Math 1172

Midterm 1

Spring 2015

Form A

Name: _____

OSU username (name.nn): _____

Lecturer: _____

Recitation Instructor: _____

Recitation Time: _____

Instructions

•) SHOW ALL WORK!!! Incorrect answers with work shown may receive partial credit, but unsubstantiated answers may receive NO credit.

•) Give EXACT answers unless asked to do otherwise.

•) You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{5}{\sqrt{12}}$ unless asked to do otherwise.

•) NO CALCULATORS. NO CELL PHONES. NO ELECTRONIC DEVICES.

•) The exam duration is 55 minutes.

•) The exam consists of 5 problems starting on page 2 and ending on page 6. Make sure your exam is not missing any pages before you start. Page 7 contains formulas. Pages 7 and 8 may be used for extra work space.

Problem Number	Maximum Point Value	Score
1	18	
2	22	
3	20	
4	20	
5	20	
Total	100	

Problem 1

[18 pts] True or False. You do not need to show work for problems on this page.

a) [2 pts] $\int xe^x\,dx = \left(\int x\,dx\right)\left(\int e^x\,dx\right)$

b) [2 pts] Given a spring that obeys Hooke's Law, the work required to stretch the spring from equilibrium to 2 cm is twice the work required to stretch the spring from equilibrium to 1 cm.

c) [2 pts] $\int \tan^5\theta \sec^4\theta\,d\theta = \int u^3(u^2-1)^2\,du$, where $u = \sec\theta$.

d) [2 pts] $\int \frac{1}{x^2+3}\,dx = \ln|x^2+3| + C$

e) [2 pts] A thin wire over $0 \le x \le 6$ (in *meters*) with the density function (in $\frac{kg}{m}$) $\rho(x) = \begin{cases} x, & \text{if } 0 \le x \le 2 \\ 3, & \text{if } 2 < x \le 6 \end{cases}$, has a mass of 14 kg

f) [2 pts] Assuming Hooke's law is obeyed, a spring that requires 7 J of work to be stretched 1 m from its equilibrium position would have a spring constant of $k = 14\ N/m$.

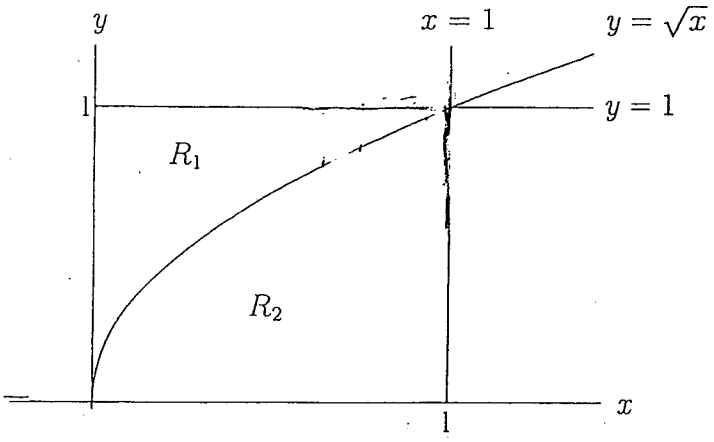
g) [2 pts] $\int xf'(x)\,ax = xf(x) - \int f(x)\,dx$ where $f'(x)$ is a continuous function.

h) [2 pts] $\int \frac{x}{x^2+5}\,dx = \int \frac{x}{x^2}\,dx + \int \frac{x}{5}\,dx$

i) [2 pts] $\int \frac{e^{2t}}{e^t+2}\,dt = \int \left(1+\frac{2}{u}\right)\,du$, where $u = e^t + 2$.

Problem 2

[22 pts] Let R_1 be the region bounded by the y-axis, and the graphs of $y = 1$ and $y = \sqrt{x}$. Let R_2 be the region bounded by the x-axis, and the graphs of $x = 1$ and $y = \sqrt{x}$. [see graph.]



a) [7 pts] Set up an integral **with respect to y** that represents the area of region R_1. DO NOT EVALUATE THE INTEGRAL.

b) [7 pts] Set up an integral that represents the volume of the solid whose base is the region R_2 and whose cross sections perpendicular to the y-axis are squares. DO NOT EVALUATE THE INTEGRAL.

c) [8 pts] Set up an integral **using the shell method** that represents the volume of the solid of revolution that is formed by revolving the region R_1 about the line $x = -2$. DO NOT EVALUATE THE INTEGRAL.

Problem 3

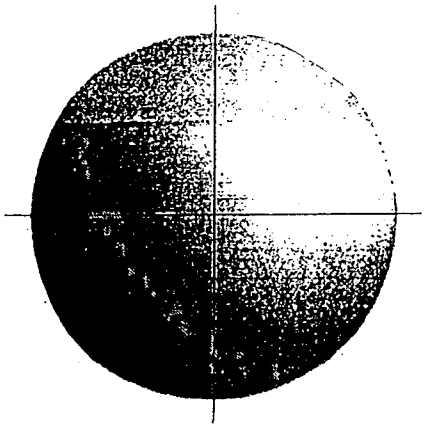
[20 pts] Evaluate the following integrals. Show your work.

a) [10 pts] $\displaystyle\int \frac{\cos^3 x}{\sin x}\, dx$

b) [10 pts] $\displaystyle\int \theta \sin(3\theta)\, d\theta$

Problem 4

[20 pts] A tank in the shape of a sphere with a diameter of 5m is partially filled with water. The tank is filled to a depth of 7 m. How much work is required to pump all of the water to an exit pipe at the top of the tank? [Note: The density of water is 1000 kg/m^3.]



[Note: Most of the credit for this problem comes from setting up the correct integral.]

Problem 5

[20 pts] Find the length of the curve $y = \frac{a}{3}\left(x^2 + \frac{2}{a}\right)^{3/2}$ on the interval $[0, 3]$, where $a > 0$ is a constant. Your final answer might involve the constant a.

A Few Trigonometric Identities

1) $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

2) $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

3) $\cos^2\theta + \sin^2\theta = 1$

4) $\sec^2\theta - \tan^2\theta = 1$

5) $\csc^2\theta - \cot^2\theta = 1$

A Few Reduction Formulas

Assume n is a positive integer.

1) $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

2) $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

3) $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx, \quad n \neq 1$

4) $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx, \quad n \neq 1$

EXTRA WORKSPACE

Do not remove this page.