Use of Stokes Parameters in Hard X-Ray Polarimetry

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Scientific Motivation

• Binary Black Holes (Schnittman & Krolik 2010)
  – Probe accretion disk inclination and spin
  – The shape of the x-ray corona

• Pulsars
  – Discriminate between physical processes—curvature vs. synchrotron emissions (different phase morphology)
  – Constrain/ determine shape of pulsar beam (oscillations in amount of x-ray polarization shape dependent)

• Active Galactic Nuclei (AGN) and their Jets (Krawczynski 2012)
  – Information about magnetic field near jet base (hard x-ray emission in uniform field near base)
X-Calibur Principles

Photons interact with materials by Compton scattering (low-Z) or photo absorption (hi-Z).

X-Calibur combines both. Low-z Scintillator Compton scatters the incident photon; hi-Z CZT detector photo-absorbs it.

This scattering is governed by the Klein-Nishina Cross Section

\[
\frac{d\sigma}{d\Omega}(\theta) = \frac{r_0^2 k_1^2}{2 k_0^2} \left[ \frac{k_0}{k_1} + \frac{k_1}{k_0} - 2\sin^2\theta \cos^2\chi \right]
\]

Incident photon wave-vector

Scattered photon wave-vector

Angle between \( k_0 \) and \( k_1 \)

Angle between E-field vector and scattering plane. (azimuthal angle)

Stokes Parameters
CZT detector panel

ASIC board (read-out CZTs)

4 ASIC boards around the scintillator (CZTs face inwards)

InFOCµS gondola with 8m optical bench (Tueller et al., NASA GSFC)

255 shell Al mirror with 50 cm² area at 30 keV (Pt/C coating)
Characterizing Polarization

- Polarization—oscillations in the plane perpendicular to a wave’s direction of travel
- Can be linearly or elliptically polarized
  - Cannot measure elliptical polarization with apparatus like X-Calibur
  - That’s ok, galactic and extragalactic objects exhibit linear polarization mostly, anyway (Krawczysnki 2012)

- Linear polarization characterized by polarization angle and fraction
- Fraction ($\pi$)—portion of the wave that is polarized
- Angle ($\chi$)—the angle of the projection of the polarization vector relative to the vertical axis (in the case of polarimetry)
- Modulation factor ($\mu$)—defined as the observed polarization fraction for a 100% polarized beam
  - Dependent on physics of Compton scattering and design of polarimeter
  - For X-Calibur, $\mu \sim 0.5$
Astrophysical source has some unknown polarization \((\pi_{\text{true}}, \chi_{\text{true}})\)

We have limited experimental statistics (cannot observe the source forever!)

Even in a beam with 0 polarization fraction, always measure something; \(\pi_{\text{computed}}\) must be \(\geq 0\)

Therefore, for a small \(\pi_{\text{true}}\), distribution of measured fraction is not centered about the true value

Using Bayesian statistics:

- Find better guess \((\pi_{\text{guess}})\) of \(\pi_{\text{true}}\)
- Create probability distribution for this \(\pi_{\text{guess}}\)
- Use the probability distribution to set error bars on a \(\pi_{\text{guess}}\) given \(\mu\)
Analysis Techniques—Stokes Parameters

• Unique to my project: propose analyze the signal photon-by-photon, using a Stokes decomposition
• Decompose wave into its intensity, horizontal, vertical, and circular polarization components
  – We only work with the first three

For single photon, have azimuthal scattering angle ($\chi$), define $q$ and $u$ (the horizontal and vertical components of polarization):

$$q = \cos(2\chi) \quad u = \sin(2\chi) \quad i = n$$

For a collection of photons, define fractional degree of polarization ($\pi_0$) and polarization angle ($\chi_0$):

$$\pi_0 = \frac{2}{\pi} \frac{\sqrt{Q^2 + U^2}}{I} \quad \tan(2\chi_0) = \frac{U}{Q}$$
$$Q = \sum_j q_j \quad U = \sum_j u_j \quad I = \sum_j i_j = N$$
Analysis Techniques—Statistics

- I simulate a source with \((\pi_{\text{true}}, \chi_{\text{true}})\).
- Model the detection of these photons in a generic detector with modulation factor \(\mu\).
- Compute best guess for true polarization, call it \((\pi_{\text{guess}}, \chi_{\text{guess}})\).

The probability of measuring a random \((Q, U)\) given we know the true values \((Q_0, U_0)\).

\[
P(Q, U \mid Q_0, U_0) = P(Q \mid Q_0)P(U \mid U_0)
\]

\[
= \frac{1}{2\pi\sigma^2} \exp \left[ - \frac{(Q - Q_0)^2 + (U - U_0)^2}{2\sigma^2} \right]
\]

\[
P(\pi, \chi \mid \pi_0, \chi_0) =
\]

\[
= \frac{1}{2\pi\sigma^2} \exp \left[ - \frac{\pi_0^2 + \pi^2 - 2\cos[2(\chi - \chi_0)]}{2\sigma^2} \right]
\]
This is $P(\pi, \chi \mid \pi_0, \chi_0)$.  
Want $P(\pi_0, \chi_0 \mid \pi, \chi)$.  

Apply Bayes’ Theorem to “invert” the distribution.  

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

$$P(\pi_0, \chi_0 \mid \pi, \chi) = \frac{P(\pi, \chi \mid \pi_0, \chi_0)P(\pi_0, \chi_0)}{\int_0^\pi \int_0^\pi d\pi_0 d\chi_0 [P(\pi, \chi \mid \pi_0, \chi_0)P(\pi_0, \chi_0)]}$$

The prior (what we assume we know about the polarization of the signal).  
For me, $=1$ (know nothing, worst case scenario)

$P(\pi_0, \chi_0)$
Methods
Steps 1→3: Simulating Measurement Runs

1- Generation of photons for one fixed ($\pi_0, \chi_0$), and given $\mu$, according to Klein-Nishina.

2-Extraction of polarization parameters ($Q, U, \pi, \chi$) for each measurement run.

3- Simulate many measurement runs (repeat 1-2), plot the distribution of ($\pi, \chi$)

\[ \pi_0 = 0.5, \ \chi_0 = 0, \ \mu = 0.5 \]

500 events
1000 measurements
Methods

Steps 4→5: Evaluation for Many $\pi_0$ and Normalization

3- Repeat steps 1-3 (generation of a “slice” of $\pi_0$) for all $\pi_0$, $0 \rightarrow 1$.

4- Normalize each slice (so sum of all probabilities is 1).

$\pi_0 = 0.5$, $\chi_0 = 0$

5000 events, 10000 measurements

y-projection (down the pol angle axis)
Methods
Steps 6 → 7: Inversion and Interval Extraction

6- Invert the 3-D distribution according to Bayes’ Theorem (is a rank 4 tensor, stored in an N-dimensional sparse matrix)

7- Extract confidence interval for a \((\pi, \chi)\) of interest—tells you with what confidence your measured \((\pi, \chi)\) represent \((\pi_0, \chi_0)\).
Conclusion

• Successfully applied Bayes theorem to generate confidence intervals and set error bars on a measured \((\pi, \chi)\)

• Future research interest and next steps
  – Raising the number of measurements
  – Assuming a non flat prior

Thank You!
Questions?
EXTRA SLIDES
Polarization determined by fitting sine waves to distributions of azimuthal scattering angles, and extracting relevant fit parameters.

Polarization Fraction $\pi = \frac{\text{Amplitude}}{(\text{Mean Value} \times \mu)}$

Minimum = Polarization Angle
Amplitude
Mean Value

Source of the sinusoidal signal in the azimuth: the Klein-Nishina

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{r^2}{2} \frac{k_1^2}{k_0^2} \left[ \frac{k_0}{k_1} + \frac{k_1}{k_0} - 2\sin^2 \theta \cos^2 \chi \right]$$
Analysis Techniques—Stokes Parameters

• Utilizes the “maximum amount of available information”
  – Working directly with the discrete photons avoids information lost when they are binned in the azimuthal histogram

• Avoids pitfalls of binning and fitting
  – Measured fraction distribution systematically broaden as histogram bin number rises
  – Computed fraction and angle depend intimately on the goodness of fit of the wave—problematic for low event numbers

• Note: Should always plot the azimuth to ensure signal is actually sinusoidal; the Stokes cannot tell you that

\[
\mu = 0.5
\]