Waveguide-QED-Based Photonic Quantum Computation

Huaixiu Zheng, Daniel J. Gauthier, and Harold U. Baranger

Department of Physics, Duke University, Post Office Box 90305, Durham, North Carolina 27708, USA

(Received 8 November 2012; revised manuscript received 11 June 2013; published 28 August 2013)

We propose a new scheme for quantum computation using flying qubits—propagating photons in a one-dimensional waveguide interacting with matter qubits. Photon-photon interactions are mediated by the coupling to a four-level system, based on which photon-photon π-phase gates (CONTROLLED-NOT) can be implemented for universal quantum computation. We show that high gate fidelity is possible, given recent dramatic experimental progress in superconducting circuits and photonic-crystal waveguides. The proposed system can be an important building block for future on-chip quantum networks.

DOI: 10.1103/PhysRevLett.111.090502

PACS numbers: 03.67.Lx, 42.50.Ct, 42.50.Ex, 42.79.Gn

Quantum computers hold great promise for outperforming any classical computer in solving certain problems such as integer factorization [1], as well as in efficiently simulating quantum many-body systems [2,3]. While quantum computation schemes often encode information in stationary qubits, such as atoms, trapped ions, quantum dots, and superconducting qubits [4], flying qubits—photons—have several appealing features as carriers of quantum information [4,5]. Most importantly, photons have long coherence times because they rarely interact and yet can be readily manipulated at the single-photon level using linear optics. Furthermore, photonic quantum computation is potentially scalable [5] in view of the recent controlled generation of single-photon pulses [6–10] and demonstration of stable quantum memories [11,12]. However, weak photon-photon interaction makes it very challenging to realize the two-qubit gates necessary for universal computation between single photons [5,13,14]. Several schemes have been proposed to circumvent this difficulty. The linear optics scheme [15] uses quantum interference between qubit photons and auxiliary photons to generate an effective non-linear interaction between qubit photons. Other approaches include employing trapped atoms in a cavity [16,17] or Rydberg atoms [18,19] to realize two-qubit gates.

In this work, we propose an alternative scheme for photonic quantum computation: using strong coupling between local emitters and photons in a one-dimensional (1D) waveguide. Because of recent tremendous experimental progress [8–10,20–26], 1D waveguide systems are becoming promising candidates for quantum information processing. A variety of capabilities has been proposed [27–35], particularly at the single-photon level, yet protocols compatible with current waveguide setups for some important tasks, notably, two-qubit gates, have rarely been investigated [36]. In our proposal, we construct photonic two-qubit gates solely based on scattering in a waveguide system that is accessible in current experiments. The photonic qubits are initialized by and returned to quantum memories [11,12] in order to realize long-term storage. Compared with the cavity approach, our setup is simplified and avoids the complexity of stabilizing the resonance between the cavity modes and the atom. The gate has a wide bandwidth, and its operation time is determined solely by the coupling strength. Combining the simplicity of the system and the scalability of photons, our waveguide-QED-based scheme opens a new avenue towards scalable quantum computation and distributed quantum networks [37] in a cavity-free setting.

The photonic qubits are encoded in the frequency degree of freedom \( |\omega_0\rangle \) and \( |\omega_1\rangle \) for simplicity; a straightforward generalization of our scheme is applicable to polarization-encoded qubits [38]. Single photons can be generated from the emission of quantum dots [8,9,25] or using circuit-QED systems [10], and single-qubit rotations can be realized using a Mach-Zehnder interferometer [39,40]. Hence, we focus on two-qubit gates and, in particular, a π-phase (CONTROLLED-NOT) gate. We consider a semi-infinite 1D waveguide side coupled to a four-level system that is located a distance \( a \) from the end (Fig. 1). Such a setup can be realized in a variety of experimental systems using superconducting transmission lines [10,23,24,41], diamond nanowires coupled to nitrogen-vacancy centers [22], photonic-crystal waveguides coupled to quantum dots [9], hollow fibers with trapped cold atoms [21], or plasmonic nanowires [20]. We now show that a π-phase gate between two photons \( A \) and \( B \) can be realized by reflecting them from the end of the waveguide.

The Hamiltonian of the system (Fig. 1) is given by

\[
H = H_{\text{wg}} + \sum_{i=2}^{4} \hbar (\Omega_i - i\gamma_i/2) \sigma_{i,0}^+ + \Sigma_{a=R,L} \int dx V_\delta(x)[a^+_a(x)\sigma_{12}(x) + \sigma_{32} + \sigma_{34}] + \text{H.c.},
\]

\[
H_{\text{wg}} = \int dx \frac{\hbar c}{i} \left[ a^+_R(x) \frac{d}{dx} a_R(x) - a^+_L(x) \frac{d}{dx} a_L(x) \right],
\]

where \( a_{R,L}(x) \) are the propagation modes along the \( x \) axis of the waveguide, \( \sigma_{i,j} = |i\rangle \langle j| \), and the energy of ground state \( |1\rangle \) is the energy reference. An imaginary term models
Step 1

![Diagram of Step 1: Trapping](image1)

Step 2

![Diagram of Step 2: π phase](image2)

Step 3

![Diagram of Step 3: Retrieval](image3)

FIG. 1 (color online). Gate operation: (1) trapping, (2) π phase, and (3) retrieval. The gate sequences here illustrate the case of both photons $A$ and $B$ being in state $|\omega_1\rangle$. For this case, step (4) does not cause any change and hence is not shown. The left and right sides show the initial and final states, respectively. Inactive transitions in each step are gray colored.

The photon qubit consists of two distinct frequencies. Frequency $\omega_1$ is chosen to be on resonance with the transitions $1 \rightarrow 2$ and $3 \rightarrow 4$, i.e., $\omega_1 = \Omega_{12}$. In contrast, $\omega_0$ is far off resonance from all of the atomic transitions—an $\omega_0$ photon does not interact with the four-level system (4LS). The 4LS is initialized in $|1\rangle$. Here, we assume that we have quantum memories [11,12] available, one for qubit photon $A$ and another for $B$, and we give an example in the Supplemental Material [38].

As illustrated in Fig. 1, a π-phase gate between photon pulses $A$ and $B$ can be realized via the following steps. (1) Trapping: photon $A$ (of frequency $\omega_A$) is sent into the system. If $A$ is in state $|\omega_1\rangle$, it is trapped: the 4LS makes the transition from $|1\rangle$ to $|3\rangle$ with photon $A$ being stored in state $|3\rangle$. In addition, an auxiliary photon $C$ of frequency $\Omega_{32}$ is emitted, and the 4LS is put into $|3\rangle$. Otherwise, $A$ will come out without interacting with the 4LS. We call the output the $A'$ photon. (2) π phase: a second qubit photon $B$ (of frequency $\omega_B$) is sent into the system; it gains a π phase if both $\omega_B = \omega_1$ and the 4LS is in state $|3\rangle$. Otherwise, $B$ will either pass through without any change if $\omega_B \neq \omega_1$, or be trapped followed by the emission of a $C'$ photon of frequency $\Omega_{32}$ if $\omega_B = \omega_1$ and the 4LS is in state $|1\rangle$. (3) Retrieval of $A$: by time reversal arguments, sending in the output photon $A'$ retrieves photon $A$, which is further directed to and stored in quantum memory $A$. (4) Retrieval of $B$: in the case of $\omega_A = \omega_0$ and $\omega_B = \omega_1$, photon $B$ will be trapped by the 4LS in step (2)—we retrieve it by simply sending in the auxiliary photon $C$. In all the other cases, photon $C$ simply passes through the 4LS without any change. Photon $B$ from either step (2) or (4) is directed to and stored in quantum memory $B$. Therefore, only when $\omega_A = \omega_B = \omega_1$ is a π phase generated by their interaction with the 4LS. We now analyze each of these steps.

**Step 1: Trapping.**—For an incoming single photon $A$ in mode $|\omega_A\rangle$ and initial state $|1\rangle$ of the 4LS, the output state of the system, obtained by imposing wave function matching [35] and a hard-wall boundary condition at the end of the waveguide (we assume perfect reflection from the end of the waveguide) [36,38], is

$$|\psi^{\text{out}}_1(\omega_A)\rangle = r_{11}(\omega_A)|\omega_A\rangle \otimes |1\rangle + r_{13}(\omega_A)|\tilde{\omega}_A\rangle \otimes |3\rangle, \quad (2)$$

where

$$\tilde{\omega} = \omega - \Omega_{13},$$

$$r_{11}(\omega) = e^{2i\omega_0} - \Omega_{12} + \frac{i\Gamma}{2} + \omega + \frac{i\Gamma}{2}[e^{2i\tilde{\omega}_0} - e^{-2i\tilde{\omega}_0}],$$

$$r_{13}(\omega) = \frac{(i\Gamma/2)(e^{2i\tilde{\omega}_0} - 1)(e^{2i\tilde{\omega}_0} - 1)}{\Omega_{12} - \frac{i\Gamma}{2} - \omega + \frac{i\Gamma}{2}[e^{2i\tilde{\omega}_0} + e^{2i\tilde{\omega}_0} - 2]]. \quad (3)$$

We first illustrate the operation principle for the lossless case $\Gamma' = 0$ and then later analyze the effect of loss in detail. We assume that the key condition $2(\Omega_{12} + \Omega_{32})\alpha = 2n_1\pi$ is satisfied; in addition, we can make the trivial choice $2\omega_0\alpha = (2n_0 + 1)\pi$ ($n_0$ and $n_1$ are integers). Then, if the incoming qubit photon $A$ is in mode $|\omega_0\rangle$, $r_{11}(\omega_0) = 1$ and $r_{13}(\omega_0) = 0$ because $\omega_0$ is far off resonance from all the transitions. Hence, it will reflect from the system without change, leaving the 4LS in $|1\rangle$. On the other hand, if photon $A$ is in mode $|\omega_1\rangle$, the on-resonance interaction with the $1 \rightarrow 2$ transition gives $r_{11}(\omega_1) = 0$ and $r_{13}(\omega_1) = -1$. As a result, it will be trapped and stored in level $|3\rangle$ of the 4LS, emitting an auxiliary $C'$ photon at frequency $\omega_1 - \Omega_{13} = \Omega_{32}$.

**Step 2: π phase.**—Now send in the second qubit, photon $B$ in mode $|\omega_B\rangle$. The output state after scattering reads [38]
\[ |\phi_2^{\text{out}}(\omega_A, \omega_B)\rangle = r_{11}(\omega_A)|\omega_A\rangle \otimes |\phi_0^{\text{out}}(\omega_B)\rangle + r_{13}(\omega_A)R_3(\omega_B)|\tilde{\omega}_A\rangle|\omega_B\rangle \otimes |3\rangle, \tag{4} \]

where

\[ R_3(\omega) = \frac{-i(\Omega_{12} - \frac{2\pi}{2} - \omega)e^{2i\omega t} + \frac{2}{2} (1 - e^{2i\omega t})}{\Omega_{12} - \frac{2\pi}{2} - \omega - \frac{2}{2} (1 - e^{2i\omega t})}. \tag{5} \]

Here, we neglect the transition 3 \rightarrow 2 because \omega_{01} is chosen to be far detuned from \Omega_{32}. If photon B is in mode \ket{\omega_0}, it is far off resonance from the transitions, and, using the same value of a as above, \( r_{11}(\omega_0) = R_3(\omega_0) = 1 \) while \( r_{13}(\omega_0) = 0 \). Hence, the output state in this case is \( |\phi_1^{\text{out}}(\omega_A)\rangle \otimes |\omega_B\rangle \) — photon B is unaffected. However, if photon B is in \ket{\omega_1}, the state after scattering is

\[ |\phi_2^{\text{out}}(\omega_A, \omega_B = \omega_1)\rangle = r_{13}(\omega_A)R_3(\omega_B)|\tilde{\omega}_A\rangle|\omega_B\rangle |3\rangle + r_{11}(\omega_A)r_{13}(\omega_B)|\omega_A\rangle|\tilde{\omega}_B\rangle |3\rangle. \tag{6} \]

Two possible outcomes exist: (i) if the 4LS is in state \ket{1} after step (1), photon B will be trapped, but (ii) if the 4LS is in state \ket{3}, photon B is on resonance with transition 3 \rightarrow 4 and gains a \( \pi \) phase \( |R_3(\omega_1) = e^{i\pi}| \). The 4LS being in state \ket{3} is, of course, conditioned upon photon A in step (1) being in \ket{\omega_1}. Notice that the \( \pi \)-phase shift is independent of coupling strength \( \Gamma \), which only determines the operation bandwidth of photon pulses (for details, see the discussion of fidelity below). The robustness of the \( \pi \)-phase shift of the reflected photon is the result of a Fano resonance [27,42]: the interference of paths that bypass the 4LS with those which go through it causes the wave function to vanish to the right of the 4LS \((x > 0)\) — analogous to the well-known perfect destructive interference in transmission past a 2LS [27,42] — and at the same time cause the \( \pi \)-phase shift of the reflected photon.

\textbf{Step 3: Retrieval of A.}—By sending in the output photon from step (1), we retrieve photon A. This process is the time reversal of photon trapping. The full wave function that results is increasingly complicated; accordingly, we focus on the specific case needed — using the two conditions \( 2(\Omega_{12} + \Omega_{32})a = 2n_1 \pi \) and \( 2\omega_0a = (2n_0 + 1)\pi \) — and relegate the full wave function, useful for other cases, to the Supplemental Material [38]. The state after this step reads

\[ |\phi_2^{\text{out}}(\omega_A, \omega_B)\rangle = r_{11}(\omega_A)r_{11}(\omega_B)|\omega_B\rangle \otimes r_{11}(\omega_A)|\omega_A\rangle |1\rangle + r_{11}(\omega_A)r_{13}(\omega_B)|\tilde{\omega}_B\rangle \otimes R_3(\omega_B)|\omega_A\rangle |3\rangle + r_{13}(\omega_A)R_3(\omega_B)|\omega_B\rangle \otimes r_{13}(\omega_A)|\omega_A\rangle |1\rangle. \tag{7} \]

In our case, the factors \( r_{ij} \) and \( R_3 \) are all either 0 or \( \pm 1 \) (see Table I in the Supplemental Material). The first line of Eq. (7) corresponds to input qubits in the 00 state, line two is for 01, and the last line covers both 10 and 11.

\textbf{Step 4: Retrieval of B.}—In the case \( \omega_A = \omega_0 \) and \( \omega_B = \omega_1 \), photon B is trapped in the 4LS in step (2). Time reversal arguments imply that sending in a C photon of frequency \( \Omega_{12} \) will release photon B in this case but will simply pass through the system without interacting in the other cases. The final state after all four steps is

\[ |\phi_4^{\text{out}}(\omega_A, \omega_B)\rangle = f_1(\omega_A, \omega_B)|\omega_A\rangle|\omega_B\rangle|\omega_c\rangle |1\rangle - f_2(\omega_A, \omega_B)|\omega_A\rangle|\tilde{\omega}_B\rangle|\omega_c\rangle |1\rangle. \tag{8} \]

where

\[ f_1(\omega_A, \omega_B) = r_{11}^2(\omega_A)r_{11}(\omega_B) + r_{13}^2(\omega_A)R_3(\omega_B), \quad f_2(\omega_A, \omega_B) = r_{11}(\omega_A)R_3(\omega_A)r_{13}(\omega_B). \tag{9} \]

The second line in Eq. (8) corresponds to an input \( \omega_A = \omega_0 \) and \( \omega_B = \omega_1 \); the state \( |\omega_c\rangle \) signifies that the frequency of the C photon is now \( \omega_1 \) — it is the retrieved B photon. By filtering out the frequency \( \Omega_{32} \) and relabeling \( |\omega_c\rangle \) as \( |\omega_b\rangle \), we obtain the final state \( |\phi_f(\omega_A = \omega_1, \omega_B = \omega_b) = (-1)^j|\omega_b\rangle|\omega_c\rangle |1\rangle \), \( i, j = 0 \) or 1.

Thus, we see that the above steps give rise to the desired \( \pi \)-phase gate:

\[ U_{AB} = \exp(i\pi|\omega_A\rangle\langle\omega_1| \otimes |\omega_1\rangle\langle\omega_B|). \tag{10} \]

Here, we assume the use of quantum memories and direct photon A from step (3) to quantum memory A. Photon B from either step (2) or (4) is directed to quantum memory B after filtering out frequency \( \omega_C \).

We now analyze the gate performance by considering photon pulses with a finite spectral width \( \sigma \) and including atomic loss \((\Gamma' > 0)\). In particular, we consider Gaussian input pulses \( A, B, \) and \( C \) centered at frequencies \( \omega_1, \omega_1, \) and \( \Omega_{32} \), respectively:

\[ |\phi_{A,B} = \int d\omega\rho_{\omega A,B}g_{\sigma}(\omega_{A,B} - \omega_1)|\omega_{A,B}\rangle, \quad |\phi_C = \int d\omega\rho_{\omega C}g_{\sigma}(\omega - \Omega_{32})|\omega_C\rangle. \quad g_\sigma(\omega) \propto e^{-\omega^2/2\sigma^2}. \tag{11} \]

The corresponding temporal width is \( \Delta T = 1/(2\sigma) \). After the scattering, the final state of the system is \( |\phi_f = \int d\omega d\omega B d\omega C g_{\sigma}(\omega_{A,B})g_{\sigma}(\omega_B)g_{\sigma}(\omega_C)|\phi_4^{\text{out}}(\omega_A, \omega_B, \omega_C)\rangle. \) The fidelity of the photon-atom gate is given by

\[ F \equiv |\langle \psi |\phi_f\rangle|^2, \tag{12} \]

where \( |\psi = -|\phi_A\rangle|\phi_B\rangle|\phi_C\rangle \otimes |1\rangle \) is the target state.

The atomic loss is characterized by introducing the effective Purcell factor \( P = \Gamma/\Gamma' \). We note that large values of \( P (\gg 20) \) have been demonstrated in recent experiments using either superconducting circuits [26], photonic-crystal waveguides [9], or semiconductor nanowires [43]. To quantify the effect of loss, we define the probability of leakage \( P_L \) to be the probability of losing the photon during the operation through spontaneous emission
coherence times, which are on the order of Such an operation time is compatible with current qubit leakage probability variation. As large, and so the fidelity is limited by the large frequency of the pulse width $/C_1$.

We now make a rough estimate of the gate operation time. Since the gate fidelity is insensitive to the pulse width variation once $\Delta T$ is sufficiently large [Fig. 2(a)], we choose $\Delta T = 10\Gamma^{-1}$ for practical estimation. Using a superconducting circuit as an example, we estimate the duration of our photon-photon $\pi$-phase gate to be $30\Gamma^{-1} \sim 300$ ns for a superconducting qubit with $\Gamma = 2\pi \times 100$ MHz [26]. Such an operation time is compatible with current qubit coherence times, which are on the order of $1\mu$s [44].

An alternative $\pi$-phase gate using only a three-level system (3LS) is possible by adapting the cavity-based proposal of Ref. [16]. First, one constructs a $\pi$-phase gate between a photon qubit and the local qubit (3LS).

Then, using the photon-atom $\pi$-phase gate as a building block, a $\pi$-phase gate between two photons $A$ and $B$ can be implemented by sending them into the system successively. This proposal has the additional advantage of naturally realizing a photon-atom $\pi$-phase gate, which can be used to entangle distant quantum nodes in a large quantum network [47]. Details of the 3LS scheme can be found in the Supplemental Material [38].

In summary, we demonstrate that two-qubit gates for photonic quantum computation can be designed in 1D waveguide-QED systems. Our waveguide-based proposal has several potential advantages over quantum computation based on cavity photons or stationary qubits. The operation time here is limited only by the coupling strength, while in the cavity case, the cavity line width is the bottleneck. Also, our scheme does not require fine-tuning of the interaction time, which is often a significant source of error. In addition, our proposal is different from and has advantages over schemes based on cross-Kerr nonlinearity [13]: a real transition ($1 \rightarrow 3$) occurs in the four-level system rather than just transient induced polarization, and the $\pi$-phase shift is robust as the result of a Fano resonance [42], independent of the coupling strength and interaction time. Overall, the system proposed here can be an important building block for future on-chip quantum networks: taking superconducting circuits as an example, we can envision such a network with (i) single photons generated using microwave resonators [10], (ii) qubit photons stored in quantum memories formed from the $M$-level scheme [38], (iii) photon flow regulated by single-photon routers [26], and (iv) two-photon operations realized by our 4LS-waveguide system.

This work was supported in part by the U.S. Office of Naval Research and U.S. NSF Grant No. PHY-10-68698. H.Z. is supported by a John T. Chambers Fellowship from the Fitzpatrick Institute for Photonics at Duke University.

*hz33@duke.edu
†baranger@phy.duke.edu
[38] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.111.090502 for the generalization to a polarization qubit, derivations of results in Eqs. (2)–(8), a M-level scheme for a qubit-photon quantum memory, and an alternative 3LS scheme for π-phase gates.
[42] A. E. Miroshnichenko, S. Flach, and Y. S. Kivshar, Rev. Mod. Phys. 82, 2257 (2010).
[44] Recently, even higher coherence times (10–20 μs [45] and ~0.1 ms [46]) have been achieved by placing the qubit in a 3D cavity.