Ralph’s Regulation

Ralph is a risk-averse owner of an asset. He recognizes that gains to trade are available since there exist risk-neutral buyers for the asset. However, the price that buyers are willing to pay depends on their perception of how well they will be able to manage the asset and this depends on their perception of the reliability of Ralph’s information report regarding the asset. In particular, the gross value of the asset $V$ is normally distributed with expectation 1,000 and variance 100. Ralph’s report $y$ is a noisy (imperfect) signal of $V$, $y = V + \varepsilon$ where $\varepsilon$ is normally distributed (and independent of $V$) with mean zero and variance $\sigma^2$ (inverse of reliability) to be chosen from the set \{125, 128.4, 131.7, 133.5, 139.1, 143\}. Hence, the equilibrium price $P$ that buyers are willing to pay reflects the expected gross value conditional on the report less a discount for remaining uncertainty regarding the buyer’s ability to manage the asset if acquired.

$$P = E[V \mid y] - 7\text{Var}[V \mid y] = 1,000 + 100/(100+\sigma^2)(y - 1,000) - 7*100\sigma^2/(100+\sigma^2)$$

The first two terms are collectively the conditional expectation of $V$ given information signal $y$, and the third term is seven times the conditional variance of $V$ given information signal $y$.

Public reliability choice setting

Ralph chooses $\sigma^2$ from the set \{125, 128.4, 131.7, 133.5, 139.1, 143\} to maximize his expected value of utility. Ralph has mean-variance preferences with increasing utility in

$$E[P \mid \sigma^2] = 1,000 + 100/(100+\sigma^2)(E[y \mid \sigma^2] - 1,000) - 7*100\sigma^2/(100+\sigma^2)$$

and decreasing utility in

$$\text{Var}[P \mid \sigma^2] = 100^2(100+\sigma^2)/(100+\sigma^2)^2 = 100^2/(100+\sigma^2).$$

In particular, Ralph’s expected payoff is $E[P \mid \sigma^2]$ less $2.5*\text{Var}[P \mid \sigma^2]$ (Ralph’s risk premia) less the information production cost which is a function of the reliability choice.

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\( \sigma_2^2 \). The information production cost is \( 0.02(150 - \sigma_2^2)^2 \). Collecting terms, Ralph’s expected payoff in the public reliability choice setting is

\[
1,000 - 7 \times 100 \sigma_2^2/(100 + \sigma_2^2) - 2.5 \times 10,000/(100 + \sigma_2^2)^2 - 0.02(150 - \sigma_2^2)^2
\]

**Private (or regulated) reliability choice setting**

The public information setting is a benchmark (common information) setting. The real issue is how to handle asymmetric information. When reliability is privately chosen (or chosen via a regulator), Ralph knows that the buyers share the above data and (when the reliability choice is not publicly available) will conjecture reliability choice \( \bar{\sigma}_2^2 \) based on these data. In equilibrium, the conjecture equals Ralph’s optimal reliability choice \( \sigma_2^2 \).

That is, Ralph chooses \( \sigma_2^2 \) to maximize his expected payoff as a best response to the buyers’ conjecture while the buyers’ conjecture matches Ralph’s optimal choice \( \sigma_2^2 \) (that each party plays best response to the other party’s best response defines a Nash equilibrium – a “fixed” point where neither party has incentives to alter strategy).

\[
1,000 - 7 \times 100 \bar{\sigma}_2^2/(100 + \bar{\sigma}_2^2) - 2.5 \times 10,000/(100 + \bar{\sigma}_2^2)^2 - 0.02(150 - \bar{\sigma}_2^2)^2.
\]

Required:

1. Suppose the reliability choice \( \sigma_2^2 \) is publicly available. Determine Ralph’s optimal choice for \( \sigma_2^2 \) and Ralph’s expected payoff.

Hereafter, the reliability choice \( \sigma_2^2 \) is private (or regulated) and \( \bar{\sigma}_2^2 \) is conjectured by the buyers. In equilibrium, \( \sigma_2^2 = \bar{\sigma}_2^2 \).

2. Consider the private reliability choice setting (Ralph privately selects reliability of the information report and buyers conjecture Ralph’s choice). Suppose buyers conjecture \( \bar{\sigma}_2^2 = 128.4 \). Determine Ralph’s optimal reliability choice. Is this an equilibrium – is \( \sigma_2^2 = \bar{\sigma}_2^2 \)? Check the remaining possible conjectures \( \{125, 131.7, 133.5, 139.1, 143\} \). Which, if any, are equilibria? What is Ralph’s expected payoff for the equilibrium reliability choice?
3. Suppose reliability is regulated (perfectly). The regulator chooses a reliability level $\hat{b}$ and the seller conforms by choosing $\sigma_2^2 = \hat{b}$. What is the regulated reliability level that maximizes Ralph’s expected payoff? (Hint: regulation is publicly observed and works perfectly in this setting.)

4. Continue the regulated setting. However, recognize that reporting discretion is necessary for Ralph to be able to convey information via his report and discretion frequently has unintended consequences. In particular, Ralph is able to mimic compliance with the regulation by transaction redesign while actually selecting a less reliable (higher variance) report. Transaction redesign is costly, in particular, the cost of redesign is proportional to the squared difference between the actual reliability choice $\sigma_2^2$ and the regulation $\hat{b}$ when $\sigma_2^2 > \hat{b}$, that is the cost of transaction design equals $0.02(\sigma_2^2 - \hat{b})^2$. Suppose the regulator chooses $\hat{b} = 128.4$. Evaluate the possible conjectures $\{125, 128.4, 131.7, 133.5, 139.1, 143\}$ to find Ralph’s equilibrium choice for $\sigma_2^2$. What is Ralph’s expected payoff at the equilibrium reliability choice?

5. As the equilibrium reliability choice in 4 is above the regulated level, the regulator may be inclined to tighten the regulation to, say, $\hat{b} = 125$. What is Ralph’s equilibrium response (again check all conjectures)? How does tightening the regulation impact Ralph’s welfare?

6. Compare Ralph’s welfare (expected payoff) for the above settings? Can regulation enhance Ralph’s welfare? Is more stringent regulation preferred? What does this suggest about the “neutrality” objective (emphasis on relevance and reliability to the exclusion of other finer details) of the FASB’s conceptual framework, or the welfare effects of “fire-dousing” responses like the Sarbanes-Oxley act?