NAME : ________________

OSU Name.# : ________________

Lecturer:: ________________

Recitation Instructor : ________

Recitation Time : ____________

INSTRUCTIONS

Do all problems. Answer each part thoroughly.

SHOW ALL WORK! Incorrect answers with work shown may receive partial credit, but unsubstantiated answers will receive NO credit.
Give EXACT answers unless asked to do otherwise. You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{3}{\sqrt{32}}$.

Calculators are permitted EXCEPT those calculators that have symbolic algebra or calculus capabilities. In particular, the following calculators and their upgrades are not permitted: TI-89, TI-92, and HP-49.
In addition, neither PDA's, laptops, nor cell phones are permitted.
Do not even have these devices out!

The exam duration is 55 minutes.

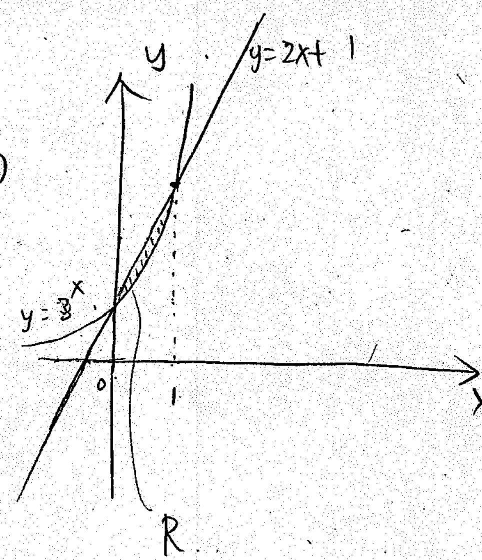
The exam consists of 5 problems starting on page 2 and ending on page 7.
Make sure your exam is not missing any pages before you start.
Page 8 may be used for extra work space.

PROBLEM	SCORE
#1	(16)
#2	(16)
#3	(30)
#4	(18)
#5	(20)
TOTAL	(100)

Midterm 1
Form A, Page 2

1. (16 pts) Let R be a region bounded by the graphs of $y = 2x + 1$ and $y = 3^x$.

[Useful tip: Both intersection points of these graphs have INTEGER coordinates.]



(a) (10 pts) Find the area of region R.

points of intersection are (0, 1) and (1, 3)

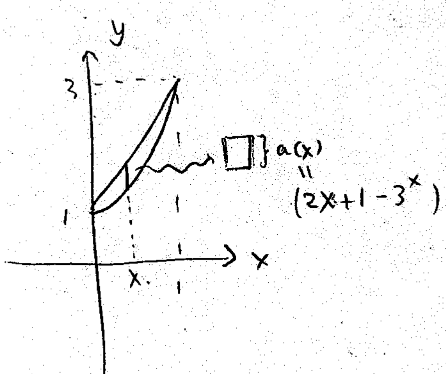
$$\text{area} = \int_0^1 2x+1 - 3^x \, dx$$

$$= x^2 + x - \frac{3^x}{\ln 3} \Big|_0^1$$

$$= 1 + 1 - \frac{3}{\ln 3} - \left(0 + 0 - \frac{1}{\ln 3}\right)$$

$$= 2 - \frac{2}{\ln 3}.$$

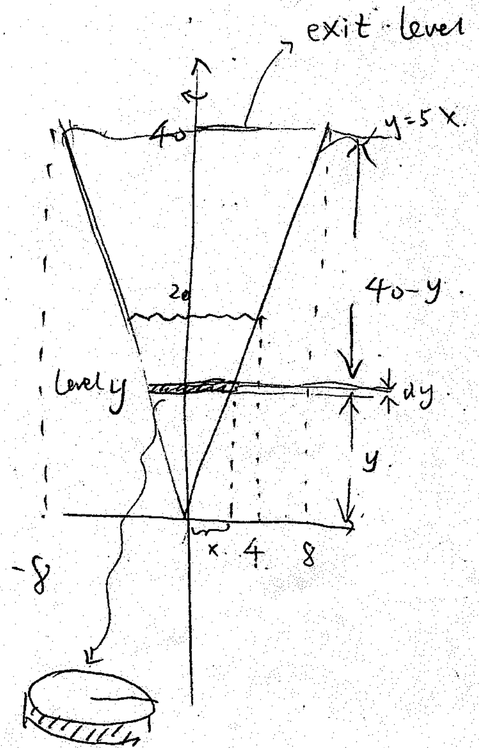
(b) (6 pts) Set up an integral that represents the volume of the solid whose base is the region R and whose cross sections perpendicular to the x-axis are squares. DO NOT EVALUATE THE INTEGRAL!



$$\text{Vol} = \int_0^1 A(x) \, dx = \int_0^1 a(x)^2 \, dx$$

$$= \int_0^1 (2x+1 - 3^x)^2 \, dx$$

2. (16 pts) A tank is shaped like an inverted cone. It is formed by revolving the graph of $y = 5x$ for $0 \le x \le 8$ (in meters) about the y-axis. The tank is filled with water to a height of 20 meters. How much work is required to pump all of the water to an exit pipe at the top of the tank? (Note: The density of water is $1000\ kg/m^3$.)



Volume of disk of water

$$= \pi x^2\, dy \qquad \left(\begin{array}{l} y = 5x \\ x = \frac{y}{5} \end{array} \right)$$

$$= \pi \left(\frac{y}{5}\right)^2 dy$$

force for disk of water

$$= \text{density} \cdot \text{volume} \cdot g$$

gravitational acceleration $= 9.8\ m/s^2$

$$= 1000 \cdot \pi \frac{y^2}{25} \cdot 9.8\, dy$$

distance lifted $= 40 - y$.

So total work

$$= \int_0^{20} 1000\pi \cdot \frac{y^2}{25} \cdot 9.8\,(40 - y)\, dy$$

$$= 40 \cdot 9.8\pi \int_0^{20} 40y^2 - y^3\, dy$$

$$= 40 \cdot 9.8\pi \left(\frac{40}{3} y^3 - \frac{y^4}{4} \right) \Big|_0^{20}$$

$$= 40 \cdot 9.8\pi \times \left(\frac{40}{3} 20^3 - \frac{20^4}{4} \right) \quad J$$

3. (30 pts) **Evaluate the following integrals. Show your work!**

(a) (10 pts) $\displaystyle\int \frac{3+\ln x}{x}\,dx$ (the expression implies $x>0$)

$$= \int \frac{3}{x}\,dx + \int \frac{\ln x}{x}\,dx$$

$$= 3\ln|x| + \int \frac{\ln x}{x}\,dx \qquad \left(u=\ln x,\quad du=\frac{1}{x}dx\right)$$

$$= 3\ln x + \int u\,du$$

$$= 3\ln x + \frac{1}{2}u^2 + C$$

$$= 3\ln x + \frac{1}{2}(\ln x)^2 + C$$

(b) (10 pts) $\displaystyle\int_0^{\frac{\pi}{6}} \theta\cos(3\theta)\,d\theta$ $\qquad u=\theta,\quad dv=\cos(3\theta)\,d\theta$

$\qquad du=d\theta,\quad v=\frac{1}{3}\sin(3\theta)$

$$= \int_0^{\frac{\pi}{6}} u\,dv$$

$$\overset{\text{IBP}}{=} uv\Big|_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{8}} v\,du$$

$$= \frac{1}{3}\theta\sin(3\theta)\Big|_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \frac{1}{3}\sin(3\theta)\,d\theta$$

$$= \frac{1}{3}\theta\sin(3\theta)\Big|_0^{\frac{\pi}{6}} + \frac{1}{9}\cos(3\theta)\Big|_0^{\frac{\pi}{6}}$$

$$= \frac{1}{3}\cdot\frac{\pi}{6}\sin\left(\frac{\pi}{2}\right) - 0 + \frac{1}{9}\underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} - \frac{1}{9}\cos(0)$$

$$= \frac{\pi}{18} - \frac{1}{9}.$$

(c) (10 pts) $\displaystyle\int \frac{2x+7}{x^2+6x+10}\,dx$

For $x^2+6x+10$,

$\Delta = 6^2 - 4\cdot 10 = -4 < 0$

So $x^2+6x+10$ is irreducible and no need for partial fraction decomposition

$$= \int \frac{2x+7}{x^2+6x+9+1}\,dx$$

$$= \int \frac{2x+7}{(x+3)^2+1}\,dx$$

$\left(u = x+3, \quad 2x+7 = 2(u-3)+7 = 2u+1 \right)$

$du = dx$

$$= \int \frac{2u+1}{u^2+1}\,du$$

$$= \underbrace{\int \frac{2u}{u^2+1}\,du}_{①} + \underbrace{\int \frac{1}{u^2+1}\,du}_{②}$$

For ①, use $v = u^2+1$, $dv = 2u\,du$,

$$① = \int \frac{dv}{v} = \ln|v| + C_1 = \ln(u^2+1) + C_1$$
$$= \ln((x+3)^2+1) + C_1$$

$$② = \tan^{-1}(u) + C_2 = \tan^{-1}(x+3) + C_2$$

So $\displaystyle\int \frac{2x+7}{x^2+6x+10}\,dx = \ln((x+3)^2+1) + \tan^{-1}(x+3) + C$

4. (18 pts) Find the length of the curve

$$y = f(x) = \frac{1}{4}\left(e^{2x} + e^{-2x}\right) \text{ on } -2 \le x \le 2.$$

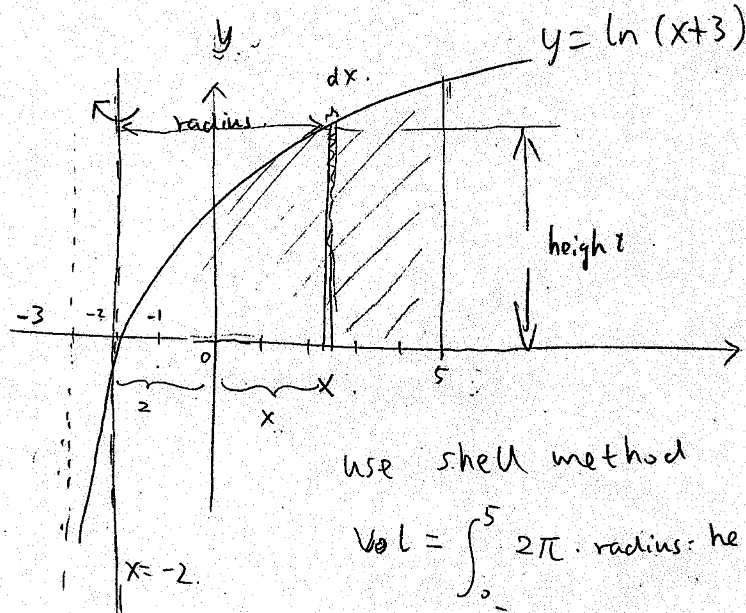
$$f'(x) = \frac{1}{4}\left(2e^{2x} - 2e^{-2x}\right)$$
$$= \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x}$$

$$1 + \left(f'(x)\right)^2 = 1 + \left(\frac{1}{2}e^{2x}\right)^2 - 2\cdot\frac{1}{2}e^{2x}\cdot\frac{1}{2}e^{-2x} + \left(\frac{1}{2}e^{-2x}\right)^2$$
$$= \left(\frac{1}{2}e^{2x}\right)^2 + \frac{1}{2} + \left(\frac{1}{2}e^{-2x}\right)^2$$
$$= \left(\frac{1}{2}e^{2x}\right)^2 + 2\cdot\frac{1}{2}e^{2x}\cdot\frac{1}{2}e^{-2x} + \left(\frac{1}{2}e^{-2x}\right)^2$$
$$= \left(\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}\right)^2$$

So

$$L = \int_{-2}^{2}\sqrt{1 + \left(f'(x)\right)^2}\,dx = \int_{-2}^{2}\sqrt{\left(\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}\right)^2}\,dx$$
$$= \int_{-2}^{2}\left(\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}\right)dx$$
$$= \left(\frac{1}{4}e^{2x} - \frac{1}{4}e^{-2x}\right)\Big|_{-2}^{2}$$
$$= \frac{1}{4}e^{4} - \frac{1}{4}e^{-4} - \left(\frac{1}{4}e^{-4} - \frac{1}{4}e^{4}\right)$$
$$= \frac{1}{2}e^{4} - \frac{1}{2}e^{-4}$$

5. (20 pts) The region under the graph $y = \ln(x+3)$ for $0 \le x \le 5$ is revolved about the line $x = -2$. Find the volume of the resulting solid.



use shell method

$$\text{Vol} = \int_0^5 2\pi \cdot \text{radius} \cdot \text{height} \cdot \text{width}$$

$$= \int_0^5 2\pi (x+2) \ln(x+3)\, dx$$

$$\begin{pmatrix} t = x+3 \\ dt = dx \end{pmatrix} = 2\pi \int_3^8 (t-1) \ln t\, dt$$

$$= 2\pi \Big(\underbrace{\int_3^8 t \ln t\, dt}_{①} - \underbrace{\int_3^8 \ln t\, dt}_{②} \Big)$$

For ①, $u = \ln t$, $dv = t\,dt$

$du = \frac{1}{t} dt$, $v = \frac{1}{2} t^2$

$$① = \int_3^8 t \ln t\, dt = \int_3^8 u\, dv$$

$$\overset{\text{IBP}}{=} uv \Big|_3^8 - \int_3^8 v\, du$$

$$= \frac{1}{2} t^2 \ln t \Big|_3^8 - \int_3^8 \frac{1}{2} t^2 \cdot \frac{1}{t}\, dt$$

$$= \frac{1}{2} t^2 \ln t \Big|_3^8 - \frac{1}{2} \int_3^8 t\, dt$$

$$= \frac{1}{2} t^2 \ln t \Big|_3^8 - \frac{1}{4} t^2 \Big|_3^8$$

$$= \frac{1}{2} 8^2 \ln 8 - \frac{1}{2} 3^2 \ln 3 - \frac{1}{4} 8^2 + \frac{1}{4} 3^2$$

For ②, $u = \ln t$, $dv = dt$

$du = \frac{1}{t} dt$, $v = t$

$$② = \int_3^8 \ln t\, dt = \int_3^8 u\, dv \overset{\text{IBP}}{=}$$

$$\overset{\text{IBP}}{=} uv \Big|_3^8 - \int_3^8 v\, du$$

$$= t \ln t \Big|_3^8 - \int_3^8 t \cdot \frac{1}{t}\, dt = t \ln t \Big|_3^8 - \int_3^8 1\, dt$$

$$= 8 \ln 8 - 3 \ln 3 - 5$$

So $\text{Vol} = 2\pi (① - ②)$

$$= 2\pi \left(\frac{1}{2} 8^2 \ln 8 - \frac{1}{2} 3^2 \ln 3 - \frac{1}{4} \cdot 8^2 + \frac{1}{4} 3^2 - 8 \ln 8 + 3 \ln 3 + 5 \right).$$