A first estimate of the expected distribution of SWOT river discharge accuracy

Michael Durand¹, Renato Frasson¹, Stephen Tuozzolo¹, Colin Gleason², Pierre-Andre Garambois³, and Pierre-Olivier Malaterre⁴

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Olentangy River, Columbus, Ohio
Survey of remote sensing of river discharge methods

- When remote sensing observations overlap a gage, substitute remote for in situ width or stage. >20 years history. Dozens of publications (e.g., Smith et al., WRR, 1995; van Dijk et al., 2016). Limited to locations with gages.

- Sensing of river bottom via optical attenuation is promising, though limited to shallow, clear water.

- Water balance methods to estimate river discharge by difference are potentially promising, though may accumulate uncertainty across the various terms in the balance.

- Assimilation of remote sensing observations into models.

- Solving an inverse algorithm, e.g. the Mass-Conserved Flow Law Inversion algorithms.
How accurately will SWOT estimate river discharge?

- SWOT observations of height, width, and slope enable discharge estimates but accurate computation of discharge is challenging.

- To estimate discharge uncertainty:
  1) review SWOT discharge methods
  2) formulate discharge uncertainty model
  3) evaluate and analyze over global rivers

Image: SWOT observable rivers

Courtesy: George Allen
General approach to estimating river discharge with SWOT

- Use simple flow laws that combine SWOT observations and calibration parameters

- Manning: discharge \( Q \) from cross-sectional area, width \( W \), slope \( S \), and friction coefficient

- Parameters estimated via solution to inverse problems, solved with advanced algorithms

\[
Q = \frac{1}{n} (\bar{A} + A')^{5/3} W^{-2/3} S^{1/2}
\]

Parameters: \( n, \bar{A} \)

SWOT observations: \( A', W, S \)

Example: discharge on the Willamette River successfully estimated via AirSWOT data (Tuozzolo et al., GRL in review)
Discharge Algorithms: Two ways of estimating flow law parameters

• Mass-conserved flow law inversion (McFLI; Gleason et al. *Eos*, 2017) is a paradigm that invokes continuity among reaches to constrain parameters. At least 5 McFLI algorithms have been proposed.

• Variational Data Assimilation (VDA) merges observations with a hydraulic model

• McFLI and VDA differ significantly from each other, but both are **discharge algorithms** produce estimates of parameters that can be used in flow laws. They are sophisticated and computationally expensive. They start from first guess discharge: $\bar{Q}$
“Integrators” make discharge algorithm results consistent across entire river networks

- McFLI and VDA are generally too computational expensive to run across entire river networks. This leads to likely mass imbalance at tributary junctions

- The integrator problem has been solved in multiple ways, e.g. Gleason et al. (in review). Steve Tuozzolo solved the problem this way (see poster OS53C-1343):

\[
\min_{n,\bar{A}} \sum_{\text{reaches}} \left[ \frac{Q(n, \bar{A}) - Q_{\text{McFLI}}}{\sigma_Q} \right]^2
\]

subject to \( Q_{in,1} + Q_{in,2} = Q_{out} \)

Estimate parameters to keep as close as possible to McFLI or VDA discharge (Qhat), while also keeping balance

Tuozzolo, PhD thesis, 2018
Presenting a linear integrator

• What if the problem were reduced to estimating **time-average** flow? You could then back out parameters to match

• This problem is linear, and has an exact solution via Lagrangian multipliers.

• This framework can be used to combine SWOT & gages

• $Q_{SWOT}$ is the **average** SWOT discharge of the period, after imposing the integrator

\[ \begin{align*}
Ax &= b \\
A &= \begin{bmatrix} E & -G^T \\ G & 0 \end{bmatrix} \\
b &= \begin{bmatrix} -2Q_{McFLI} \circ \sigma_{Q_{McFLI}}^{-1} \\ 0 \end{bmatrix} \\
E_{ij} &= \begin{cases} -2\sigma_{Q_{McFLI}}^{-1} & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}
\]

Note: $G$ chosen s.t.: $GQ = 0$ when continuity is obeyed.
Summary of Discharge Methods

1. SWOT measures global rivers: starting 2022

2. Science team uses McFLI algorithms to solve localized discharge estimates, and to estimate parameters ($Q_{\text{McFLI}}$)

3. Science team uses integrators applied to McFLI estimates and in situ gages to revise parameter estimates ($Q_{\text{SWOT}}$)

4. Science team provides these final parameters to JPL, allowing discharge to be produced
Discharge Error Budget for $Q_{SWOT}$

\[
Q = \frac{1}{n} (\bar{A} + A')^{5/3} W^{-2/3} S^{1/2}
\]

\[
\left( \frac{\sigma_Q}{Q} \right)^2 = \left( \frac{\sigma_Q}{Q} \right)_{Par}^2 + \left( \frac{\sigma_Q}{Q} \right)_{Obs}^2 + \left( \frac{\sigma_Q}{Q} \right)_{Mod}^2
\]

Consider discharge uncertainty in three parts: Parameters, observations, and model.

Following Yoon et al., WRR, 2016 showed that parameter error (shown here in purple) tends to dominate...

Part 2: Discharge Uncertainty Model
Observation Error is the Most Straightforward

\[
\left( \frac{\sigma_Q}{Q} \right)_{obs}^2 \approx \left( \frac{5}{3} \frac{\sigma_H \sqrt{2}}{c (Wa^{-1})^{\frac{d}{b}}} \right)^2 + \left( \frac{2}{3} \frac{\sigma_W}{W} \right)^2 + \left( \frac{1}{2} \frac{\sigma_S}{S} \right)^2
\]

- Height: \( \sigma_H = 0.10 \text{ m} \)
- Width: \( \sigma_W = 10 \text{ m} \)
- Slope: \( \sigma_S = 1.7 \text{ cm km}^{-1} \)

- We follow work by Renato Frasson (OS51A-04 presented in this session). We do not include layover, but errors are otherwise conservative: 100 m rivers, 10 km in length.

- Assumption 1: Uncertainty in the measured change in cross-sectional area are dominated by height errors

- Assumption 2: “Average” width-to-depth ratio, with \( a, b, c, d \) from Moody & Troutman’s 2002 global at-a-station hydraulic geometry coefficients and exponents

- Need non-Manning strategy for flat rivers; ignored in this study.
Model Error: Modified Manning’s Equation is Surprisingly Effective

Part 2: Discharge Uncertainty Model

Multiple channels
Management

Willamette
in situ:
nRMSE = 0.05

Here, assume:

\[ \left( \frac{\sigma_Q}{Q} \right)_{Mod} = 0.1 \]
Parameter error Part 1: McFLI

- Assume that background or first guess estimates of discharge have relatively accuracy of about 1.0

- Durand et al. WRR 2016 showed mixed results for blind tests across 19 rivers.

- Ongoing, new version of this experiment using more robust algorithms shows relative uncertainty about 0.67

Assume:

\[
\left( \frac{\sigma_Q}{Q} \right)_{McFLI} \approx 0.67
\]

Results across 17 rivers show that for 10-day sampling MetroMan results have typical (median) performance of normalized RMSE = 0.67
Parameter error Part 2: Integrators and “Toy” Demo

Because the integrator is linear, there is an analytical estimate of its uncertainty. Yay!

Example: one confluence, three rivers

<table>
<thead>
<tr>
<th></th>
<th>$Q_{True}$</th>
<th>$Q_{McFLI}$</th>
<th>$Q_{SWOT}$</th>
<th>$\sigma_Q Q^{-1}_{par}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>122</td>
<td>128</td>
<td>0.48</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>99</td>
<td>150</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>319</td>
<td>278</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Rewrite integrator as:

$$\begin{bmatrix} Q_{SWOT} \\ \lambda \end{bmatrix} = M \begin{bmatrix} Q_{McFLI} \\ 0 \end{bmatrix}$$

where $M = A^{-1}B$

Estimate SWOT parameter error:

$$\hat{P} = M \tilde{P} M^T$$

where:

$$\tilde{P}_{ij} = \begin{cases} \sigma_{Q_{i,McFLI}}^2, & \text{if } i = j \\ \rho Q_{i,McFLI} Q_{j,McFLI}, & \text{otherwise} \end{cases}$$

Random, ±0.67 $Q_{True}$
Implementation Details

\[
\left( \frac{\sigma_Q}{Q} \right)^2 = \left( \frac{\sigma_Q}{Q} \right)_{Par}^2 + \left( \frac{\sigma_Q}{Q} \right)_{Obs}^2 + \left( \frac{\sigma_Q}{Q} \right)_{Mod}^2
\]

- Require global topologically-consistent dataset, with width and slope. Used Hydro1K, with tweaks and modifications following Pavelsky et al. 2014. Only consider widths > 100 m
- Applied integrator iteratively, starting from mainstem, and working to smaller rivers
- Contrary to previous slide, assumed small (0.25) correlation coefficient between McFLI errors (conservative)
- Analysis global, except four areas where Hydro1K is poor: Amazon and Parana basins, Australia, and central Asian endorheic basins
- Assumed no McFLI possible for a) multi-channel rivers and if river < 30 km in length. Assumed 1/4 of rivers north of 60° are multi-channel. No McFLI \( \sigma_QQ^{-1}_{par} = 1.0 \).
Results: Mississippi Basin

Small rivers on big mainstems have highest uncertainty

Parameter uncertainty usually dominates (>80% of variance)

Uncertainty decreases as you go downstream
Results: Mississippi vs Eastern US

Compared to Mississippi, far less connected, so integrators help less. Median uncertainty for Eastern US rivers is 0.53, compared to 0.44 for Mississippi.
Results: Global* Patterns

- Median discharge uncertainty is 0.49. Note: Agrees with Tuozzolo et al., OS53C-1343.
- Dominated by parameter uncertainty
- Uncertainty is lower downstream in large river basins. It is highest for small tributaries coming into large mainstems: this leads to the “long tail”: 10% of rivers have $\sigma_Q Q^{-1} > 0.67$. 

Part 3: Discharge Uncertainty Analysis
Results: What if we used integrators to combine real-time (USGS, ECCC etc.) gages and SWOT?

Northwest Coast, including Yukon

Method: assume we can estimate parameter discharge uncertainty to 0.10
Results: What if we used integrators to combine real-time (USGS, ECCC etc.) gages and SWOT?

Northwest Coast, including Yukon

Method: assume we can estimate parameter discharge uncertainty to 0.10

Part 3: Discharge Uncertainty Analysis
Results: What if we used integrators to combine historical (GRDC) gages and SWOT?

Part 3: Discharge Uncertainty Analysis

Method: assume we can estimate parameter discharge uncertainty to 0.30

Northwest Coast, including Yukon

Medians:

- 0.44 vs 0.45
- 0.45-0.67
- 0.67-0.8
- >0.8
- flat
Summary & Outlook

• SWOT discharge uncertainty global distribution has median ~50% & long tail.

• “Integrators” are critical: talk by Colin Gleason (OS51A-08) next, and poster by Steve Tuozzolo (OS53C-1343). Integrators can leverage historical or real-time gages, or just enforce continuity.

• Future: Incorporate lateral inflows uncertainty. See poster OS53C-1346 by Cassie Nickles (Northeastern University)

• Future: Use global, high-resolution hydrography, & include layover errors. See SWOT river database poster, OS53C-1345 by Elizabeth Altenau (UNC)
Acknowledgments

...and Shameless Plug)

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Extra Slides

“Murder your darlings.” — Sir Arthur Quiller-Couch
Correlation between parameters. Does equifinality matter?

\[
Q = \frac{1}{n} (\bar{A} + A')^{5/3} W^{-2/3} S^{1/2}
\]

\[
\left( \frac{\sigma_Q}{Q} \right)^2 = \left( \frac{\sigma_Q}{Q} \right)_{Par}^2 + \left( \frac{\sigma_Q}{Q} \right)_{Obs}^2 + \left( \frac{\sigma_Q}{Q} \right)_{Mod}^2
\]

\[
\left( \frac{\sigma_Q}{Q} \right)_{Par}^2 \approx \left( \frac{\sigma_n}{n} \right)^2 + \left( \frac{5}{3} \frac{\sigma_{\bar{A}}}{\bar{A} + A'} \right)^2 - 2\rho \left( \frac{5}{3} \frac{\sigma_{\bar{A}} \sigma_n}{(\bar{A} + A')n} \right)
\]

From Yoon et al. WRR 2016
Observation Errors from Sacramento River SWOT simulation

- Combines 6-month hydraulic simulation with JPL SWOT Science Simulator (1051 build), and RiverObs

- Height errors: 12.5 cm RMSE. Width errors: 4.3 m RMSE. Slope errors: 1.1 cm/km RMSE.

- Caveats: does not include dark water, riparian vegetation, or phase unwrapping errors.

- Includes layover, instrument (i.e. roll), and wet troposphere correction errors
Integrators: Discharge uncertainty can go up, but not by much

- For the three-river scenario:
  - If mass is conserved in the inputs, then you can express uncertainty as a function of the ratio between tributary and mainstem
  - This expression has a limit: about a 20% increase in uncertainty for small tributary entering a large mainstream
  - To achieve the conservation requirement, apply integrator iteratively

\[
\sigma^2_{Q_{1,SWOT}} = \frac{\sigma^2_{Q_1} \sigma^2_{Q_2} + \sigma^2_{Q_1} \sigma^2_{Q_3} + \sigma^2_{Q_1} \left( \sigma^2_{Q_2} + \sigma^2_{Q_3} \right)}{\sigma^2_{Q_1} + \sigma^2_{Q_2} + \sigma^2_{Q_3}}
\]

\[
\lim_{Q_2/Q_1 \to \infty} \sigma_{Q_{1,SWOT}} = \sqrt{\frac{3}{2}} \sigma_{Q_1}
\]
Mass-conserved sets of reaches (McSets)

- SWOT produces height, width and slope for river reaches: discrete units ~10 km in length
- McFLI algorithms are run on McSets, generally at least three reaches over which mass is conserved
Mississippi Basin with USGS gages on SWOT rivers (n=617)

Observation & model error now dominates sometimes

Median: 0.18
Is 50% SWOT discharge error too large?

- Land surface models run in areas with poor precipitation data, and no discharge calibration data, can have significant uncertainty.
- Additionally, SWOT discharge anomaly will be far more precise than absolute discharge (Durand et al., JSTARS, 2010).

Discharge RMSE, %

C. Emmery et al., HESS, 2018
Observation error is small compared to parameter error

Long obs tail due to flat reaches…

… only affects highest quantiles
Slide on flat reaches

$$\left( \frac{\sigma_Q}{Q} \right)_{obs}^2 \approx \left( \frac{5}{3} \frac{\sigma_H \sqrt{2}}{c \left( Wa^{-1} \right) \frac{d}{b}} \right)^2 + \left( \frac{2 \sigma_W}{3 W} \right)^2 + \left( \frac{1 \sigma_S}{2 S} \right)^2$$

- For low-slope rivers, slope uncertainty will generally stay the same, leading to large discharge uncertainty due to slope. Slope < 3.5 cm/km will lead to significant discharge uncertainty.

- For low-slope rivers, will need to use rating curve on height alone, not Manning’s equation, or some other strategy.

- Should still be able to include in integrators. However, observation error budget is not yet known