The Independence of Language and Number

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1. Introduction

In this paper we will argue that grammar and number are distinct mental faculties, and that number cannot be bootstrapped from language. Two recent proposals, Bloom (1994) and Hurford (1978, 1987), argue that the number faculty is dependent on the language faculty. Hurford’s position is that with the correct formulation of recursive phrase structure rules, well-formedness conditions, and particular assumptions about semantic interpretation, the entirety of numerical cognition can be derived from grammar, as illustrated in the quote in (1).

(1) ...the number faculty largely emerges through the interaction of central features of the language faculty with other cognitive capacities relating to the recognition and manipulation of concrete objects and collections. The relevant features of the language faculty include the pairing of words with concepts by the linguistic sign (à la Saussure) and highly recursive syntax. It is therefore not necessary to postulate an autonomous ‘faculty of number’ as a separate module of mind. Hurford, 1987 (p. 3).

In Bloom’s view, the number faculty receives its property of discrete infinity through a bootstrapping process from the grammar of the count routine. Specifically, he argues that initially children do not understand the notion of cardinality of set size. They come to know the count routine, but initially fail to acquire the knowledge that it is infinite in principle. Bloom points out that the count routine (that is, 1 cat, 2 cats, 3 cats...) uses grammar, which we know has discrete infinity through processes like relativization for syntax and compounding for morphology. Hence, if children learn the linguistic count routine before they understand the numerical representations it stands for, they have "a way in" or a bootstrapping mechanism for transferring discrete infinity from grammar to number. This view is illustrated in (2).

(2) ...in the course of development, children ‘bootstrap’ a generative understanding of number out of the productive syntactic and morphological structures available in the [linguistic] counting system. Bloom, 1994 (p. 186).

We will argue against both of these views: first, by presenting evidence of a double dissociation of grammar and number during development, second, by exploring the plausibility of the argument that systemic properties can be bootstrapped across cognitive domains.
2. Dissociations Of Language And Number in Outageny

First we turn to developmental dissociations between language and number. Bloom's developmental argument holds that recursion in the number faculty is bootstrapped from recursion in the linguistic grammar of the count routine. There are several cases of developmental dissociations of the two systems providing clear evidence that such bootstrapping does not take place. In one direction, there are accounts in the literature of mentally retarded individuals who demonstrate intact grammatical development yet possess limited or no ability to calculate and perform basic arithmetical principles. Clearly, such evidence also argues against Hurnford's claim that adult state numerical competence could be derived from language.

Though there is no relevant case for lack of space we will limit ourselves here to the case of Antony. Antony showed a striking developmental dissociation between grammar and number. Antony's language indicated normal grammatical development but his cognitive performance evidenced no numerical knowledge at all. At the age of 7 years, Antony functioned cognitively at a level of approximately 18-24 months. He could not dress himself, could not draw representationally, and more pertinently, could not count, could not demonstrate an understanding of the concepts "more" or "same," nor even an ability to differentiate sets of 2 items from sets of 3, 4, or 5 items. Yet, throughout his developmental history of marked retardation, he showed surprising linguistic growth, reportedly producing 2 and 3-word utterances at age 2, and full sentences at age 3. Antony's language clearly possessed the properties of discrete infinity and recursive enumeration, as illustrated by sentences containing small clauses, embedded participial clauses, infinitival clauses, WH-complement clauses, and relative clauses, shown in (3)-(7).

(3) Jeni, will you help me draw pictures of Susie? [small clause]
(4) I don't want Bonnie coming in here [participial clause]
(5) He wants to chase the cat [infinitival clause]
(6) I don't know who he got [WH-complement with object-extraction]
(7) A stick, that we hit peoples with [relative clause with object extraction]

Antony, as well as others, displayed a striking disparity between knowledge of number and knowledge of grammar, supporting a position which holds that these are developmentally autonomous faculties. Even stronger evidence that the number faculty is developmentally autonomous from the language faculty is found perhaps, comes from individuals displaying disparities in the other direction; that is, individuals with developed number faculties; i.e., who know how to perform arithmetic operations such as counting and multiplication, possessing language. A number of such cases are documented. The double dissociation of these two faculties suggests that number and language are separate faculties in the adult state, contra Hurnford. The existence of Antony in which numerical cognition appears to have developed normally in the absence of a grammar, however, pointedly argues against Bloom's proposal that the discrete infinity found in number comes from grammar.

Chelsea is one individual who shows such a dissociation. A hearing-impaired woman who grew up in a then small, rural community, without learning any natural language, Chelsea was "discovered" in her early thirties and has been the subject of much habilitation, instruction, and study (Curtiss, 1988; 1994; 1996; Glusker, 1987; Dronters, 1987). With aids, her hearing falls within the normal range, and she now possesses a substantial spoken, sign, and written vocabulary, which continues to increase. However, after 13 years of language instruction and exposure, she still does not possess the rudiments of natural language grammar, such as knowledge of phrasal or clausal structure, recursive rules, morphological features, even syntactic properties such as the C-selection features or theta structure requirements of words which have long been in her productive vocabulary. Note, for example, the sample utterances in (8)-(12), all constructed of words which have been in her productive vocabulary for years.

(8) Missy girl same both girl (1987) [comparing the gender of 2 animals]
(9) Cat chasing cat (1992) [She had been asked: What is the cat chasing? Answer: A dog.]
(10) Fort Bragg Fort Bragg L.A. your (1992) [a comment about where we were each from]
(11) P. broken. see F. (1995) [P's car had broken down. Chelsea could see that P was nervous]
(12) I me pay money I mess money G (1995) [C. paid G money, situation unclear]

The persistent absence of the basic structural principles of grammar suggests an inability of the language faculty to develop or be instantiated in relevant respects, at this stage in her cognitive-neurological life. Yet, despite the absence of the properties of language which Bloom suggests are relevant for triggering the development of the number faculty, Chelsea can perform all basic mathematical operations. She can perform such operations in her head as well as on paper. She understands and uses money correctly and even balances a checkbook! (Glusker, p.c.). As far as can be determined, she acquired all of this knowledge as an adult. What's more, she uses number words and expressions correctly, as illustrated in (13)-(17). She can tell time. She can talk about specific times, as in (16), and can talk about numbers and money, such as in discussing the costs of things, as in (17).

(13) (Pretending she's the tester) How many apples? Seven apples
(14) (C is staying in a house with 3 bathrooms, but has seen only the 2 upstairs. She is downstairs, speaking to J, one of the inhabitants)
C: I go bathroom (C turns away and starts to go upstairs. J calls after her)
J: There's a bathroom down here! (C turns around. J points)
C: Three. Three bathroom.
(15) Baby. Hive 2 (re my having two children)
(16) Go work 8:30
(17) (Re needing to buy a new battery for her hearing aid.)
C: Change. Throw away. Battery no good. Pay less.
S: How much do they cost?
C: Three dollar. Pay less. Fifty cent. (She only paid $2.50.)

Most importantly, in performing calculations on or reading numbers in the hundreds or thousands, she uses number words and expressions that require numerical computation. Thus Chelsea is capable of numerical cognition in the absence of a linguistic grammar.

Taken together, the cases cited demonstrate a double dissociation between knowledge of language and number. In the tradition of cognitive neuropsychology a double dissociation between two mental faculties constitutes clear evidence that they are distinct, autonomous systems. The double dissociation between language and number evidenced in these and other similar cases, then, constitutes strong empirical evidence of the developmental autonomy of the two faculties. In particular, the case of Chelsea shows that knowledge of number is not dependent upon knowledge of grammar, since Chelsea displays normal numerical cognition but no recursive syntax. These results suggest that accounts which assume a total or developmental dependency of numeracy on grammar are mistaken.

Now we turn to the question of what bootstrapping a property across mental domains might mean and what the plausibility of such a proposal is for bootstrapping discrete infinity from the syntax of counting into the domain of number. First let us examine what we mean by discrete infinity.

3. Recursion

Language and number both have the property of discrete infinity, as has been observed on numerous occasions. Of language, Chomsky (1955) observed that this property could be formalized using recursive function theory. In a recursive system, many structures may be built up from finitely many discrete units because the rules or principles of the system allow structures to recur within one another, or be properly included within one another, without limit.

In language, recursion is pervasive. Recursion is possible through nearly every major phrasal category. While such recursion has traditionally been formalized in phrase structure rules, the same properties can be captured in more recent formulations.

For counting, the property of discrete infinity might be represented in terms of a successor function s, such that for any number n, s(n) is n+1. Using such a function, (18) builds (unary) recursive structures, each properly included in its predecessor, as in (19).

(18) Recursion in the counting system

| l | is a number,
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<td>i</td>
<td>If n is a number, so is s(n)</td>
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(19) |

- s(5) = 6
- s(4) = 5
- s(3) = 4
- s(2) = 3
- s(1) = 2

Notice that discrete infinity, in this case and in all cases, is epiphenomenal of a system of rules. It emerges because the rules of the system have a particular character, namely, their form is that of a recursive function which builds primitives into particular structures. Hence, the existence of such a rule system is logically prior to the emergence of the property of discrete infinity. While there has been some success in the cognitive sciences in characterizing such systems in a number of cognitive domains (for instance, vision, language, number and music), there is no known operation which can derive a new discretely infinite rule system from another discretely infinite rule system as a result of the original system possessing this abstract, epiphenomenal property. The observation that two systems share a common property may lead us in some cases to speculate about a common evolutionary path, but this fact alone in no way suggests that one system is derived from the other in the course of ontogenesis. Many objects in the natural world share properties (any two organs of the human body, for instance), but such observations do not generally lead us to think that there is a developmental interdependence of some kind between them. With the epiphenomenal nature of discrete infinity in mind, let us now address more specifically the question of whether such a property could be bootstrapped across domains.

4. Modularity and Bootstrapping Theories

There are a number of different criteria used to define mental domains or autonomous mental faculties. We will concentrate on two: domain-specificity and informational encapsulation. We follow Fodor (1983), as well as Jackendoff's (1972) refinement of Fodor's conception of informational encapsulation and domain-specificity.

Jackendoff proposes a refinement of Fodor's (1983) formulation, taking the domain specificity of mental modules to be "a consequence of the formal representations they operate on," so that "modularity of processing is determined precisely by the forms of the mental representation being processed" (1992: vii).
Jackendoff’s representation-based notion of modularity permits us to make precise claims about interface relations that hold between domains. Extracting from recent work by Kenstair and Stabler (1994) on general grammars, we take a mental faculty to be as in (20), a fourtuple \( MF = \langle V, C, L, F \rangle \)

(20) \( MF \) stands for Mental Faculty. \( V \) is a terminal vocabulary (such as acoustic arrays, retinal arrays, or numerons, depending on the faculty in question). \( C \) is a set of categories: (Verb, Noun, Preposition, Adjective, Adverb) in the case of language, (edge, angle, color, bore...) in the case of vision. \( L \) is a set of paired expressions \( \langle V, C, L, F \rangle \) (terminal vocabulary with its category), and \( F \) is the set of structure building (partial) functions which takes tuples of structures to other structures. Thus, the set of expressions defined by such a system is the lexicon \( L \) plus everything which can be built using the generating functions \( F \). The resulting set of expressions is the closure of the lexicon under the structure building functions, as in (25): \( E(MF) = Cl(L, F) \).

(21) \( V \) is a terminal vocabulary (acoustic arrays, retinal arrays, or numerons)
(22) \( C \) is a set of categories (V, N, P, Adj, Adv...
(23) \( L \) is a set of paired expressions \( \langle V, C, L, F \rangle \)
(24) \( F \) is the set of structure building (partial) functions which takes tuples of structures to other structures
(25) \( E(MF) = Cl(L, F) \)

This system captures the property of domain-specificity for mental faculties because the structure building functions \( F \) can only operate on the lexicon of a single mental domain. Hence, the successor function operates on a numerical concept to render its successor. It makes no sense, from the perspective of natural languages, to speak of the successor function, or of any other operation or principle peculiar to the number faculty, as operating on (say) a syntactic string to compute its meaning, structure, or even its length.

In addition, the more formal definition of mental faculty or domain also captures the property of encapsulation, since it does not allow structures in \( E(MF) \) (all the expressions of \( MF \)) to be built by applying \( F \) to \( L \). In other words, because the input values in different domains are different types of objects (morphological features and structural representations in the case of language, sets and numerons in the case of number), they cannot be used by a single “general-purpose” function to build structures. There is no operation in \( F \) that can use elements of \( L \) to construct expressions of \( MF \). These operations are domain-specific and the domains are informationally encapsulated.

Notice, too, that although the set of expressions built in this way is discrete and infinite for both number and language, discrete infinity is a property of the respective r e systems. The claim that this property is in some way “extracted” across domains, therefore, must be a claim about “extracting” objects and rules across domains. Moreover, in addition to there being no known operation whereby the property of discrete infinity could flow from one domain into another, there is the additional problem that by virtue of the domain-specificity and informational encapsulation of mental faculties, objects and rules of one domain are not equipped to operate in another. Thus, even if such an operation existed, the donor faculty could not provide the other with useable mechanisms.

5. Bootstrapping Theories

Now, bootstrapping theories, attempt to explain gaps between inputs and outputs by positing a particular “coordination” of information across mental modules, but never across mental faculties or domains. Bootstrapping theories map modules onto modules within domains, for example, the components of grammar. It is reasonable to propose such theories since there is a clear interaction between modules of the grammar. As reflected in the diagram in (26), one grammatical module serves as the input or output of another.


While language and number appear to have in common the property of discrete infinity, there is no known input/output relation to directly link these two systems formally. This is exactly what should be expected, in virtue of domain-specificity and informational encapsulation. Rather, as in the case of other concepts, it is only the lexicon that links language to the external world of numerons.
6. Language-Number Interface Conditions

While we have suggested that it is impossible for their to be direct contact between number and grammar in the way necessary to allow bootstrapping to take place as it does between syntax and phonetics, for example, we do not deny that there is any contact between the two domains. So counting, for example, is a process which clearly recruits the resources of both domains, among others. Thus it would appear that there must exist an interface between number and grammar, in the same sense in which Landau and Jackendoff (1993) suggest that there exists an interface between spatial language and spatial cognition. In this regard, we find that the grammar of the count routine differs strikingly from the grammar of what we might call clausal syntax. The significance of this fact is that these differences likely reflect properties of the number domain, rather than the other way around.

For example, if you count five women at a bus stop using the count routine, as in (27), you assign an order to that set of women, in addition to calculating their cardinality.

(27) one woman...two women...three women...four women...five women

However, if you use clausal syntax to express the idea that there are five women at a bus stop, as in (28) - (30) one generally cannot express this same relationship without a great deal of circumlocution, although the cardinal value of the set is easily expressed in these simple clauses.

(28) I saw [five women] at the bus stop.
(29) The five women with blue hats] stood at the bus stop.
(30) The bus crashed into [all five women] at the bus stop.

We would like to suggest that this fact follows from properties of the number domain, as expressed by grammar. The idea would be that the number domain carries out a computation of the kind illustrated with the successor function in (18). The result is then encoded into an intermediate, "higher order" representation which can be interpreted by the grammatical system as a lexical item, namely a quantifier. This quantifier can then participate in the computations of the grammatical domain to produce clausal utterances such as those in (28) - (30). However, it can also participate in the kind of computation which produces (27). The count routine uterrache represented in (27) appears to have fundamentally different properties than the clausal utterances in a number of ways. The fact, just mentioned, that these representations carry cardinal as well as ordinal meaning is an example of such a property. This appears to be a numerical property because counting is a serial process. It is a property of the successor function in (18) that each successive representation is computed by using the previous representation as an input. Thus cardinality is simply a property of the successor function.

Another property particular to count routine DPs is that they are universally quantified. This follows from a property of the number faculty which Gelman and Gallistel (1978, 1992) refer to as the Cardinality Principle. The idea is that the last number in the count sequence gives you the cardinality of the set. This means for grammar that at every step in the counting routine, as in (31), in fact, the number given, universally quantifies over the set of objects counted up to that point. This is the exact opposite of what numeral quantifiers do in clausal syntax, in which they existentially quantify over the unit set, as in (32).

(31) one orange...two oranges...three oranges...four oranges...
(32) John ate four oranges.

That is, in (32) the existential quantification of the DP merely tells us that there exist some four oranges which John ate. The "four oranges" in the counting routine, however, have all been included in a set that is quantified over exhaustively.

7. Conclusion

In conclusion, we have argued that the number faculty is neither derivative of the system of grammar, as Burford maintains, nor does its development depend upon any bootstrapping relation with the language faculty, as Bloom maintains. If number depended on language in either of these ways, the double dissociation which obtains between these two mental faculties in the subjects described here should not exist.

Second, the notion of cross-faculty bootstrapping is difficult to make sense of. While it makes no sense to talk of epiphenomenal properties of rule systems floating from one domain to another, it makes even less sense to suggest that lexical and computational objects could be transferred directly from one domain to another in the light of domain-specific conceptions of mental architecture such as Jackendoff's. There is no other sense in which a property of one domain could be transferred to another.

On the other hand, it appears that an interface between the two domains appears possible and, that at some higher level of integration, cognitive architecture allows insertion of lexical items into the computational system of grammar which carry properties particular to the numerical domain with them. Thus, the lexicon allows quantifiers and nouns to express properties of numerical domain. While this "locally" properties of the products of certain computations, it does not appear to hold of entire cognitive processes, such as the successor function, or their properties, such as discrete infinity, such that they may move from one domain to another.

Endnotes

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1. Chelsea has learned a rule for marking plural on nouns and does so, although not consistently. However, subject-verb number agreement marking is neither comprehended nor produced, nor is number marking on pronouns. Thus, her plurals are not a reflex of grammatical agreement.

References


