Chapter 7
Causality in Superluminal Pulse Propagation

Robert W. Boyd, Daniel J. Gauthier, and Paul Narum

7.1 Introduction

The theory of electromagnetism for wave propagation in vacuum, as embodied by Maxwell’s equations, contains physical constants that can be combined to arrive at the speed of light in vacuum $c$. As shown by Einstein, consideration of the space–time transformation properties of Maxwell’s equations leads to the special theory of relativity. One consequence of this theory is that no information can be transmitted between two parties in a time shorter than it would take light, propagating through vacuum, to travel between the parties. That is, the speed of information transfer is less than or equal to the speed of light in vacuum $c$ and information related to an event stays within the so-called light cone associated with the event. Hypothetical faster-than-light (superluminal) communication is very intriguing because relativistic causality would be violated. Relativistic causality is a principle by which an event is linked to a previous cause as viewed from any inertial frame of reference; superluminal communication would allow us to change the outcome of an event after it has happened.

Soon after Einstein published the theory of relativity, scientists began the search for examples where objects or entities travel faster than $c$. There are many known examples of superluminal motion [1]. One example arises when observing radio emission in certain expanding galaxies known as superluminal stellar objects. This motion can be explained by considering motions of particles whose speed is just below $c$ (i.e., highly relativistic) and moving nearly along the axis connecting the object and the observer [2]. Hence, these are not superluminal motions after all.

R.W. Boyd (✉)
The Institute of Optics and Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA, boyd@optics.rochester.edu

D.J. Gauthier
Department of Physics, Duke University, Durham, NC 27708, USA, gauthier@phy.duke.edu

P. Narum
Norwegian Defence Research Establishment, NO-2027 Kjeller, Norway, Paul.Narum@ffi.no

DOI 10.1007/978-3-642-03174-8_7 © Springer-Verlag Berlin Heidelberg 2009
Explaining, in simple terms, why apparent superluminal motions do not violate the special theory of relativity or allow for superluminal communication can be exceedingly difficult. Also, approximations used to solve models of the physical world can lead to subtle errors, sometimes resulting in predictions of superluminal signaling. For these reasons, studying superluminal signaling can be an interesting exercise because it often reveals unexpected aspects of our universe or the theories we use to describe its behavior.

One example of apparent superluminal behavior occurs in the transfer of information encoded on optical pulses propagating through a dispersive material. Under conditions where the dispersion of the medium is anomalous over some spectral region, defined in greater detail below, it is possible to observe the peak of a pulse of light apparently leaving a dispersive material before a pulse peak enters! Does such a situation imply that information flows outside the light cone and thus that relativistic causality is violated?

The possibility of such “fast-light” behavior has been known for nearly a century and has been the source of continued controversy and confusion. Yet a rather simple mathematical proof shows that such behavior is completely consistent with Maxwell’s equations describing pulse propagation through a dispersive material and hence does not violate Einstein’s special theory of relativity. While the proof is straightforward, great care is needed in interpreting the special theory of relativity and in determining whether experimental observations are consistent with its predictions.

This chapter reviews the history of fast-light research, describes one approach for understanding how information encoded on optical beams flows through a dispersive material, and describes how these results can be interpreted within the framework of the special theory of relativity. In this chapter we do not discuss the issue of the optical analogue of the tunneling of particles through a potential barrier. It is known that this process can also lead to superluminal behavior, but for reasons quite distinct from the situation treated here, the propagation of light through dispersive media. Superluminal effects based on tunneling have been reviewed recently by Winful [3]; see also “Time in Quantum Mechanics – Vol. 1.”

### 7.2 Descriptions of the Velocity of Light Pulses

Before we delve more deeply into the implications of superluminal propagation velocities, it is crucial to define exactly what we mean by the “velocity of light” in a dispersive material. Because a pulse disperses, its motion cannot be described rigorously using a single velocity. For this reason, there is in fact more than one way to define the velocity of light, depending on what aspect of light propagation is being considered. In this section, we review some of these definitions. Additional discussions of these points can be found in [4] or on page 58 of [5].
1. Oftentimes, the velocity of light is taken to mean the velocity of light in vacuum, the universal constant $c$. Since 1983, $c$ has been defined to have the value 299,792,458 m/s. This is both the velocity at which points of constant phase move through the vacuum and the velocity at which disturbances in the electromagnetic field move.

2. In a material medium the points of constant phase move with the velocity $v_p = c/n$ where $n$ is the refractive index and $v_p$ is called the phase velocity.

3. When the refractive index varies with frequency, the velocity at which disturbances in the field move through the medium will, in general, be different both from the phase velocity and from $c$. One velocity that is often associated with the motion of a disturbance is called the group velocity and is given by $v_g = c/n_g$, where $n_g$ is the group index which is related to the refractive index by $n_g = n + \omega \frac{dn}{d\omega}$. Clearly, the group index differs from the refractive index for a dispersive medium, that is, a medium for which the refractive index is frequency dependent. If the refractive index varies nearly linearly with frequency, at least over the frequency range of interest, the group velocity will itself be frequency independent. In this case, a pulse will propagate with negligible distortion, and the group velocity can be interpreted as the velocity with which the peak of the pulse moves. For the case of a medium with loss, there are some subtleties regarding the interpretation of the group velocity. We shall return to this point in Sect. 7.7.

4. There is also an energy velocity, defined by $v_E = S/u$, where $S$ is the magnitude of the Poynting vector and $u$ is the energy density. Loudon [6] shows that $v_E = c/[n' + n''(\omega/\Gamma)]$ for an absorbing medium comprised of a collection of Lorentz oscillators, where $n'$ and $n''$ denote, respectively, the real and imaginary parts of the refractive index and $\Gamma$ is the transition linewidth. The reason why one must specify the type of medium is that, near resonance, much of the energy density resides in the Lorentz oscillators. While it is not clear from naive inspection, the energy velocity reduces to the group velocity when damping is negligible (that is, when $\Gamma$ goes to zero). Lysak [7] shows that the energy velocity is always less than $c$ for a non-inverted atomic medium. Additional discussions of this topic have been presented by Sherman and Oughstun [8, 9].

5. There are three somewhat related velocities known as the front velocity, signal velocity, and information velocity. Let us suppose that initially the optical field vanishes in all space and at a certain moment of time $t_0$, it is suddenly turned on. The initial turn-on of the field will propagate at a velocity known as the front velocity. It can be shown theoretically that the front velocity is equal to $c$, because the abrupt turn-on must possess extremely high-frequency components that cannot induce a response in the optical medium.

Let us now assume further that, following the front, the source emits a well-defined pulse, and that the field vanishes entirely both before the pulse begins and after the pulse ends. We refer to a pulse of this sort as a signal, and the velocity at which the peak of the pulse moves is known as the signal velocity. Under many practical circumstances, the signal velocity will equal the group velocity.
Any radiation that arrives before the main body of the pulse is known as a precursor. The precursors arrive after the arrival of the pulse front. A precursor consisting of high-frequency components is known as a Sommerfeld precursor, and it arrives at a time determined by the velocity \( c/n(\infty) \approx c \). Any precursor associated with low-frequency components is called a Brillouin precursor, and it arrives at a time determined by the velocity \( c/n(0) \). Here we symbolically represent the refractive index at high frequencies by \( c/n(\infty) \) and the refractive index at low frequencies by \( c/n(0) \).

Finally, the information velocity is the velocity at which the information content of the pulse is transmitted. From a practical point of view, useful information usually arrives at the peak of the pulse, which travels approximately at the group velocity. However, the information velocity is usually defined in terms of the earliest moment at which, even in principle, the information content of a pulse could be determined and hence is associated with the pulse front. This velocity is then associated with the velocity \( c \). This thought can be made more precise by noting that information resides at points of discontinuity of an optical waveform [10, 11], which propagate at the vacuum velocity \( c \) because of their broad frequency content. This result follows from the argument that smooth parts of a pulse cannot carry information, since the future evolution of the pulse can, in principle, be predicted by performing a Taylor series expansion of the pulse amplitude. It is the difference between the practical and precise definitions of the information velocity that has given the greatest confusion regarding the tension between fast-light pulse propagation and the special theory of relativity.

6. There is still another velocity known as the centroid velocity. This approach goes back to Smith in 1970 [4], although the recent advocates have been Peatross et al. [12] and Cartwright and Oughstun [13]. The centroid velocity is the velocity with which the time center of mass moves through the material. The paper by Cartwright and Oughstun is especially interesting. They study the centroid velocity as a function of optical thickness of a material. They show that the centroid velocity is equal to the group velocity for thin media and is equal to the velocity of the Brillouin precursor (that is, \( c/n(0) \)) for thick media.

7.3 History of Research on Slow and Fast Light

Ideas about slow and fast light go back at least 170 years. The distinction between the group velocity and phase velocity was recognized by Hamilton [14] as early as 1839. A full theoretical treatment of the group velocity was presented by Lord Rayleigh [15, 16] in 1877. The classic book Theory of Electrons by Lorentz [17] provides formulas for the refractive index of an atomic vapor. Straightforward evaluation of these formulas shows that the group velocity can become very small (slow light) or very large (fast light). Early in the 20th century people were especially intrigued at the prediction of superluminal group velocities, as such velocities
seemed at odds with the theory of relativity. Work by Brillouin and Sommerfeld helped to resolve this dispute by showing that, even though the group velocity could become superluminal, this did not mean that the front velocity (associated with the turn-on of a wave) could become superluminal. Interest in slow and fast light continued during the 1980s. Significant work during this period included the experiments [18, 19] aimed at verifying earlier predictions.

Many of the procedures that can be used to produce superluminal propagation are in fact variations of methods that were first used to slow down the velocity of light. We therefore review both slow- and fast-light methods in the remainder of this section.

A great impetus for more recent research was the experiment of Hau and coworkers [20] in 1999. This work captured the popular imagination by showing that light could be slowed down to the “human” speed of 17 m/s. The breakthrough in laboratory implementation behind this achievement was the use of electromagnetically induced transparency (EIT). EIT is a quantum coherence effect that induces transparency in a material while allowing it to retain large nonlinear properties. The use of EIT methods was crucial to this study. Without the use of EIT, the transmitted pulse of light, while significantly delayed, would have been attenuated so strongly as to be immeasurably weak. In order to implement EIT, Hau et al. applied both the signal field and a strong coupling field to their atomic medium. This coupling field induces a narrow transparency window that both allowed the signal to be transmitted and created a large spectral variation of the refractive index. The material medium used in the experiment of Hau et al. was an atomic ensemble in the form of a Bose–Einstein condensate. This experiment was soon followed by that of Kash [21], who showed that ultraslow-light speeds could also be obtained in a hot atomic vapor of rubidium. This observation dispelled the notion that the use of ultra-cold atoms was essential to ultraslow-light propagation.

More recently there has been enormous interest in developing additional tools for controlling the group velocity of light. In broad scope, there are two procedures that can be used to control the group velocity. One of these procedures is to make use of material resonances, such as the sharp absorption resonances of an atomic such as those used by Hau et al. Control can be achieved, for example, by applying a strong optical field to modify through nonlinear optical methods the optical response experienced by the signal field.

From a practical point of view, it is desirable to find means for producing slow light that avoid the need to use atomic ensembles held at exotic temperatures. One approach is to make use of slow light based on the concept of coherent population oscillations (CPO). This process is quite insensitive to the presence of dephasing processes, and thus can operate in room temperature solids, media that hold particular promise for use in practical applications. This CPO process leads to a very narrow spectral hole in the absorption profile of a saturable absorber and consequently to a very large value of the group index. Slow light based on the CPO effect was demonstrated first in ruby [22]. Later both slow and fast light based on the CPO effect was observed in an alexandrite crystal [23] and in an erbium-doped fiber amplifier [24].
In addition to EIT and CPO, a wide variety of other sorts of resonances have been used to produce slow-light effects. There has been particular success with the use of stimulated Brillouin scattering [25, 26] and stimulated Raman scattering [27]. In each of these processes, the strong gain feature induced by the presence of a strong pump field will also produce (as a consequence of Kramers–Kronig relations) a rapid spectral variation in the refractive index which in turn leads to strong slow-light effects.

The other procedure for producing slow and fast light is to use structural resonances to modify the optical response. For example, in a photonic crystal the group velocity of light can be slowed dramatically near the edge of the photonic Brillouin zone. This approach has also been quite exciting. One particular example of this approach is the work of Vlasov [28] in producing a group index of 300 by appropriate patterning of a silicon waveguide, with important implications for silicon photonics.

Historically, the greatest challenge in slow-light research has been to find situations in which a data packet could be delayed by many pulse lengths. Delay measured in units of pulse length is often referred to as the delay-bandwidth product. For several years, the largest delay-bandwidth product was limited to the value of approximately five observed in the initial experiment of Hau et al. More recently, a delay-bandwidth product of 80 was observed in one specific situation [29]. Nonetheless, the need to develop methods for producing large delay-bandwidth products at arbitrary wavelengths and for arbitrary pulse lengths remains an active area of research within the slow-light community.

Fast light possesses some similarities but some differences from slow light. First of all, fast light is conceptually very intriguing. People can accept the suggestion that many physical processes can be slowed down to an arbitrary degree. But it is more difficult to accept the suggestion that the same process can be speeded up to an arbitrary degree. For instance, there is no limit to how slowly one can walk across a lecture hall, but there are obvious limits to how quickly one can do so. But what is surprising is that, at a formal mathematical level, there seems to be almost complete symmetry between slow light and fast light. Since the group velocity is given by \( \frac{c}{n_g} \) where \( n_g = n + \omega \frac{dn}{d\omega} \), we see that slow light occurs if \( \frac{dn}{d\omega} \) is large and positive (known as normal chromatic dispersion) whereas fast light occurs if \( \frac{dn}{d\omega} \) is large and negative (anomalous chromatic dispersion). Both types of behavior occur regularly in nature. The study of fast light is conceptually important as it allows us to examine the nature of the modification of the velocity of light. In addition, fast light can lead to applications of its own. One application of fast light is in the construction of regenerators for optical telecommunication. One form of regeneration requires that optical data pulses be actively centered in their time windows. The ability to advance as well as delay a data packet greatly facilitates this form of regeneration.

As described above, Sommerfeld and Brillouin investigated a step-modulated pulse propagating through a collection of Lorentz oscillators in a spectral region of anomalous dispersion. Based on this investigation, Brillouin suggested that the group velocity is not physically meaningful in this situation because the pulse
becomes severely distorted [30]. For this reason, several textbooks on electromagnetism state that \( v_g > c \) or \( v_g < 0 \) is unphysical.

An interesting twist to this story came about in 1970 when Garrett and McCumber [31] published a theoretical study on propagation of smooth-shaped pulses through a resonant absorber in a region of anomalous dispersion. They showed that the group velocity does have meaning even in the “fast light” case as long as the medium is thin enough so that pulse distortion is not too severe. In fact, they predicted that it is possible that the peak of a light pulse may exit the optical material before a pulse peak passes through the entrance face, which is the physical interpretation of a negative group velocity. The first indirect measurement of a group velocity exceeding \( c \) was made by Faxvog and collaborators [32, 33], who studied mode pulling in a self-mode-locked helium–neon laser containing a neon absorption cell.

Some years later, Chu and Wong [18] studied experimentally both slow and fast light for picosecond laser pulses propagating through a GaP:N crystal as the laser frequency was tuned through the absorption resonance arising from the bound A-exciton line. Both positive and negative group delays were observed. Because they were using short pulses, an autocorrelation method was used to measure pulse delay, which can obscure possible pulse distortions. Their experimental results were found to be in good agreement with theoretical predictions, which were obtained from a model that is a slight generalization of the model studied by Garrett and McCumber. Somewhat later, work by Ségard and Macke [34] on microwave pulse propagation through a resonant absorber made direct measurements of the field envelope, thus demonstrating directly that there was only minor pulse distortion. Negative group velocities have since been observed by others [35].

In these experiments, fast-light pulse advancement was accompanied by substantial pulse attenuation. Steinberg and Chiao [36] predicted that it is possible to use two adjacent gain lines to obtain fast light, where anomalous dispersion occurs when the carrier frequency of the pulse is set in the middle of the gain doublet. Chiao and collaborators published several other works that described why fast-light pulse propagation does not violate the special theory of relativity [37–39]. In particular, they focus on the idea that information is encoded on points of non-analyticity on optical waveforms, and it is these points that move at \( c \). More recently, Parker and Walker [40] suggest that the very act of encoding information on a waveform necessarily creates points of non-analyticity.

The prediction of Steinberg and Chiao [36] was verified in an experiment by Wang et al. [41], where the gain doublet was produced in a laser-pumped cesium vapor. They observed measurable pulse advancement in combination with small pulse amplification. While they were careful to point out that their experimental observations were consistent with the special theory of relativity, they did not give a detailed explanation of why this was the case. Unfortunately, some of the popular press cast their experiment as violating Einstein’s theory, giving rise to considerable confusion and controversy.

Soon thereafter, Stenner et al. [42] designed an experiment to measure directly the speed at which information propagates through a fast-light material. They used
an experimental setup similar to that used by Wang et al., but with large dispersion that gave rise to larger pulse advancement. Figure 7.5 shows an example of their data for the case of a smooth Gaussian-shaped pulse propagating through the fast-light medium in comparison to the same pulse propagating through vacuum. The larger advancement relative to the pulse width obtained in their experiment made it easier to distinguish the different velocities describing pulse propagation. From this data, they inferred that \(n_g = -19.6 \pm 0.8\), indicating that they were operating in the highly superluminal regime.

![Fig. 7.1 Fast-light pulse propagation. Temporal evolution of a 263.4-ns-long (full width at half maximum) pulse propagating through a laser-pumped potassium vapor (dashed line) and vacuum (solid line) [42]](https://example.com)

To determine the information velocity, Stenner et al. encoded new information on the waveform at the top of the Gaussian-shaped pulse by rapidly turning the pulse off or by switching it to a higher value. Such an approach enhanced their ability to estimate the location of a non-analytic point in the presence of noise. The moment when a decision was made to switch between the communication symbols (either pulse high or pulse low) corresponded to the point of non-analyticity.

They detected the location of the point of non-analyticity by determining the arrival of new information using a receiver that distinguished between symbols to a desired level of certainty, characterized by the bit error rate (BER). Using this method, they found that the information velocity is always less than but nearly equal to \(c\), even for a medium where \(n_g\) is highly superluminal. Thus, they demonstrated that the peak of the advanced pulse at the exit face of the medium is not causally connected to the peak at the entrance face. Follow-up studies showed that information also travels nearly at \(c\) even for a material where \(v_g \ll c\) [43] and that information propagation is connected to the front velocity and optical precursors [44].

Other analyses of the relation between superluminality and information transfer have been reported as well in the literature. Diener [45] has concluded that superluminal group velocities do not imply superluminal information velocities because the pulse shape can always be determined by analytic continuation of the pulse shape within the light cone. Kurizki et al. [46] have shown that the injection of
spectrally narrow wavepackets into quantized amplifying media can give rise to transient tachyonic wavepackets. Kuzmich et al. [47] have studied limitations on information transfer in fast-light situations based on quantum effects. Wynne [48] has argued theoretically that information cannot be transmitted superluminally and that claims to the contrary are the result of incorrect reasoning. Tanaka et al. [49] have observed negative group velocities in a Rb vapor. Ruschhaupt and Muga [50] have shown theoretically that the peak of an electromagnetic pulse can arrive simultaneously at different positions in an absorbing waveguide. Clader et al. [51] have shown that instabilities often associated with superluminal propagation can be avoided through use of sufficiently short pulses.

The surprising behavior discussed above can be illustrated with some examples. Under conditions of sufficiently large anomalous dispersion, the group index can take on negative values (recall the definition $n_g = n + \omega \frac{dn}{d\omega}$). This possibility raises the question of what it means for a group velocity $v_g = c/n_g$ to become negative. Figure 7.2 shows a numerical simulation of the propagation of a pulse through a material possessing a negative value of the group velocity. The influence of gain, absorption, or group velocity dispersion is not included in this model, and thus the simulation is based simply on performing a numerical integration of the reduced wave equation.

Fig. 7.2 Numerical simulation of pulse propagation through a material with a negative value of the group velocity.
\[
\frac{\partial A}{\partial z} - \frac{1}{v_g} \frac{\partial A}{\partial t} = 0
\]  
(7.1)

for the pulse amplitude \( A(z, t) \) for the situation in which the group velocity is negative. One sees from Fig. 7.2 that the pulse appears to leave the material before it enters and that the pulse appears to move backward within the material. How behavior of this sort can possibly be physical and consistent with the concept of causality is a matter that we will deal with in later sections of this chapter. For the present, we simply point out that an input pulse in the form of a Gaussian waveform has wings that extend from plus infinity to minus infinity. In this sense, even in the top frame of Fig. 7.2, the input pulse already has a contribution at the output, and there is no possibility for a violation of causality to occur. Alternatively, we can consider superluminal pulse propagation to represent a special form of pulse reshaping, in which the pulse form is retained but shifted earlier in time.

Experimental verification of this sort of behavior has been reported by Gehring et al. [24] in an experiment that studied pulse propagation through an erbium-doped optical amplifier. A negative value of the group index was obtained by means of the CPO effect described above. Some of the experimental results are shown in Fig. 7.3.

![Experimental results demonstrating the reality of negative group velocities](image)

**Fig. 7.3** Experimental results demonstrating the reality of negative group velocities [24]. The arrows point to the peak of each pulse.
Note that the peak of the pulse clearly is moving in the backward direction (right to left) inside the material, even though outside the material the pulse is moving left to right. Because the experiment was performed through use of an amplifying medium, the output pulse is larger than the backward-going pulse within the material.

7.4 The Concept of Simultaneity

As we mentioned earlier, a primary concern of this chapter is to examine how the superluminal pulse propagation can be compatible with the concept of causality. But to understand what is meant by causality, one must first understand what it means for events to occur simultaneously. In this section, we present an examination of the concept of simultaneity. Much of the subtlety involved in considerations of causality involves the concept of what it means for two events to be simultaneous [52].

We begin by distinguishing local simultaneity from distant simultaneity. There are subtleties involved in each concept. We first consider the case of local simultaneity, as it is the more basic concept. One says that two events, A and B, located at the same point in space, are simultaneous if they occur at the same time. The subtlety of this definition is that it presupposes that one understands what one means by time. If the events are not simultaneous, either A occurs before B or it occurs after B. The distinction between “before” and “after” implies that there is a direction of the flow of time. We know from everyday experience that time flows in one direction (from the past to the future), but this point remains unsettling in part because it is not obvious what physical process breaks the symmetry between earlier and later times. This thought can be made more precise by noting that most of the laws of physics are symmetric upon the reversal of the sign of the time coordinate. The existence of certain laws which do not obey this property, such as the tendency of entropy to increase monotonically, may lead to some understanding of the origin of the direction of the flow of time [53].

In considerations of causality and simultaneity, one conventionally defines an “event” as a process that occurs at a given location with coordinates $x$, $y$, and $z$ at a given time $t$. One can thus define an event in terms of its space–time coordinates $(x, y, z, t)$. As noted above, the time coordinate of the event has a very different character from the spatial coordinates, in that there is a sense of directionality to the time coordinate not present in the spatial coordinates.

So how does one define time? One definition is that put forth by Kant, as reported by Jammer [52]. Kant says that if event A could cause event B, then A is said to occur before B. This thought is very much consistent with modern views of physical causality, but has the disadvantage that one cannot then examine the relation between causality and the flow of time if time has been defined in terms of causality. Perhaps a better procedure is to follow the lead of Einstein and define time simply to be what a clock measures. We assume that we place a “clock” at the point $(x, y, z)$ and define the unit of time to be the interval between successive ticks of the clock. From this point of view, a good clock is one for which the laws of classical
mechanics are well satisfied with time defined in this manner. To summarize this point, we conclude that two events both occurring at the same spatial point \((x, y, z)\) are said to be simultaneous if both events occur at the same time \(t\), with \(t\) defined as above.

Somewhat more subtle is the concept of distant simultaneity. The underlying question here is, what does it mean for two events to be called simultaneous if they do not occur at the same point in space? Philosophical discussion of this point goes back at least as far as the golden age of Greece. This issue certainly possesses a technological component (How could one hope to measure distant simultaneity without the help of reliable clocks?), but also addresses the conceptual issue of what it means for two separated events to be said to be simultaneous.

One impetus for these discussions occurred within the field of astrology, which holds that one’s future depends on the configuration of the stars and planets at the time of one’s birth. It thus became important to know the state of the heavens at the moment of a child’s birth, even if the heavens were obscured by cloud cover or rendered unobservable by daylight. It is interesting to note that Saint Augustine argued against the validity of astrology by means of the following argument [54]. He considered the hypothetical situation in which two women located in different households were to give birth at approximately the same time. One woman had great wealth, whereas the other was a servant. The child of the wealthy woman would almost certainly be more successful in life than the child of the poor woman. If these children were born simultaneously, this occurrence would contradict the predictions of the laws of astrology. But how would one establish the simultaneity of the two births, occurring at separated points? In a manner that foreshadows that of Einstein some 1500 years later, Augustine proposes the following procedure. Two messengers are employed and they are selected so that they run at the same speed. One messenger is stationed near each expectant mother, and at the moment that the child is born the messenger is told to run to the other household to announce the birth. If the messengers meet en route, the exact location of their meeting is recorded, and if this spot is exactly equidistant between the two households the births are said to have occurred simultaneously.

Within modern physics, one defines distant simultaneity in terms of synchronized clocks. One assumes that two clocks of identical construction are located at spatial points A and B. Being of identical construction, these clocks are, therefore, assumed to run at the same rate. If the clocks can be synchronized, then the concept of distant simultaneity becomes meaningful, in the sense that two events are said to be simultaneous if the event at A occurs at a time measured by the clock at A, that is, the same time as the time of event at B as measured by the clock at B.

Eddington [55] describes two possible procedures for synchronizing distant clocks. One method is to transport clock A to point B, set the clocks to read the same time, and then transport clock A back to its original location. Of course, an auxiliary clock can alternatively be used for this purpose. Because of relativistic time dilation, the clock needs to be moved very slowly in order for this procedure to be valid. In principle, one can always perform this procedure, because time dilation effects are second order in the ratio \(\nu/c\) (here \(\nu\) is the velocity of the clock), whereas
the time required for the transport scales as $1/v$. This method also presupposes that the clock maintains its accuracy during the time intervals of acceleration needed to change its velocity.

The second method, described by Einstein in his 1905 paper, is based on signaling using light beams. Several variations of this method exist. One is for A to send a light pulse to B, where it is reflected back to A. B sets its clock to some reference time (for instance $t = 0$) at the moment that the pulse arrives at B. A waits until the light pulse returns, and then sets its clock to the reference time at a moment exactly halfway between the time $t_1$ at which the pulse left A and the time $t_2$ at which the pulse returned. Many authorities have argued that while this procedure provides an acceptable procedure for synchronizing the two clocks, the synchronization thus achieved is simply one of arbitrary convention. They argue that A could set the reference time of its clock to any time in the interval $(t_1, t_2)$ and thereby establish an entirely consistent form of clock synchronization. The reason for this arbitrariness is that there appears to be no definitive proof that the velocity of light $c_+$ along the positive $x$-direction (for instance) is the same as the velocity $c_-$ along the negative $x$-direction. The argument is that measurements of the velocity of light actually yield only the average of $c_+$ and $c_-$, and it is this quantity that is conventionally known as $c$. This unexpected conclusion follows from the fact that it is possible to measure the one-way velocity of light only if one already has synchronized clocks at both ends of the beam path, which cannot be possible if one’s intent is to develop a procedure for clock synchronization.

7.5 Causality and Superluminal Pulse Propagation

The key to understanding why fast-light pulse propagation is consistent with the special theory of relativity is to investigate what is meant by a signal, as described above in Sect. 7.2 and its connection to an event. In Einstein’s public discussions of the theory [56], he focuses on the concept of an “event,” such as a spark caused by a lightning bolt, and how the event (or multiple events) would be observed by people at various locations. He was especially interested in observers moving with respect to a coordinate system that is stationary with respect to the events. A detailed description of his findings is not needed for our present discussion, as it is necessary to consider only the properties of a single event in a single coordinate system.

A convenient way to discuss the flow of information from an event is to use a space–time diagram (Minkowski diagram), where the horizontal axis is a single spatial coordinate and time is plotted along the vertical axis (see Fig. 7.5). According to the special theory of relativity, the fastest way that knowledge of the event can reach an observer is if it travels at the speed of light in vacuum; the lines that connect points in a space–time diagram that follow vacuum speed-of-light propagation define the light cone – the shaded region in Fig. 7.5(a). The inverse of the slope of lines drawn in a space–time diagram is equal to the velocity. Observers at space–time points within the shaded cone (e.g., observer A in Fig. 7.5(b)) are able
to see the event and those outside the cone cannot (e.g., observer B in Fig. 7.5(b)). Note that the cone extending for times preceding the event represents the space–time regions where light could reach the location of the event. That is, an observer (not shown) in this region can affect the event but cannot see the event.

On the other hand, hypothetical faster-than-light propagation of information is relativistically acausal. Acausal means that there is no direct time-ordered link between a cause and an effect. An example of a hypothetical faster-than-light communication scheme is shown in Fig. 7.5(c), where we assume that it is possible to transmit information with a speed that is less than zero (negative velocity). If such superluminal signal was possible, information could be transmitted from the positive-time light cone to a person at position D. This observer could change the outcome of the event (e.g., prevent it from happening) because she is located within the light cone leading to the event, but at a time before the event happens. Thus, she can change the outcome of the event.

![Fig. 7.4 The light cone associated with an event. (a) Space–time diagram of an event. (b) Observers at space–time points A and B. In a world that is relativistically causal, A observes the event, but B does not. (c) Communication in a hypothetical acausal world. If relativistic causality could be violated, a person at C could observe the event and transmit information to a person at D using a superluminal communication channel. The person at D could then change the outcome of the event.](image)

To address whether fast-light pulse propagation provides a mechanism for relativistically acausal communication, it is necessary to define a signal. As discussed briefly in Sect. 7.2, Sommerfeld [30] defined a “signal” as a wave that is initially zero and suddenly turns on to a finite value, which is known as a step-modulated pulse. In terms of the special theory of relativity, the moment that the wave turns on corresponds to the event.

In an analysis conducted by Sommerfeld and Brillouin [30], they used Maxwell’s equations to predict the propagation of a step-modulated electromagnetic wave, which was coupled to a set of equations that described how it modifies the dispersive material. For the dispersive material, they assumed that it consisted of a collection of Lorentz oscillators. A Lorentz oscillator is a simple model for
an atom that describes its resonant behavior when interacting with light. Each oscillator consists of a massive (immoveable) positive core and a light negative charge that experiences a restoring force obeying Hooke’s Law. Furthermore, they assumed that the negative charge experiences a velocity-dependent damping so that any oscillations set in motion will eventually decay in the absence of an applied field.

Conceptually, an incident electromagnetic wave polarizes the material (causes a displacement of the negative charges away from their equilibrium position), and this polarization acts back on the electromagnetic field to change its properties (e.g., amplitude and phase). The coupled Maxwell–Lorentz-oscillator model is known to possess spectral regions of anomalous dispersion where the group velocity $v_g$ takes on negative values or values greater than $c$ thus should be able to address the controversy. The model is so good that it is still in use today for describing the linear optical response of dispersive materials.

Using asymptotic methods to solve the pertinent inverse Fourier integral, Sommerfeld was able to predict what happens to the propagated field for times immediately following the sudden turn-on of the wave, that is, what happens in the vicinity of the pulse front. He was able to show that the velocity of the front is always equal to $c$. In other words, the front of the pulse coincides with the boundary of the light cone shown in Fig. 7.5, even though the pulse is propagating through the dispersive dielectric material.

Sommerfeld gave an intuitive explanation for his prediction. When the electromagnetic field first starts to interact with the oscillators, they cannot immediately act back on the field via the induced polarization because they have a finite response time. Thus, for a brief moment after the front passes, the dispersive material behaves as if there is nothing there – as if it were vacuum. From the point of view of information propagation, one should be able to detect the field immediately following the front and hence observe information traveling precisely at $c$.

After the front passes, mathematical predictions are very difficult to make because of the complexity of the problem. Brillouin extended Sommerfeld’s work to show that the initial step-modulated pulse, after propagating far into a medium with a broad resonance line, transforms into two wavepackets (now known as optical precursors) and is then followed by the bulk of the wave (what Brillouin called the “main signal”). They found that the precursors tend to be very small in amplitude and thus it would be difficult to measure information transmitted at $c$; rather, it would be easiest to detect at the arrival of the main signal, which they found travels slower than $c$. The term precursor is somewhat confusing because it implies that the wavepacket comes before something; in this usage, the precursors come before the main signal, but after the pulse front.

One aspect of Sommerfeld and Brillouin’s result that can lead to confusion is the possible situation when one or more of these wavepackets travel faster than $c$. What is implied here is that a velocity can be assigned to the precursors and the main signal to the extent that they do not distort and that these velocities can all take on different values. In a situation where the velocity of a wavepacket exceeds $c$, it will eventually approach the pulse front (which travels at $c$), become much distorted (so
In a typical experiment, the emitted waveform has the shape of a well-defined pulse. Nonetheless, the waveform has a front, the moment of time when the intensity first becomes non-zero. When such a waveform passes through a fast-light medium, the peak of the pulse can move forward with respect to the front, but can never precede the front. Thus, even though the group velocity is superluminal, no information can be transmitted faster than the front velocity, which is always equal to $c$.

That assigning it a velocity no longer makes sense), and either disappear or pile up at the front.

Sommerfeld and Brillouin’s research appeared to satisfy scientists in the early 1900s that “fast light” does not violate the special theory of relativity. Yet there continue to be researchers who question various aspects of their work. One point of contention is that some people believe that it is impossible to generate a waveform in the lab that has a truly discontinuous jump (e.g., there is no electromagnetic field before a particular time and then a field appears). Yet having something appear at a particular space–time point is precisely what is meant by an event described above. Thus, if one does not believe in discontinuous waveforms, then the very conceptual framework of the special theory of relativity and the associated light cone shown in Fig. 7.5 would need to be thrown out. Many scientists are unwilling to do so. Also, the existence of optical precursors has been questioned because Sommerfeld and Brillouin made some mathematical errors in their analysis concerning the propagated field for times well beyond the front, although recent research suggests that precursors can be readily observed in experimental setups similar to that used in recent fast-light research [44, 57].

So how can the data shown in Fig. 7.1 be consistent with the special theory of relativity? To answer this question, we need to make a connection between Sommerfeld’s idea of a signal and the data shown in the figure. In the experiment, a pulse was generated by opening a variable-transmission shutter (an acousto-optic modulator); only a segment of the pulse is shown in the figure. At an earlier time not shown in the figure, the light was turned from the off state to the on state, but
with very low amplitude. The moment the light first turns on coincides with the pulse front (the event). At a later time, the pulse amplitude grows smoothly to the peak of the pulse and then decays.

As far as information transmission is concerned, all the information encoded on the waveform is available to be detected at the pulse front (although it might be difficult to measure in practice). The peak of the pulse shown in Fig. 7.5 contains no new information. Thus, the fact that the peak of the pulse is advanced in time is not a violation of the special theory of relativity, so long as it never advances beyond the pulse front. Figure 7.6 shows a schematic of the light cone for such a fast-light experiment.

![Fig. 7.6 Pulse propagation in a fast-light medium with a negative group velocity. The space–time diagram shows that the peak is advanced as it passes through the medium, but the pulse front is unaffected. The opening angle of the light cone is drawn differently from that in Fig. 7.5 for illustration purposes only.](image)

In our opinion, all experiments to date are consistent with the special theory of relativity, even though it may be difficult to show this. In some experiments, the pulse shape is such that it is exceedingly difficult to detect the pulse front and hence it may appear that the special theory has been violated. In other experiments especially designed to accentuate the pulse front, it has been shown that the information velocity is equal to $c$ within the experimental uncertainties in both fast-light [42] and slow-light regimes [43].

### 7.6 Quantum Mechanical Aspects of Causality and Fast Light

Quantum mechanics provides a mechanism that at first glance seems to imply the possibility of superluminal communication, even for propagation through vacuum. This mechanism is the simultaneous collapse of the wave function at all points in
space caused by a measurement performed on the system at one particular point in space. This is an effect that does not occur in classical physics and hence deserves further consideration with regard to the possibility of superluminal communication.

Let us consider a hypothetical superluminal communication system based on this effect [58–60], as illustrated in Fig. 7.7. We consider entangled particles generated by an Einstein–Podolsky–Rosen (EPR) source. For concreteness, we consider a system that generates two correlated photons that travel in opposite directions and carry zero total spin angular momentum. Furthermore, two observers, Alice and Bob, are located on opposite sides of and at large distances from the source. They are equipped with optical components that can analyze the state of polarization of the arriving photons. Bob is slightly further away from the source than Alice, and we want to establish a one-way superluminal communication link from Alice to Bob.

![Fig. 7.7 Potential superluminal quantum communication scheme. A source produces pairs of entangled photons, where one photon is sent to Alice and the other to Bob. In the linear measurement basis, a polarizing beam splitter and two single-photon detectors determine whether the photons are horizontally (H) or vertically (V) polarized. In the circular measurement basis, a quarter-wave plate is inserted before the beam splitter, allowing for the measurement of left (L) or right (R) circularly polarized photons. Bits are communicated by inserting or not the quarter-wave plate in the setups, as described in the text.](image)

In one scenario, Alice places a polarizing beam splitter that spatially separates one state of linear polarization (say vertical, V) from the other state of polarization (horizontal, H). The output ports of the polarizing beam splitter are directed to single-photon detectors. Bob has an identical apparatus, which is at a great distance from Alice, and he aligns the axis of his polarizing beam splitter the same as Alice’s. Because of the fact that their total angular momentum of the photons is zero, whenever Alice measures V, the bi-photon wave function collapses and Bob is assured of measuring H essentially instantaneously after Alice performs her measurement.
Similarly, Bob will measure $V$ whenever Alice measures $H$. In this configuration, the polarizing beam splitters and single-photon detectors perform measurements in what we call the “linear” basis.

Alice and Bob can also perform measurements in the “circular” basis, where the analysis apparatus will determine whether the photons are left circular ($LC$) polarized or right circular ($RC$) polarized. This measurement can be performed by placing a quarter-wave plate in front of the polarizing beam splitters, where the optical axis of the plate is orientated at 45° to the axis of the linear polarizing beam splitter. With the wave plate in the system, Bob is assured to measure $RC$ ($LC$) whenever Alice measures $RC$ ($LC$).

The hypothetical superluminal communication scheme is based on a change of measurement basis. By inserting the wave plate into the setup or not, Alice can force Bob’s photon to be either linearly or circularly polarized (more precisely, she can force Bob’s photon to be an eigenstate of either linear or circular polarization). Thus, it appears as if Alice can transmit binary information to Bob by inserting or not inserting the wave plate into her apparatus. All Bob has to do is to determine with certainty whether Alice was using the linear or circular basis. The first problem with this scheme is a well-known classical result: it is only possible to measure whether an optical beam is linear or circular polarized by analyzing it both with linear and circular polarizers. In other words, Bob would have to send the photon through the linear-basis apparatus and the circular-basis apparatus. Unfortunately, one apparatus destroys the incident photon as a result of the measurement and hence it is unavailable to send on the other.

One way around this problem is for Bob to “clone” the incident photon so that there are two copies, where one copy will be sent to a linear-basis apparatus and the other is sent to a circular-basis apparatus [58, 59]. The process of stimulated emission of radiation can in a sense clone an incident photon, so one might think that an optical amplifier in the path of the photon would be useful for this communication scheme. Unfortunately, an optical amplifier adds additional photons to the beam path via the process of spontaneous emission. These additional photons have an arbitrary state of polarization [61]. They destroy the benefits of the amplifier and hence prevent Alice from communicating with Bob via the nonlocal characteristics of quantum mechanics.

The problem with the superluminal communication scheme is much deeper that it appears from this discussion. The very linearity of quantum mechanics prevents the cloning of an arbitrary quantum state, a result of the no-cloning theorem [62, 63]. Thus, any device – not just an optical amplifier – fails to clone the incident photon and hence the communication scheme fails.

Other researchers have wondered whether an imperfect copy of the incident photon would be sufficient for superluminal communication. The best or optimal quantum copying machine has been identified; even with the best possible copying apparatus, the quantum communication scheme just barely fails. This failure is nicely summarized by N. Gisin [64] in his 1998 paper: “Once again, quantum mechanics is right at the border line of contradicting relativity, but does not cross it. The peaceful coexistence between quantum mechanics and relativity is thus re-enforced.”
7.7 Numerical Studies of Propagation Through Fast-Light Media

In order to explore further some of the features of fast-light propagation described above, we have performed numerical studies of the propagation of optical pulses through fast-light media. For these studies, we assume that the medium consists of a single Lorentzian absorption line set on a broad gain background. We assume the presence of the broad gain background to prevent the transmitted pulse from becoming so weak as to be immeasurable. The absorption coefficient of the material is thus taken to be of the form

\[ \alpha(\delta) = \alpha_b + \frac{\alpha_l}{1 + (\delta^2/\gamma^2)} . \]  

(7.2)

Here \( \alpha_b \) is the value of the background absorption coefficient (assumed negative), \( \alpha_l \) is the line-center absorption coefficient of the absorption line, \( \gamma \) is its linewidth, and \( \delta \) is the frequency detuning from line center. According to the Kramers–Kronig relations, the refractive index associated with this absorption is given by

\[ n(\delta) = n_b + \frac{\alpha_1}{4\pi} \frac{\gamma/\delta}{1 + (\delta^2/\gamma^2)} , \]  

(7.3)

where \( \lambda \) is the optical wavelength and \( n_b \) is the background refractive index, which we shall subsequently set equal to unity. From this result, we then find that the group index \( n_g = n + \omega \frac{dn}{d\omega} \) is given by

\[ n_g = 1 + \frac{\alpha_1 c}{2\gamma} \frac{1 + (\delta^2/\gamma^2)}{1 + [(\delta^2/\gamma^2)]^2} . \]  

(7.4)

The expression for the group velocity \( v_g = c/n_g \) then follows directly.

The input pulse is taken to be a transform-limited Gaussian pulse of the form

\[ A(z, t) = A_0 e^{-t^2/T^2} . \]  

(7.5)

Here \( T \) is the pulse width defined as the amplitude half-width to \( 1/e \) or the intensity half-width to \( 1/e^2 \). Equivalently, we can describe the pulse in the frequency domain as

\[ \tilde{A}(z, \omega + \delta) = \tilde{A}_0 e^{-\delta^2/\xi^2} , \]  

(7.6)

where \( \xi = 2/T \) is the frequency-domain pulse width. This pulse will be advanced compared to vacuum propagation as it passes through the medium. Neglecting dispersion in the group velocity and the fact that the frequency-varying absorption will cause spectral reshaping of the pulse, the amount of pulse advancement \( \Delta T = L/v_g - L/c \) resulting from propagation through a length \( L \) of the medium is found to be
\[ \Delta T = -\frac{\alpha_L L}{2\gamma} \frac{1 + (\delta^2/\gamma^2)}{1 + [(\delta^2/\gamma^2)]^2} . \]  

(7.7) 

As can be seen, the amount of pulse advancement increases with increasing absorption \(\alpha_L L\) and also with decreasing linewidth \(\gamma\). However, a line that is too narrow compared to the spectral width of the input pulse will lead to pulse distortion either by group velocity dispersion or by spectral reshaping of the pulse [65–67]. In the cases studied here, pulse narrowing by spectral reshaping is the dominant effect. If the limit on pulse narrowing is set so that to the first order the pulse duration becomes infinitesimally small, we find by means of the procedure described in [66] that the following inequality must be satisfied:

\[ 2\alpha_L L \leq (\gamma T)^2 . \]  

(7.8) 

The allowable total integrated absorption \(\alpha_L L\) is limited by two factors. First, in order to be able to detect the output pulse, the transmission at the center frequency of the pulse must not be too small. Second, in order to avoid instabilities, the gain of the background must not be too large. Taking (somewhat arbitrarily) the minimum allowable transmission at the center frequency of the pulse to be \(e^{-32}\) and the maximum allowable gain to be \(e^{32}\), we find that the maximum relative pulse advancement \(\Delta T/T\) that can be obtained in the system studied here is given according to Eqs. (7.7) and (7.8) by

\[ \Delta T/T = 2\sqrt{2} . \]  

(7.9) 

This maximum pulse advancement is obtained when the line-center absorption of the absorption line is set equal to \(e^{-64}\) and the gain of the broadband background is set equal to \(e^{32}\). Other authors have deduced similar limits on the maximum possible pulse advancement [68].

A more detailed description of the propagation of the optical pulses through the material medium can be obtained by performing a numerical simulation of the propagation process. Because we are considering the situation in which the material responds linearly to the optical pulse, we model the propagation by means of the following procedure. The input pulse is decomposed into a Fourier integral, and each frequency component is allowed to propagate through the medium, acquiring phase and amplitude modifications in accordance with its frequency-dependent refractive index (Eq. 7.3) and absorption coefficient (Eq. 7.2). The time evolution of the output pulse is then obtained by performing the inverse Fourier transform on the output spectrum.

Some of the results of this procedure are shown in Fig. 7.8. The parameters for the advanced pulse were chosen to give the maximum possible advancement in that sense that according to the first-order analytic model the pulse duration would shrink to zero. We see that the pulse has narrowed but not as much as the first-order theory would predict. In addition, the amount of pulse advancement is 12% smaller than predicted by Eq. (7.9). For comparison, in this figure we also
Fig. 7.8 Pulse advancement in a medium consisting of an absorption line with a total attenuation of $e^{-64}$ set on a broad gain background with a gain of $e^{64}$. The ratio between the width $\gamma$ (half-width at half maximum) of the Lorentzian line to the spectral width $\xi$ (half-width to 1/e in amplitude) of the input pulse is 5.7. The input/reference pulse shows what the output pulse would be if the medium were replaced by an equal length of vacuum. Also shown for comparison is the pulse delay that experienced upon propagation through a slow-light medium consisting of an $e^{64}$ gain line set on a broad $e^{-32}$ absorptive background. Note that pulses tend to compress in time for fast-light propagation and broaden in time for slow-light propagation

give an example of pulse propagation through a slow-light medium. In the configuration studied here, there is no fundamental limit on how strong the broad absorptive background could be, and therefore there is no limit on how much the pulse can be delayed [66]. In choosing the parameters for this example, we required that the pulse broaden by no more than a factor of $\sqrt{2}$. Other configurations using multiple lines and operating far from line center can also give rise to large delays [65, 67, 69].

No other choice of line strengths and linewidths has been found that gives significantly more pulse advancement than that shown in Fig. 7.8. The reason for this behavior is that fast light occurs when the center frequency of the light pulse is at a local minimum of the transmission. Therefore, if one tries to obtain a larger pulse advancement, either the transmission at the center frequency will be too low for detection of the pulse at the output or the gain away from the center frequency will be so high that it leads to instabilities [19, 68].

The fact that the peak arrives at the output earlier than it would have arrived had the medium been replaced by an equal length of vacuum suggests that the peak of the pulse travels faster in the medium than the speed of light in vacuum. A situation of this sort is shown in Fig. 7.9. Here the material parameters are chosen such that the group velocity from first-order theory is twice the speed of light in vacuum. As can be seen, at $t = 0.4$ (in all of our numerical work we normalize the time in units of the vacuum transit time through the medium) the pulse peak has traveled approximately twice as far as the reference pulse. Due to spectral reshaping of the pulse by the frequency-varying absorption, the apparent pulse velocity then begins to slow down and pulse distortion starts to occur. This simulation is performed with a ratio of the width $\gamma$ of the absorption line to the spectral width $\xi$ of the input pulse of 4. For this ratio, we have found that the maximum allowable integrated line strength (that is, $\exp(-\alpha L)$) before the occurrence of significant pulse distortion by spectral reshaping is $e^{32}$. The integrated line strength of the example shown in Fig. 7.9 is $e^{48}$ and consequently after propagating approximately two-thirds of the
Fig. 7.9 A sequence of frames showing a Gaussian pulse propagating through a fast-light medium. The medium is comprised of a Lorentzian absorption line of integrated line strength $e^{-48}$ set on a broad gain background with a gain of $e^{45.3}$ chosen to give an output pulse equal in amplitude to the input pulse. The ratio $\gamma/\xi$ of the width of the absorption line to the spectral width of the input pulse is 4. For comparison we also show as the dashed curve how the pulse would propagate through an equal length of vacuum.

way through the medium (at $t = 0.8$) severe pulse distortion sets in and the pulse breaks up into several parts.

The advancement $\Delta T$ in units of input pulse width $T$ depends only on the integrated line strength $\exp(\alpha L)$ and on the linewidth $\gamma$ but not on the physical length of the medium. Through use of the concept of group velocity, the pulse advancement shown in Fig. 7.8 can be considered to be the difference in the transit times of a pulse moving at $c$ and of a pulse moving through the medium at the effective group velocity $v'_g$ such that

$$\Delta T = \frac{L}{c} - \frac{L}{v'_g}.$$  \hfill (7.10)

This effective group velocity is, in general, different from the group velocity $v_g$ given by first-order theory. Solving this equation for the effective group velocity, we find that

$$\frac{c}{v'_g} = 1 - \frac{\Delta T}{L/c}.$$  \hfill (7.11)
Fig. 7.10 Pulse advancement in a medium consisting of a Lorentzian absorption line of integrated line strength $e^{-32}$ set on a broad gain background of gain $e^{30.7}$. Depending on the length of the medium compared to the spatial advancement of the pulse, the pulse advancement can be considered to be the result of a larger-than-$c$, infinite, or negative group velocity as shown in the left, middle, and right columns, respectively.

When the length $L$ of the medium lies in the range $c\Delta T < L < \infty$, $\Delta T$ will be positive and hence the group velocity will be larger than the speed of light in vacuum, for $c\Delta T = L$, the group velocity becomes infinite, and when $L$ is smaller than $c\Delta T$ the group velocity is negative. Examples of these three situations are shown in Fig. 7.10.

When the group velocity is larger than $c$ but finite, the pulse propagation takes the form of a distinct pulse that moves through the medium at a superluminal velocity. When the group velocity is infinite, the propagation through the medium occurs instantaneously in the sense that the peak at the output occurs at the same time that the peak of the input pulse reaches the input face of the medium. For negative group velocities, a well-defined pulse moves backward through the medium, as has been observed experimentally [24]. In this case the peak of the output pulse actually leaves the medium before the input pulse enters. For the three cases shown in Fig. 7.10, the group velocities obtained from the numerical simulation and from Eq. (7.11) are very close to the group velocity predicted by the first-order theory.

In the simulations described above, the strength of the broad gain background has been chosen such that the peak intensities of the input and output pulses are equal. When this is the case, we observe a distinct pulse propagating through the medium. We can then think of the pulse advancement as occurring as a result of superluminal pulse propagation. However, in many experimental situations, a large net absorption is experienced in passing through the medium. An example of such behavior is shown in Fig. 7.11. Here we assume the presence of an absorption line of integrated line strength $e^{-32}$ and no gain background.

In this case the pulse decays approximately exponentially as it propagates through the medium, and the output peak is approximately $e^{32}$ times smaller than that of the input. In order to make the pulse visible in the plots, the pulse intensities are
normalized in each panel to fill the vertical scale. The intensity inside the medium initially decays exponentially, as would be expected because of the large absorption. Later, a clear pulse is formed. However, until \( t = 0.6 \) the pulse lags behind the reference pulse and in a sense propagates with a group velocity smaller than \( c \). Later there are multiple peaks within the medium, and only at the output does a clear, relatively undistorted, advanced pulse appear. In this case the pulse advancement cannot be regarded as occurring as the result of superluminal propagation because there is no well-formed pulse propagating faster than \( c \) within the medium. Rather, the apparent superluminal behavior occurs as the result of spectral reshaping.

The appearance of an undistorted output pulse that propagates faster than the vacuum speed of light \( c \) might initially seem to imply a violation of the laws of the special theory of relativity. One can even find situations in which the peak of the output pulse leaves the medium before the peak of the input pulse reaches the medium, apparently violating causality.

The next two figures illustrate this sort of behavior. In Fig. 7.12 we show that the moderately distorted output pulse is not the result of the peak of the input pulse propagating faster than the speed of light in the medium, but rather is the result of
Fig. 7.12 (a) The propagation of a Gaussian pulse through a medium consisting of an absorption line of integrated line strength $e^{-44}$ on a gain $e^{41.6}$ background. The output peak arrives before the reference peak, indicating superluminal propagation. (b) Same as in (a), but in this case only the very leading edge of the Gaussian pulse is launched into the medium. There is still an output peak identical to the peak in (a) showing that this peak is not caused by the peak in the Gaussian input pulse propagating faster than the vacuum speed of light $c$, but is the result of distortion and amplification of the leading edge of the pulse, a fully causal process.

distortion and amplification of the early part of the input pulse. We first note that the Gaussian input pulse in Fig. 7.12(a) has, in principle, been in existence for all times, even if it falls off very rapidly as we move away from the peak. The second point to note is that we have been very careful in placing the center frequency of the input pulse at a frequency where the transmission through the medium is at a minimum and has the value 9%. Close to this frequency, the gain is very large, on the order of $10^{18}$. Third, we note that, like previous workers, we use Gaussian input pulses. Gaussian pulses have the property of having a clear peak in the time domain while at the same time being well contained in the frequency domain. In Fig. 7.12(b) we show the result of launching only the very early parts of the input pulse. Up until the time labeled “turn off of input pulse” the input in Fig. 7.12(b) is the same as the Gaussian input in Fig. 7.12(a). After this point the Gaussian input is rapidly ramped down to zero as shown. On the output side of the medium, we see that the peak of Fig. 7.12(a) is still present, unchanged. This result indicates that this peak occurs as the result of the unchanged part of the input pulse, the leading edge, being amplified and distorted [42, 70]. Thus there is no causal connection between the peak of the output pulse and the peak of the input pulse. The relative gain required to turn the weak leading edge into the output peak shown is only $10^7$, much below the $10^{18}$ gain that is available. The turning-off of the input pulse in Fig. 7.12(b) introduces frequency components on the input that are far from line center and see high gain.
on propagating through the medium, resulting in the later parts of the output going off scale in the plot shown.

From the type of simulation shown in Fig. 7.9, it is tempting to regard the pulse advancement as occurring as the result of the peak of the pulse propagating faster than \(c\), moving gradually ahead of the reference pulse while suffering very little pulse distortion. This picture leads to concern about how this behavior can be consistent with the special theory of relativity. The resolution of this concern is that this is a very misleading picture. It is more correct to regard what happens on propagation as a continuous distortion of the pulse that moves the peak of the distorted pulse ahead of the peak of the undistorted reference pulse.

The information velocity can be considered to be the velocity at which the outcome of some “choice travels.” In many circumstances the information velocity is, for practical purposes, equal to the group velocity. The simulation shown in Fig. 7.13 demonstrates that this is certainly not the case in a fast-light system. Here, the input vanishes up to the point marked “pulse front.” Then a decision is made to launch a pulse into the medium with the shape and peak position as shown. The information velocity is the velocity within the medium at which this information travels. The decision to launch the pulse forces the input to jump from zero up to a value that is consistent with the rest of the pulse being Gaussian. This jump forms the pulse front [70, 42].

![Fig. 7.13](image)

**Fig. 7.13** Up to the point marked “pulse front,” the input vanishes. At that point a decision is made to launch the pulse. The jump in pulse amplitude at this point forms the pulse front. This pulse front arrives at the output after propagating through the medium with velocity \(c\) and heralds the fact that the pulse has been launched. The subsequent arrival of the peak carries no new information, even though it can be thought of as arriving after superluminal propagation. These numerical results are consistent with the conceptual picture presented in Fig. 7.5.

Under these conditions, we see that there are two peaks in the output, the “normal” advanced pulse and a second feature labeled “front.” In this numerical simulation, the various parameters have been chosen such that these two peaks have equal amplitude. The first peak marks the arrival of the pulse front. This peak arrives with the same speed as the reference pulse, the speed of light in vacuum. Later the advanced pulse arrives. This peak arrives earlier than the peak of the pulse would have arrived had the medium been replaced by an equal length of vacuum. It can, therefore, be regarded as consequence of the group velocity being greater than \(c\).
But this peak carries no new information, as we already know from the arrival of the front that the rest of the pulse must arrive. The arrival of the superluminal pulse does, therefore, not violate causality.

7.8 Summary

We have reviewed recent theoretical and experimental research that establishes that pulses can propagate through material systems with superluminal or even negative group velocities. Nonetheless, these exotic propagation effects are fully compatible with established notion of causality.

At a fundamental level, the nature of slow and fast light seems fairly well understood. But there still may be some surprises on the horizon. We noted in the body of this chapter that there seems to be no fundamental limit on how much one can delay a light pulse using slow-light methods, and in fact pulse delays as great as 80 pulse lengths have been observed [29]. Conversely, there seem to be severe limitations that limit the amount of advancement for a fast-light system to at most several pulse widths [71].1 But can these limitations be overcome? This is an intriguing question that merits further examination.

Acknowledgments  RWB and DJG gratefully acknowledge support from the DARPA/DSO Slow Light Program, and RWB from the NSF.

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