Math 1172

Midterm 2

Autumn 2014

Form A

Name: ____________________

OSU username (name.nn): ____________________

Lecturer: ____________________

Recitation Instructor: ____________________

Recitation Time: ____________________

Instructions

•) SHOW ALL WORK!!! Incorrect answers with work shown may receive partial credit, but unsubstantiated answers may receive NO credit.

•) Give EXACT answers unless asked to do otherwise.

•) You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{5}{\sqrt{12}}$.

•) **NO CALCULATORS. NO CELL PHONES. NO ELECTRONIC DEVICES.**

•) The exam duration is 55 minutes.

•) The exam consists of 5 problems starting on page 2 and ending on page 6. Make sure your exam is not missing any pages before you start. Page 7 contains formulas. Pages 7 and 8 may be used for extra work space.

Problem Number	Maximum Point Value	Score
1	21	
2	19	
3	20	
4	18	
5	22	
Total	100	

Problem 1

[21 pts] True or False. Give a brief explanation or example to justify your answer.

a) [3 pts] Suppose $f(x) = 2x^3 - 3x$. Then the 3^{rd}-order Taylor polynomial for f centered at $a = 1$ is $p_3(x) = -1 + 3(x-1) + 6(x-1)^2 + 2(x-1)^3$.

b) [3 pts] $\sum_{k=1}^{\infty} \frac{2^k}{k^{10}}$ diverges.

c) [3 pts] $\int \tan^5\theta \sec^6\theta \, d\theta = \int (u^2-1)^2 u^5 \, du$, where $u = \sec\theta$.

d) [3 pts] If $\lim_{k\to\infty} a_k = 0$ then the series $\sum_{k=0}^{\infty} a_k$ converges.

e) [3 pts] $\sum_{k=2}^{\infty} \frac{6}{(-3)^k} = \frac{1}{2}$.

f) [3 pts] The sequence $a_n = \frac{(-1)^n n}{3n^2+1}$ converges as $n \to \infty$.

g) [3 pts] $x = 2 - 4t$, $y = 3 + 8t$, for $-\infty < t < \infty$ are parametric equations for a line with $\frac{dy}{dx} = -2$.

Problem 2

[19 pts] Integrals and Series.

a) [10 pts] Evaluate the indefinite integral.

$$\int \frac{1}{x^2+6x+8}\,dx$$

b) [9 pts] Determine whether the series converges or diverges. If it converges, find the value.

$$\sum_{k=1}^{\infty} \frac{1}{k^2+6k+8}$$

Problem 3

[20 pts] Consider the function f and its power series $f(x) = \sum_{k=0}^{\infty} \frac{5}{3^k}(x+1)^k$.

a) [2 pts] Compute $f(-1)$.

b) [8 pts] Find the radius of convergence for the power series.

c) [4 pts] Find the interval of convergence for the power series.

d) [4 pts] Find a power series representation for $f'(x)$.

e) [2 pts] Compute $f'(-1)$.

Problem 4

[18 pts] Taylor Series.

a) [10 pts] Find the first four nonzero terms of the Taylor series for $f(x) = \frac{1}{x^2}$ centered at $a = 1$.

b) [8 pts] Use the Taylor series $\frac{1}{1+x} = \sum_{k=0}^{\infty}(-1)^k x^k$ to find the first four nonzero terms of the Taylor series centered at 0 for the function $g(x) = \frac{x^2}{1+5x^3}$.

Problem 5

[22 pts] Integration.

a) [14 pts] Evaluate the integral. Write your answer in terms of w and **simplify**.

$\displaystyle\int \frac{1}{w(a^2+w^2)}\,dw,$ where $a > 0$ is a constant.

b) [8 pts] Determine whether the improper integral converges or diverges.
If it converges, find its limit (which may involve the constant a.)

$\displaystyle\int_a^{\infty} \frac{1}{w(a^2+w^2)}\,dw,$ where $a > 0$ is a constant.

A Few Trigonometric Identities

1) $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

2) $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

3) $\cos^2\theta + \sin^2\theta = 1$

4) $\sec^2\theta - \tan^2\theta = 1$

5) $\csc^2\theta - \cot^2\theta = 1$

A Few Reduction Formulas

Assume n is a positive integer.

1) $\displaystyle\int \sin^n x\,\mathrm{d}x = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,\mathrm{d}x$

2) $\displaystyle\int \cos^n x\,\mathrm{d}x = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,\mathrm{d}x$

3) $\displaystyle\int \tan^n x\,\mathrm{d}x = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,\mathrm{d}x, \quad n \neq 1$

4) $\displaystyle\int \sec^n x\,\mathrm{d}x = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,\mathrm{d}x, \quad n \neq 1$