

HAFS 2017 @ OSU

Schedule and Abstracts

The conference will take place February 3-5 in the Cockins Hall 240 (adjacent to the math tower). Breakfast and Lunch will be provided on Saturday and Sunday outside Cockins Hall 240. The conference dinner and poster session on Saturday evening will take place on the 7th floor of the Math Tower.

Friday

- 1:45 - 2:30 PM *Greetings, coffee, and refreshments.*
2:30 - 3:15 PM *Conference Plenary Talk I:* Karoly Simon, Budapest University of Technology
3:30 - 4:15 PM Malabika Pramanik, University of British Columbia
4:15 - 4:45 PM *Break*
4:45 - 5:30 PM Erin Pearse, Cal Poly San Luis Obispo

Saturday

- 8:30 - 9:30 AM *Catered breakfast and coffee.*
9:30 - 10:15 AM Alex Iosevich, University of Rochester
10:30 - 11:15 AM Pablo Shmerkin, Universidad Torcuato di Tella
11:30 - 12:15 PM Laura Cladek, University of British Columbia
12:30 - 2:00 PM *Catered lunch and coffee.*
2:00 - 2:45 PM Azita Mayeli, The Graduate Center, City University New York
3:00 - 3:45 PM Semyon Dyatlov, Massachusetts Institute of Technology
3:45 - 4:15 PM *Break*
4:15 - 5:00 PM Eyvindur Palsson, Virginia Tech
5:15 - 7:15 PM *Poster session and conference dinner - by invitation.*

Sunday

- 8:00 - 9:00 AM *Catered breakfast and coffee.*
9:00 - 9:45 AM Mahya Ghanderhari, University of Delaware
10:00 - 10:45 AM *Conference Plenary Talk II:* Karoly Simon, Budapest University of Technology
10:45 - 11:15 AM *Coffee break*
11:15 - 12:00 PM Sze-Man Ngai, Georgia Southern University
12:15 - 1:00 PM Mariusz Urbanski, University of North Texas

Friday Talks

On the algebraic sum of a planar set and the unit circle

Karoly Simon
Budapest University of Technology

1:45 - 2:30 PM

Given a set $A \subset \mathbb{R}^2$. We study the set of those points on the plane which are at a distance 1 from at least one of the elements of A , where "distance" means either the Euclidean distance or some other natural distances on the plane. This set is $A + S^1$, where S^1 is the unit circle in the given distance. Our goal is to understand for which A is the set $A + S^1$ big in the sense that it is a set of positive Lebesgue measure or even if it contains interior points. This is a work in progress, joint with Krystal Taylor.

Sets and configurations

Malabika Pramanik
University of British Columbia

3:30 - 4:15 PM

This talk will offer a general overview of results in geometric measure theory focusing on the existence or absence of patterns in sets.

Tube formulas for fractals: complex dimensions and Minkowski (non)measurability

Erin Pearse
Cal Poly San Luis Obispo

4:45 - 5:30 PM

A classical result of Steiner shows that for convex compact subsets $A \subseteq \mathbb{R}^d$, the volume of the region within ϵ of A takes the form of a polynomial, and the coefficients in this polynomial are key geometric invariants of A . I will describe an analogue of this formula for self-similar sets; the formula is given naturally in terms of the poles of a zeta function associated to the underlying iterated function system. These poles are called "complex dimensions" and their geometric configuration in \mathbb{C} provides information about the geometry of the fractal. In particular, they indicate whether or not the fractal is Minkowski measurable.

Saturday Talks

Tiling, bases and Erdős combinatorics

Alex Iosevich
University of Rochester

9:30 - 10:15 AM

We are going to discuss a very simple combinatorial problem: how large does a subset of a d -dimensional vector space over a finite field need to be to ensure that it determines every possible direction? We are also going to discuss a simple question in number theory: what does the zero set of the Fourier transform of an indicator function of a set in a d -dimensional vector space over a finite field tell us about the structure of this set? We are going to see that in combination, these two problems lead to the resolution of the Fuglede conjecture in two-dimensional vector spaces over \mathbb{Z}_p . Continuous variants of these problems will also be discussed in the same context.

Furstenberg's intersection conjecture and the L^q norm of convolutions

Pablo Shmerkin
Universidad Torcuato di Tella

10:30 - 11:15 AM

I will present some of the ideas involved in the recent solution of a conjecture of Furstenberg on the dimension of the intersections of $\times 2$ and $\times 3$ -invariant sets. In particular, I will discuss a result on the flattening of the L^q norm under convolutions of discrete measures, which is one of the main tools of the proof and may have other applications.

Radial Fourier multipliers

Laura Cladek
University of British Columbia

11:30 AM - 12:15 PM

Let m be a radial multiplier supported in a compact subset away from the origin. For dimensions $d \geq 2$, it is conjectured that the multiplier operator T_m is bounded on $L^p(\mathbb{R}^d)$ if and only if the kernel $K = \hat{m}$ is in $L^p(\mathbb{R}^d)$, for the range $1 < p < 2d/(d+1)$. Note that there are no a priori assumptions on the regularity of the multiplier. This conjecture belongs near the top of the tree of a number of important related conjectures in harmonic analysis, including the Local Smoothing, Bochner-Riesz, Restriction, and Kakeya conjectures. We discuss new progress on this conjecture in dimensions $d = 3$ and $d = 4$. Our method of proof will rely on a geometric argument involving sizes of multiple intersections of three-dimensional annuli.

Tight wavelet frames in finite vector spaces

Azita Mayeli
The Graduate Center, City University New York

2:00 - 2:45 PM

In this talk we shall show how to construct tight wavelet frames in $L^2(\mathbb{F}_q^d)$, where q is prime and \mathbb{F}_q^d , $d \geq 1$, is the vector space over the cyclic space of prime order.

Fractal uncertainty principle

Semyon Dyatlov
Massachusetts Institute of Technology

3:00 - 3:45 PM

Let $X, Y \subset [0, 1]$ be two Ahlfors-David regular sets of dimension $\delta < 1$, and denote by $X(h), Y(h)$ their h -neighborhoods. In this talk I show the following estimate uniform as $h \rightarrow 0$: each function which is Fourier supported on $h^{-1} \cdot Y(h)$ has at most $\sim h^\beta$ of its L^2 mass localized on $X(h)$, for some $\beta > \max(0, 1/2 - \delta)$. This implies that no quantum state can be localized near a fractal set of dimension > 1 in both position and frequency. The case $\beta > 0$ uses the Beurling-Malliavin multiplier theorem and harmonic measure bounds and the case $\beta > 1/2 - \delta$ is an adaptation of the methods of Dolgopyat and Naud.

I will also explain how the above result implies that every convex co-compact hyperbolic surface has an essential spectral gap, i.e. a strip to the left of the unitarity axis $\Re s = 1/2$ which only has finitely many zeroes of the Selberg zeta function, and discuss applications to quantum chaos and partial differential equations.

This talk is based on joint works with Jean Bourgain, Long Jin, and Joshua Zahl.

Falconer type theorems for simplices

Eyvindur Palsson
Virginia Tech

4:15 - 5:00 PM

Finding and understanding patterns in data sets is of significant importance in many applications. One example of a simple pattern is the distance between data points, which can be thought of as a 2-point configuration. Two classic questions, the Erdős distinct distance problem, which asks about the least number of distinct distances determined by N points in the plane, and its continuous analog, the Falconer distance problem, explore that simple pattern. Questions similar to the Erdős distinct distance problem and the Falconer distance problem can also be posed for more complicated patterns such as triangles, which can be viewed as 3-point configurations. In this talk I will present recent progress on Falconer type problems for simplices. The main techniques used come from analysis and geometric measure theory.

Sunday Talks

Does $\ell_w^1(S)$ have stable character property?

Mahya Ghandehari
University of Delaware

9:00 - 9:45 AM

Let \mathcal{A} be a Banach algebra. A bounded linear functional ϕ on \mathcal{A} is said to be *approximately multiplicative* if

$$\sup\{|\phi(xy) - \phi(x)\phi(y)| : x, y \in \mathcal{A}, \|x\|, \|y\| \leq 1\} \quad (1)$$

is small. For example if ϕ is multiplicative then the quantity defined in (1) is zero. In this talk, we consider approximately multiplicative functionals on weighted convolution algebras of semilattices, and study whether all such functionals arise as small perturbations of multiplicative functionals. A Banach algebra with such a property is said to have *stable characters*. For a semilattice S and a weight function $w : S \rightarrow [1, \infty)$, we give intrinsic conditions which answer the question whether $\ell_w^1(S)$ has stable characters. Our main result states that if S is a semilattice with “infinite breadth” then one can construct a weight w on S such that $\ell_w^1(S)$ does not have stable characters.

This talk is based on a joint work with Yemon Choi and Hung Le Pham.

Hausdorff dimension of certain self-affine attractors

Karoly Simon
Budapest University of Technology

10 - 10:45 AM

Self-affine Iterated Function Systems (IFS) are defined by a finite list of contracting affine transformations

$$\mathcal{S} = \{S_i(x) = A_i \cdot x + t_i\}_{i=1}^m, \quad x, t_i \in \mathbb{R}^d, \quad (2)$$

and A_i is a $d \times d$ matrix having $\|A_i\| < 1$. We can always choose a sufficiently large closed ball B for which $S_i(B) \subset B$ for all $i = 1, \dots, m$. So, $\cup_{i=1}^m S_i(B) \subset B$. The attractor of \mathcal{S} is the set of those points of B which remain if we iterate this system infinitely many times. In spite of the simplicity of the construction, it is a very difficult problem to find the Hausdorff dimension of self-affine attractors. However, in the last three years there have been a very intensive development on this field. In this talk I focus on the special case when the matrices in the formula above are triangular or even diagonal matrices. Even in these special cases we are far from complete understanding of the problem.

Spectral asymptotics of a class of one-dimensional fractal Laplacians

Sze-Man Ngai
Georgia Southern University

11:15 AM - 12:00 PM

We observe that some self-similar measures defined by finite or infinite iterated function systems with overlaps satisfy certain “bounded measure type condition”, which allows us to extract useful measure-theoretic properties of iterates of the measure. We develop a technique to obtain, under this condition, a closed formula for the spectral dimension of the Laplacian. This is a joint work with Wei Tang and Yuanyuan Xie.

Asymptotic counting in conformal dynamical systems

Mariusz Urbanski
University of North Texas

12:15 - 1 PM

In this lecture, based on joint paper with Mark Pollicott, I will consider a general setting of conformal dynamical systems (conformal graph directed Markov systems) modelled by countable state symbolic subshifts of finite type. I will address two types of problems. The first is an asymptotic counting problems for preimages or periodic orbits ordered by a natural geometric/dynamical (multipliers/diameters) weighting. The second is a central limit theorem describing further features of the distribution of the weights. The proofs are based on detailed analysis of spectral properties of associated complexified Perron-Frobenius operators and Tauberian theorems. These will have direct applications to a variety of examples, including the case of Apollonian Circle Packings, expanding and parabolic rational functions, and Kleinian groups of Schotky type.

Posters

Non-autonomous conformal graph directed Markov systems

Jason Atnip
University of North Texas

In 1979 Rufus Bowen made the connection between the zero of the pressure function and the Hausdorff dimension of quasi-circles. In this poster we generalize the works of Rempe, Urbański (2016) and Mauldin, Urbański (2003) to introduce the notion of non-autonomous conformal graph directed Markov systems. In this same vein as Bowen, we show that the Hausdorff dimension of the limit set of such a system can be found by examining the associated pressure function as it transitions from positive to negative.

Measures with large Fourier dimension avoiding all 4-term arithmetic progressions

Marc Carnovale
The Ohio State University

In recent years there has been increasing interest in which geometric patterns and finite point configurations exist within fractional sets obeying “nice” assumptions, where “nice” refers usually to a regularity or Fourier dimension condition. Though most work has focused on the positive direction, it is natural to ask about the limits to what kinds of patterns a Fourier dimension condition can guarantee. Viewing Fourier dimension through the lens of pseudorandomness, we show that “linear” pseudorandomness as guaranteed by Fourier dimension is insufficient to guarantee the existence of 4 term arithmetic progressions (4APs) within a fractional subset of the real line by constructing a set of fixed dimension arbitrarily close to 1 obeying the conditions of Laba and Pramanik’s 2009 result guaranteeing 3APs, yet containing no 4APs.

Fourier dimension of the well-approximable numbers in Euclidean and p-adic settings

Robert Fraser
University of British Columbia

We obtain a pointwise Fourier decay estimate for a measure supported on the well-approximable numbers in \mathbb{Z}_p . We compare the proof of this estimate to the proof of a similar estimate obtained by Kaufman for the well-approximable numbers in $[0, 1]$. This is joint work with Kyle Hambrook.

Packing measure of super separated iterated function systems

James Reid
University of North Texas

In the context of fractal geometry, the natural extension of volume in Euclidean space to include fractal sets is given by Hausdorff and packing measures. If J is the limit set of an iterated function system in Euclidean space satisfying the open set condition, then J is often a fractal set. It is well known that the h -dimensional packing measure of J is positive and finite when h is given by Hutchinson’s formula. Feng was able to find exact formulas for the h -dimensional packing measure for a large class of Cantor sets in the interval $[0, 1]$.

In this presentation, we introduce a super separation condition, which allows us to reduce the problem of computing the h -dimensional packing measure to checking densities of a finite number of balls around each point in J . We then use this fact to find formulas for the packing measure of classes of examples in higher dimensions.

The norm of the Fourier transform on compact or discrete abelian groups

Peng Xu
University of Delaware

We calculate the norm of the Fourier operator from $L^p(X)$ to $L^q(X)$ when X is an infinite locally compact abelian group that is, furthermore, compact or discrete. This subsumes the sharp Hausdorff-Young inequality on such groups. In particular, we identify the region in (p, q) -space where the norm is infinite, generalizing a result of Fournier, and setting up a contrast with the case of finite abelian groups, where the norm was determined by Gilbert and Rzeszotnik. As an application, uncertainty principles on such groups expressed in terms of Rényi entropies are discussed.