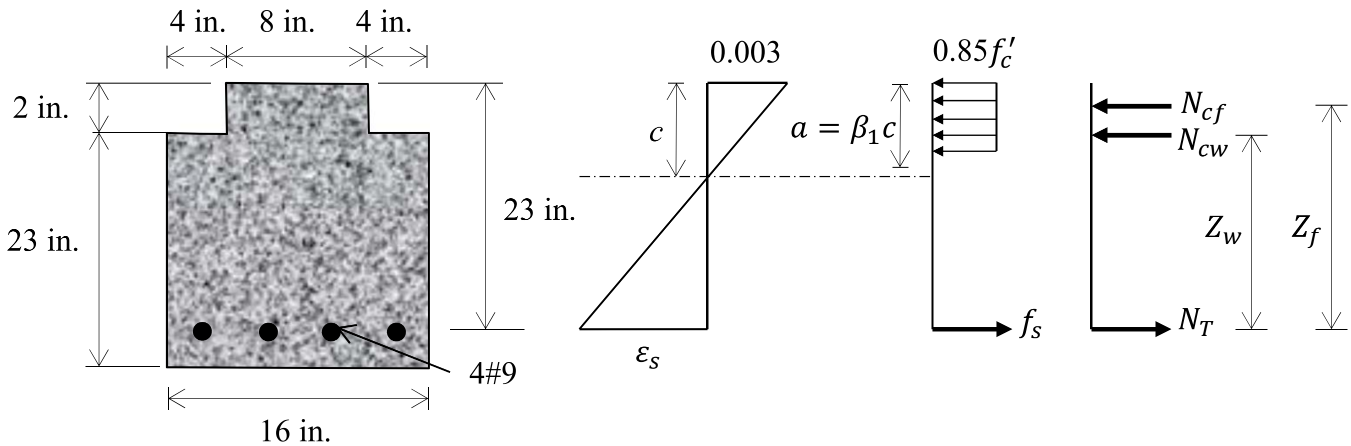


Fig. 8–32
Failure of a tension lap splice. (Photograph courtesy of J. G. MacGregor.)

Prepare for mid-term
T-beam Example

Calculate design moment of this irregular shape T-beam as shown below. Use $f_c' = 4$ ksi, $f_y = 60$ ksi.



$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \geq \frac{200}{f_y} b_w d$$

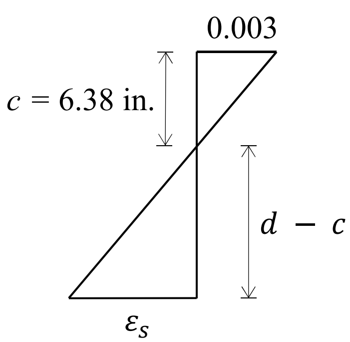
- Check ACI Code limits:

$A_{s,min} = 0.0033 b_w d$
$A_{s,min} = 0.0033(16)(23) = 1.26 < A_s$, OK

$N_T = A_s f_y = 4(60) = 240$ kips (Assuming steel yields!)
$N_{cf} = 0.85 f_c'(8)(2) = 54.4$ kips $< N_T$ (N.A is within web $a > h_f$)

$N_{cw} = N_T - N_{cf} = 240 - 54.4 = 185.6$ kips

$$N_{cw} = (a - h_f)0.85 f_c' b_w \Rightarrow a = \frac{185.6}{0.85(4)(16)} + 2 = 5.42 \text{ in.}$$
$$c = \frac{a}{0.85} = 6.38 \text{ in.}$$



Check: $\varepsilon_s = (23 - 6.38)\dfrac{0.003}{6.38}$
$\varepsilon_s = 0.0078 > \varepsilon_y$
$f_s = f_y$

$M_n = Z_w N_{cw} + Z_f N_{cf}$
$M_n = (185.6)(19.3) + (22)(54.4) = 4770$ in − k
$\phi M_n = 0.9(4770)$
$\phi M_n = 4293$ in − k

$$Z_f = d - \frac{2}{2} = 22 \text{ in.}$$
$$Z_w = \mathrm{d} - 2 - \frac{\mathrm{a} - 2}{2} = 19.29 \text{ in.}$$

Kai Li, EIT, LEED GA, Ph.D. Candidate