A Note on the Roles of Aggregation and Delay in Management Control

Anthony D. Nikias, Steven T. Schwartz, and Richard A. Young

ABSTRACT: Accounting information is often produced in an aggregate format and is delayed in its arrival. This teaching note examines the effects of aggregation and delayed arrival in the context of management control. One might expect that a reduction of information about managerial performance would reduce the efficiency of incentive schemes designed to increase goal congruence within the firm. Contrary to this intuition, aggregation, which reduces information, in some circumstances increases the efficiency of incentive schemes. This potential improvement in efficiency results because under aggregation the superior can exploit the subordinate’s uncertainty about his future compensation. However, the resultant information loss from aggregation is generally costly, and these costs may outweigh the benefits of aggregation. We further illustrate that delayed arrival of information may allow an owner to enjoy the benefits of aggregation without incurring its costs. The approach taken in this note is similar to Antle and Demski (1988) and Arya et al. (1998), in that the discussion is centered on numerical examples and placed within the context of a simple model of management control that is accessible to upper-level undergraduate and master’s students. The advantage of this approach over having students go directly to the academic literature is that the simple linear structure of the illustrations facilitates construction and solution of numerical examples using spreadsheet analysis and also exposes the intuition more easily.

INTRODUCTION

In this instructional note we examine the effects of two fundamental characteristics of accounting information: aggregation and delay. Accounting information is aggregated, to a greater or lesser degree, due to the enormity of the data produced and the limited cognitive resources of users. Delay is inherent in the data because accounting is generally historical in nature and transactions must be completed, recorded, and summarized before accounting information becomes available.

Our examination of aggregated and delayed information takes a management control perspective, which considers how accounting information can be used to motivate employees. For two reasons an instructional note on aggregation and delay from this perspective is warranted. First, the issues concerning aggregation and delay from a management control perspective are

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perspective differ from those arising from an external reporting perspective, wherein the degree of aggregation and delay is chosen to balance the often-competing needs of external users such as comprehensibility, relevance, and reliability. Second, while management accounting texts discuss the general nature in which aggregation and delay of accounting information affect its usefulness to internal users, recent theoretical research that explicitly examines these attributes from a management control perspective has not yet been made accessible for those below the advanced graduate level. Therefore, this note complements the existing instructional material on these attributes of accounting, while introducing a broader audience to important new research findings.

Modern theories of the firm consider effective management control systems crucial to the success of the organization. The most prominent reasons for this belief are the inherently dissimilar goals of owners and employees and the inability of firm owners to monitor and direct the actions of every employee. In response, employee performance evaluation and compensation schemes are established to motivate behaviors desired by ownership. The importance of such schemes is highlighted in Milgrom and Roberts (1992) and Jensen (2003), who provide numerous examples of improper evaluation and compensation schemes adversely affecting organizations. Well-designed employee performance evaluation and compensation are an integral component of an effective management control system.

Research in management control has shown that, absent costs to acquire and process information, evaluation systems should use any and all obtainable data that provides information regarding employees’ unobservable actions (Antle and Demski 1988; Demski 1977; Holmstrom 1979). Much attention has been given to why accounting data is a useful source of information about the unobservable actions of employees, with verifiability and reliability cited as the two primary reasons. Given these findings, it would appear that accounting data should be disaggregated as much as possible for use as a performance evaluation tool.

However, earlier research focused on relatively simple settings, usually with employees performing a single task. In contrast, employees are often assigned multiple tasks that are completed in sequence. For example, an executive might be responsible for launching a product in the domestic market and also for subsequently launching the same product in an international market. Recent research has demonstrated that under such circumstances aggregation can improve the efficiency of control systems (Arya et al. 2004; Gigler and Hemmer 2002). This belief is seemingly contrary to prior findings and common intuition, because aggregation necessarily destroys information. The benefits to aggregation result from superiors’ ability to exploit the uncertainty of employees at the time they perform subsequent tasks. Delaying the arrival of information also provides this benefit, without incurring the costs associated with aggregation due to lost information.

Our objective is to synthesize and summarize the relatively recent theoretical results regarding aggregation and delay in a way that makes them intuitive and accessible to first-year doctoral students, master’s students, and upper-level undergraduate students. In order to implement this learning objective, we employ a simple setting wherein a superior is concerned with designing an accounting information system that minimizes the expected compensation necessary to induce desired behavior from a subordinate. Our approach is to provide an intuitive discussion centered on numerical examples, rather than to present formal theorems and proofs.

1 Demski (1997), a graduate-level text, presents a numerical example on aggregate and disaggregate information using a setting different than the one in this paper.

2 Other research that has studied aggregation and timing issues from a management accounting perspective includes Chang et al. (2003), Christensen et al. (2003), Dutta and Zhang (2002), and Dye and Sridhar (2004).
In addition to numerical examples included in the text, we provide in Appendices A and B a more general formulation of the management control problems in this note. In Appendix C we provide exercises to help students further develop their understanding. The exercises can be completed using widely available spreadsheet software that is capable of solving linear programs. However, for those not interested in mathematical exercises, the text is sufficient to gain an understanding of the intuition for why aggregation and delay of accounting information can have such pronounced effects on managerial compensation.

The outline of the paper is as follows. The second section demonstrates that aggregation can reduce the expected compensation necessary to motivate an employee. In the third section, we illustrate that aggregation is not always costless—sometimes the reduction in information resulting from aggregated accounting data decreases the informativeness of the performance measure to the extent that the benefit of aggregation is outweighed by its cost. In the fourth section we demonstrate that if information only becomes available late, the superior may enjoy the benefits of aggregation without incurring its costs. The fifth section provides a summary of the results as well as limitations of our model. The sixth section provides some guidelines for the instructor as to how these materials can be introduced into the classroom. We also provide technical details and student exercises in the appendices.

THE BENEFITS OF AGGREGATION

In order to clearly expose the main effects of aggregation on performance evaluation, we employ a model of the firm. By definition, a model of the firm does not capture all of its aspects, for if it did so it would be both intractable and pointless (since one might as well just observe an actual firm). Therefore, simplified models are a useful tool for discovering and communicating the most important forces affecting the phenomena of central interest. We begin our illustration with a simple, single-task setting.

**Single-Task Setting**

The phenomenon of interest in this instructional note is the role of accounting information in management control. For such a model to be useful, several features are crucial:

1) There must be at least two people—we will assume exactly two people, and shall refer to them as owner and manager.
2) The owner and manager must have different preferences regarding the actions of the manager.
3) The owner must be unaware of the actions of the manager.
4) The model must be rich enough so that the management control problem is not trivially solved.

In the model we assume that a firm consists of an owner who hires a single manager (see (1) above). The manager performs a single task that results in one of two outcomes: 100 or 0. The likelihood of obtaining 100 or 0 is determined by the manager’s effort. The manager can either put forth high effort or low effort on the task. In our example, the probability of obtaining 100 (0) is .8 (.2) if the manager chooses high effort, and the probability of obtaining 100 (0) is .48 (.52) if the manager chooses low effort.

The owner and manager each maximize their own utility. The owner’s utility is defined as the firm’s expected profit. The manager’s utility is defined as his expected monetary

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3 For most of the discussion we will assume that maximization of the owner’s profits is obtained by inducing high effort from the manager. This implies that profit maximization simplifies to minimizing the expected compensation necessary to motivate high effort.
compensation, less the disutility of his effort.\textsuperscript{4} Let the manager’s disutility for high and low effort be 10 and 0, respectively. We assume that an outcome of 100 is sufficiently valuable to the owner that she wishes the manager to provide high effort despite the necessity of higher compensation. These assumptions ensure that the owner and manager have differential preferences over the manager’s choice of effort (see (2) above).

The owner cannot observe the manager’s effort (see (3) above). The manager is assumed to have no resources outside of his compensation, so he can never be required to make payments to the firm. It is natural to further assume the manager can obtain employment elsewhere. We operationalize this simply by assuming the manager can obtain an expected utility of zero from this alternative employment.\textsuperscript{5} These assumptions prevent the management control problem from being trivially resolved (see (4) above).

In order to obtain high effort from the manager, the owner must commit to a compensation scheme, or contract, that satisfies three constraints. The first constraint ensures the manager is willing to join the firm. We refer to this constraint as participation, denoted (P). The second constraint ensures the manager receives at least as much compensation from providing high effort as from providing low effort. We refer to this constraint as incentive compatibility, denoted (IC). The third constraint ensures that the manager does not receive negative compensation. We refer to this constraint as limited liability, denoted (LIM). The owner’s goal is to set the payments to the manager for each potential outcome such that all three constraints are satisfied with the least expected compensation.

Below we provide a mathematical formulation of the contracting problem. We use the notation \(s_1, s_0\) to denote the payment to the manager for outcomes 100 and 0, respectively. The objective function is equal to the expected compensation paid to the manager given he provides high effort, which is a linear function of the outcome-contingent payments.

Single task compensation contract program:

\[
\begin{align*}
\text{Objective: Minimize } & \quad 0.8 s_1 + 0.2 s_0 \\
\text{Subject to: } & \\
& 0.8 s_1 + 0.2 s_0 - 10 \geq 0 \quad (P) \\
& 0.8 s_1 + 0.2 s_0 - 10 \geq 0.48 s_1 + 0.52 s_0 \quad (IC) \\
& s_1, s_0 \geq 0 \quad (LIM)
\end{align*}
\]

The optimal solution to this contracting program is \(s_1 = 31.25\) and \(s_0 = 0\).\textsuperscript{6} Notice that if the manager puts forth high effort, then he receives an expected utility of \(15 = 0.8(31.25) + 0.2(0) - 10\). If the manager puts forth low effort, then he receives an expected utility of \(15 = 0.48(31.25) + 0.52(0)\). Hence, (IC) is satisfied. The fact that (P) and (LIM) are also satisfied is immediate. The expected payment to the manager under this compensation plan is 25 = 0.8(31.25).

\textsuperscript{4} One might argue that managers like to work. The misalignment of goals within the firm due to the disutility of managerial effort is just one interpretation. The important issue is that, without proper incentives, the manager would choose an action other than that desired by the owner.

\textsuperscript{5} Assuming that the manager’s personal cost of low effort is zero and the outside opportunity yields an expected utility of zero is the simplest way to guarantee that an interesting management control problem obtains in this setting. In particular, these assumptions prevent the problem from being resolved trivially by a franchise agreement wherein the manager pays a fixed fee to obtain the rights to make all decisions and keeps all residual profits.

\textsuperscript{6} The solution to this and all succeeding formulations can be found using spreadsheet software capable of solving linear programs.
Even with these extreme simplifications, this setting is still a powerful investigative tool, producing two salient results. The first result is that the manager cannot be paid a fixed wage and be expected to perform as the owner wishes, unless the owner is satisfied with a minimal level of effort. The second result is that the owner must pay the manager so that his expected utility (15) is greater than what he would receive from his next best opportunity (0). The first result helps explain why employees who perform distasteful and difficult-to-observe tasks (such as salespeople) are given incentive contracts. The second result helps explain why employees who can perform specialized tasks for the firm earn more than the opportunity cost of their time.

Two-Task Setting

Although the preceding example is useful in illustrating the fundamental concepts of management control, there is not enough complexity to explore the role of aggregation. With a single-task outcome, there is nothing to aggregate. In order to explore aggregation, we now assume that the manager is hired to perform two sequential tasks. Each task is identical to the task introduced in the above example.

Before proceeding to a mathematical formulation of the two-task setting, we explain what we mean by disaggregate and aggregate information. A disaggregate accounting system tracks the two task outcomes independently, so that both owner and manager observe the outcome of the first task before the manager performs the second task. That is, the outcomes are not added together. An example of disaggregation in practice is retailers that track sales for individual stores even though such data is not required to meet financial reporting standards. An aggregate accounting system reports only the sum of the outcomes from the two tasks, which implies no information is available prior to completion of the second task. An example of aggregation in practice is a weekly or monthly sales report, rather than a daily or transaction-by-transaction sales report.

We begin by examining the disaggregate case. In order to induce high effort from the manager on both tasks the owner must design a contract which specifies payments to the manager for each of the possible outcomes: (100, 100), (100, 0), (0, 100), and (0, 0). The addition of a second task increases the number of constraints the contract must satisfy. The design of the contract must ensure that: (1) the manager is willing to join the firm, (2) the manager prefers to choose high effort on the first task, (3) the manager prefers to choose high effort on the second task, regardless of the observed outcome of the first task, and (4) the manager never receives negative compensation.

We use the notation $s_{11}$, $s_{10}$, $s_{01}$, and $s_{00}$ to denote the payment for outcomes (100, 100), (100, 0), (0, 100), and (0, 0), respectively. For simplicity, we assume the task outcomes are statistically independent, given the manager’s effort levels. It is useful to denote the manager’s expected compensation assuming he chooses high effort on both tasks by $E[C|HH]$, calculated as follows:

$$E[C|HH] = .64 s_{11} + .16 s_{10} + .16 s_{01} + .04 s_{00}.$$  

Below is a mathematical formulation of the contracting problem. As before, the objective function is equal to the expected compensation paid to the manager under the assumption that the manager takes the desired actions. The first three incentive compatibility

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7 Because the outcomes on each task are independent, the probability of obtaining any outcome combination is the product of the probabilities for each outcome. For example, the probability of obtaining 100 on the first task and 0 on the second task given high effort on both tasks is $.8 (.2) = .16$.
constraints, denoted (IC-HL), (IC-LH), and (IC-LL), are to ensure high effort on both tasks is preferred to: (1) high effort on one task and low on the other, and (2) low effort on both tasks. The last two incentive compatibility constraints, denoted (IC-100) and (IC-0), are to ensure that for each possible outcome on the first task, 100 or 0, high effort is preferred to low effort on the second task. The participation constraint (P) and the limited liability constraint (LIM) are similar to those found in the single-task setting. A general description of the mathematical formulation of the two-outcome, identical task setting can be found in Appendix A.8

Two-task compensation contract program—Disaggregate case with two outcomes:

Minimize \[.64s_{11} + .16s_{10} + .16s_{01} + .04s_{00}\]

Subject to:

\[E[C|HH] - 20 \geq 0\] (P)

\[E[C|HH] - 20 \geq .3840s_{11} + .4160s_{10} + .0960s_{01} + .1040s_{00} - 10\] (IC-HL)

\[E[C|HH] - 20 \geq .3840s_{11} + .0960s_{10} + .4160s_{01} + .1040s_{00} - 10\] (IC-LH)

\[E[C|HH] - 20 \geq .2304s_{11} + .2496s_{10} + .2496s_{01} + .2704s_{00}\] (IC-LL)

\[.8 s_{11} + .2 s_{10} - 10 \geq .48 s_{11} + .52 s_{10}\] (IC-100)

\[.8 s_{01} + .2 s_{00} - 10 \geq .48 s_{01} + .52 s_{00}\] (IC-0)

\[s_{11}, s_{10}, s_{01}, s_{00} \geq 0\] (LIM)

Table 1 presents the optimal solution for both the disaggregate and aggregate cases of the two-task setting, assuming the owner wishes to motivate high effort on both tasks.9 Expected compensation in the disaggregate case for the two tasks combined is 50, which is twice the expected compensation found in the single-task setting.10 We note that due to the linearity of the contracting problem the solution is not unique. Alternative solutions exist that also yield an expected compensation of 50, which pay less for an outcome of (100, 0) but more for an outcome of (100, 100).11

Shifting our attention to the aggregate case, the most obvious difference is that there are fewer potentially observable outcomes. The three possible summed outcomes are 0 (0 on both tasks), 100 (100 on one task and 0 on the other), and 200 (100 on both tasks). It is important to note that a reduction in information results from aggregation—an aggregated outcome of 100 can be attained by an outcome of 0 on the first task and 100 on the second, or 100 on the first task and 0 on the second. Aggregation reduces the information available to both the owner and the manager.

8 The mathematical formulation for the disaggregate case can be written in a reduced form. Writing it in this way facilitates comparison with the aggregate case.
9 Exercise 1 in Appendix C asks the student to verify the solution in Table 1.
10 This is not a coincidence. The simple structure allows the disaggregate solution to be written as a “memoryless” contract (Fellingham et al. 1985). A memoryless contract is one that is equivalent to writing separate contracts for each task. The ability to write the optimal solution in the disaggregate case as a memoryless contract implies that the owner receives no benefit from combining performance on both tasks within a single contract.
11 We chose to provide the memoryless solution in Table 1. One alternative solution is for the owner to pay 70.3125 for an outcome of (100, 100) and 31.25 for an outcome of (0, 100). Shifting some payment from \(s_{10}\) to \(s_{11}\) remains optimal, because the (100, 0) and (100, 100) outcomes are equally informative in the binding constraint (IC-LH). See Kreps (1990) for an extended discussion of informativeness. If students do not arrive at the same values for the payments as those in Table 1, then we suggest they enter the Table 1 solution in their spreadsheet to verify that it also satisfies all the constraints and yields expected compensation of 50.
TABLE 1
Least-Cost Contracts with Two Identical Tasks and Two Outcomes for Disaggregate and Aggregate Accounting Systems

<table>
<thead>
<tr>
<th>Parameters for Each Task</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential outcomes</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.80</td>
<td>.20</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.48</td>
<td>.52</td>
</tr>
<tr>
<td>Disutility of high effort</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Disutility of low effort</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Manager expected utility from alternative employment</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Least-Cost Contract under Disaggregate System

<table>
<thead>
<tr>
<th>s_{11}</th>
<th>s_{10}</th>
<th>s_{01}</th>
<th>s_{00}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment</td>
<td>62.50</td>
<td>31.25</td>
<td>31.25</td>
</tr>
<tr>
<td>Expected compensation paid by owner</td>
<td>50.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Least-Cost Contract under Aggregate System

<table>
<thead>
<tr>
<th>s_{11}</th>
<th>s_{10}</th>
<th>s_{01}</th>
<th>s_{00}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment</td>
<td>48.83</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Expected compensation paid by owner</td>
<td>31.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note under an aggregate system s_{10} = s_{01} is required, which is the amount paid whenever the total outcome is 100.

As she did in the disaggregate case, the owner must ensure that the manager participates in the firm, works hard on both tasks, and receives non-negative payments. When the contract is based on the aggregated outcome, the manager’s willingness to work hard on the second task will be independent of the outcome on the first task, because the manager does not observe the outcome on the first task when making his second effort choice. In addition, the owner’s contract must reflect an inability to distinguish between the outcomes (0, 100) and (100, 0).

The contracting program for the aggregate case can be found by modifying the contracting program for the disaggregate case as follows: (1) eliminate the (IC-100) and (IC-0) constraints, and (2) add the constraint that s_{10} = s_{01}. The elimination of (IC-100) and (IC-0) follows because of the manager’s uncertainty of the first task outcome when choosing effort for the second task. The s_{10} = s_{01} constraint ensures that the payment scheme chosen by the owner matches her ability to observe outcomes.

Of particular note in the above example is that forcing the payments for outcomes (100, 0) and (0, 100) to be equal is not restrictive for the owner. Inspecting Table 1 reveals that in the disaggregate case the optimal payments for outcomes (100, 0) and (0, 100) are equal, even though the owner can distinguish between the two. Put differently, the constraint unique to the aggregation case did not impose any costs on the owner, and the removal of constraints found in the disaggregate regime may provide a benefit. In fact, examining the aggregate solution in Table 1, we see that the owner decreases expected compensation from...
50 to 31.25, while still satisfying the aggregate constraints. In the aggregate case the owner pays zero to the manager for a total outcome of 100, which would violate the (IC-100) constraint in the disaggregate case. If in the disaggregate case the owner attempted to pay zero for outcome (0, 100), equivalent to an aggregate outcome of 100, then this would not motivate high effort on the second task given a first task outcome is 0, because the manager would not obtain positive expected total compensation, regardless of the second task outcome. The above example illustrates the benefit obtained by the owner of not having the manager observe the first task outcome prior to the second task effort choice. More precisely, it shows that under aggregation benefits ensue because fewer incentive compatibility constraints must be satisfied.

Another way to illustrate the benefit of aggregation is to consider the potential strategies of the manager under both information regimes (see Figure 1). In the disaggregate case the manager has eight potential strategies, corresponding to all combinations of: (1) high or low effort on the first task, (2) high or low effort on the second given an outcome of 0 on the first task, and (3) high or low effort on the second task given an outcome of 100 on the first task. In the aggregate case, the manager has four potential strategies, corresponding to all combinations of: (1) high or low effort on the first task and (2) high or low effort on the second task. Hence, under disaggregation the owner must ensure that the desired strategy is at least as good for the manager as the other seven strategies, while under aggregation the desired strategy must be at least as good as only three other strategies. In the disaggregate case the contract must satisfy additional incentive compatibility constraints, which in turn imposes a cost on the owner.12

The preceding example demonstrates that the owner can obtain considerable savings in expected compensation under an aggregate accounting system relative to a disaggregate system. The owner is able to exploit the manager’s uncertainty about the outcome of the first task, and hence about his future compensation, to provide incentives more efficiently than in a disaggregate system.

### THE COSTS OF AGGREGATION

A natural question that arises at this point is whether the reduction in information resulting from aggregation is ever costly. We first note that in the simple setting in the previous section aggregation would always be costless and, hence, would lead to lower expected compensation. The explanation can be found in the nature of the optimal compensation scheme. Under aggregation the manager receives positive compensation only if an outcome of 100 is obtained on both tasks. Further, for any two-outcome example with a risk-neutral manager and limited liability, it can be shown that the owner would only compensate the manager for an outcome of 200, even if she could tell whether the first or second task led to an outcome of 100 (Nikias et al. 2005).13 That is, the reduction in information is not harmful to the owner because she would not use it even if she had it. Therefore, we must introduce a more complex setting in order to see whether there might be costs to aggregation, and further whether these costs ever exceed the benefits. Of course, only if there are costs to aggregation can disaggregation ever be preferred.

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12 The simple structure of our model implies that one incentive compatibility constraint in the disaggregate contracting program can rule out two of the manager’s strategies, as described in Figure 1. In particular, (IC-0) rules out HLH and LLH and IC-100 rules out HHL and LHL.

13 This statement also holds for the case where the two tasks are not identical. Exercise 2 in Appendix C provides an illustration.
FIGURE 1
Comparison of Available Manager Strategies under Disaggregate and Aggregate Accounting Systems for Two-Outcome Tasks

<table>
<thead>
<tr>
<th>Manager Strategy</th>
<th>Disaggregation (all strategies are possible)</th>
<th>Corresponding Strategy under Aggregation</th>
</tr>
</thead>
</table>
| HHH^a            | High effort task 1  
High effort task 2 given 0 on task 1  
High effort task 2 given 100 on task 1 | High effort task 1  
High effort task 2 |
| HHL              | High effort task 1  
High effort task 2 given 0 on task 1  
Low effort task 2 given 100 on task 1 (IC-100) | Not possible |
| HLH              | High effort task 1  
Low effort task 2 given 0 on task 1  
High effort task 2 given 100 on task 1 (IC-0) | Not possible |
| HLL              | High effort task 1  
Low effort task 2 given 0 on task 1  
Low effort task 2 given 100 on task 1 (IC-HL) | High effort task 1, low effort task 2 (IC-HL) |
| LHH              | Low effort task 1  
High effort task 2 given 0 on task 1  
High effort task 2 given 100 on task 1 (IC-100) | Low effort task 1, high effort task 2 (IC-LH) |
| LHL              | Low effort task 1  
High effort task 2 given 0 on task 1  
Low effort task 2 given 100 on task 1 (IC-LH) | Not possible |
| LLH              | Low effort task 1  
High effort task 2 given 0 on task 1  
Low effort task 2 given 100 on task 1 (IC-100) | Not possible |
| LLL              | Low effort task 1  
Low effort task 2 given 0 on task 1  
Low effort task 2 given 100 on task 1 (IC-LL) | Low effort task 1, low effort task 2 (IC-LL) |

^a Manager strategy motivated by owner’s contract
Parentheses indicate which incentive compatibility constraint rules out that strategy.

For costs to aggregation to be present, there must be a setting wherein aggregation causes the owner to lose information that she would wish to use in determining compensation for the manager. We create such a setting by making two enrichments to the previous model. First, we introduce a third possible outcome on the assigned tasks (Arya et al. 2004). In particular, we now assume the manager performs two tasks that can each result in one of three possible outcomes: 200, 100, or 0. Therefore, when the outcomes are aggregated the owner can only observe outcomes of 400, 300, 200, 100, or 0. Second, we assume that the tasks are not identical, in that the probability of obtaining a particular outcome given an effort level is not identical across the tasks.
There are two important implications of these enrichments. First, as in the identical task case, if an aggregated outcome of 100 occurs, the owner cannot tell which task produced an outcome of 100 and which produced an outcome of 0. The ability to discern which task produced an outcome of 100 is important for contracting in the different task case, whereas it is unimportant in the identical task case.\textsuperscript{14} Informativeness about effort is generally important in determining payments, and in the different task case the informativeness of the outcome depends on the task (Kreps 1990). Second, if an aggregated outcome of 200 occurs the owner cannot distinguish between: (1) outcomes of 100 on both tasks and (2) an outcome of 200 on one task and an outcome of 0 on the other. This aspect of aggregation is different from the two-outcome setting, where if the manager observed an aggregated outcome of 100 she knew it arose from 0 on one task and 100 on the other. Thus, in the three-outcome setting an aggregate outcome of 200 could be attributable to either excellent performance on one of the two tasks and poor performance on the other, or to intermediate performance on each of the two tasks. For these reasons the setting with three or more outcomes per task \textit{and} different tasks provides a useful platform for studying the information loss due to aggregation.

Aggregation still affords the owner the benefits described in the preceding section. However, with dissimilar three-outcome tasks, it is now possible that the costs of aggregation outweigh its benefits, causing the owner to prefer disaggregation.\textsuperscript{15} Table 2 presents a numerical example for non-identical tasks with three possible outcomes for each task.

The probability of task outcomes 200, 100, and 0 arising from high and low effort on the first task are {.1, .5, .4} and {.1, .4, .5}, respectively; on the second task they are {.1, .8, .1} and {.1, .1, .8}, respectively. The notation is identical to that used in the preceding section, except that a subscript of 2 represents an outcome of 200. For example, \( s_{02} \) denotes the payment for outcome (0, 200). The manager’s expected compensation if he chooses high effort on both tasks is as follows:

\[
E[C|HH] = .01s_{22} + .08s_{21} + .01s_{20} + .05s_{12} + .40s_{11} + .05s_{10} + .04s_{02} + .32s_{01} + .04s_{00}.
\]

Below we provide a mathematical formulation of the problem for the Table 2 setting with disaggregated outcomes. A general description of the mathematical formulation of the owner’s problem for two different three-outcome tasks is found in Appendix B.

Two-task compensation contract program—Disaggregate case with three outcomes:

Minimize \( .01s_{22} + .08s_{21} + .01s_{20} + .05s_{12} + .40s_{11} + .05s_{10} + .04s_{02} + .32s_{01} + .04s_{00} \)

Subject to:

\[
\begin{align*}
EC|HH - 20 & \geq 0 \quad (P) \\
EC|HH - 20 & \geq .01s_{22} + .01s_{21} + .08s_{20} + .05s_{12} + .05s_{11} \\
& + .40s_{10} + .04s_{02} + .04s_{01} + .32s_{00} - 10 \quad (IC-HL)
\end{align*}
\]

\textsuperscript{14} In this three-outcome setting, the owner faces similar problems when an aggregated outcome of 300 is observed.

\textsuperscript{15} It should be noted that dissimilar three-outcome tasks do not ensure that aggregation is costly. Exercise 3 in Appendix C provides an illustration.
TABLE 2
Comparison of Aggregate, Disaggregate, and Delayed Disaggregate Accounting Systems

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Task 1</th>
<th>Task 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential outcomes</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.1</td>
<td>.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Task 1</th>
<th>Task 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of high effort to manager</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Cost of low effort to manager</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Manager’s expected utility from alternative employment</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Panel A: Least-Cost Contract under Disaggregate and Aggregate Systems

Disaggregate System

<table>
<thead>
<tr>
<th>Parameters</th>
<th>s₁₂</th>
<th>s₁₁</th>
<th>s₁₀</th>
<th>s₀₁</th>
<th>s₀₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>14.29</td>
<td>14.29</td>
</tr>
<tr>
<td>Expected compensation paid by owner</td>
<td>0</td>
<td>0</td>
<td>14.29</td>
<td>61.43</td>
<td></td>
</tr>
<tr>
<td>Expected utility of manager</td>
<td>0</td>
<td>0</td>
<td>14.29</td>
<td>41.43</td>
<td></td>
</tr>
</tbody>
</table>

Aggregate System

<table>
<thead>
<tr>
<th>Parameters</th>
<th>s₁₂</th>
<th>s₁₁</th>
<th>s₁₀</th>
<th>s₀₁</th>
<th>s₀₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>142.86</td>
<td>142.86</td>
</tr>
<tr>
<td>Expected compensation paid by owner</td>
<td>0</td>
<td>0</td>
<td>142.86</td>
<td>64.29</td>
<td></td>
</tr>
<tr>
<td>Expected utility of manager</td>
<td>0</td>
<td>0</td>
<td>142.86</td>
<td>44.29</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Least-Cost Contract under Delayed Disaggregate System (information arrives only after second task outcome is obtained)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>s₁₂</th>
<th>s₁₁</th>
<th>s₁₀</th>
<th>s₀₁</th>
<th>s₀₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>125</td>
</tr>
<tr>
<td>Expected compensation paid by owner</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Expected utility of manager</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Note under an aggregate system s₀₁ = s₁₀ is required, which is the amount paid whenever the total outcome is 100; s₀₂ = s₁₁ = s₂₀ is required, which is the amount paid whenever the total outcome is 200; and s₁₂ = s₂₁ is required, which is the amount paid whenever the total outcome is 300.

\[
EC|HH - 20 \geq .01s_{22} + .08s_{21} + .01s_{20} + .04s_{12} + .32s_{11} + .04s_{10} + .05s_{02} + .40s_{01} + .05s_{00} - 10 \quad (IC-LH)
\]

\[
EC|HH - 20 \geq .01s_{22} + .01s_{21} + .08s_{20} + .04s_{12} + .04s_{11} + .32s_{10} + .05s_{02} + .40s_{01} + .05s_{00} \quad (IC-LL)
\]

\[
.1s_{20} + .8s_{21} + .1s_{22} - 10 \geq .8s_{20} + .1s_{21} + .1s_{22} \quad (IC-200)
\]

\[
.1s_{10} + .8s_{11} + .1s_{12} - 10 \geq .8s_{10} + .1s_{11} + .1s_{12} \quad (IC-100)
\]

\[
.1s_{00} + .8s_{01} + .1s_{02} - 10 \geq .8s_{00} + .1s_{01} + .1s_{02} \quad (IC-0)
\]

\[
s_{22}, s_{21}, s_{20}, s_{12}, s_{11}, s_{10}, s_{02}, s_{01}, s_{00} \geq 0 \quad (LIM)
\]
From the solution in Table 2, Panel A, we see that under the disaggregate system expected compensation over the two tasks is 61.43. In the disaggregate case the largest payment to the manager is associated with an outcome of 100 on the first task, because it is more difficult to motivate than the second task. The first task is more difficult to motivate because the owner gains less information about the manager’s actions from observing the outcome. This is due to the fact that the probability distributions arising from high and low effort are very similar—the probabilities are {.1, .5, .4} versus {.1, .4, .5}, whereas in the second task the probabilities are {.1, .8, .1} versus {.1, .1, .8}.

The contracting program for the two-task setting with aggregated outcomes can be found by modifying the contracting program for the disaggregate case as follows: (1) eliminate the (IC-200), (IC-100), and (IC-0) constraints, and (2) add the following constraints:

\[ s_{10} = s_{01}, \quad s_{02} = s_{11} = s_{20} \quad \text{and} \quad s_{12} = s_{21}. \]

From Table 2, Panel A, we find that the optimal contract pays the manager 64.29, in expectation, which is greater than that paid under the optimal disaggregate contract. To understand why the owner prefers disaggregation, we refer again to the difficulty in motivating the first task and the desire to reward the manager for an outcome of 100 on that task. We see that in the optimal aggregation contract the owner pays the manager only for a total outcome of 200. While this aggregated outcome does derive from an outcome of 100 on the first task combined with an outcome of 100 on the second task, it also derives from an outcome of 0 on the first task and 200 on the second task. This latter combination of outcomes is actually more likely if the manager provides low effort on both tasks (probability 0.05) than high effort on both tasks (probability 0.04). This payment scheme is the best that the owner can do under aggregation, and so it follows that under aggregation the owner is forced into an inefficient means of compensating the manager. The above example illustrates that when aggregation obscures both the outcome pertaining to each task and the combination of outcomes that constitute the aggregated outcome, the cost of aggregation may outweigh the benefit. Interpreting this result, there are conditions under which the owner would wish to have the greater amount of information available even though it would require additional incentives.

**THE BENEFIT OF DELAYED INFORMATION**

Thus far, we have seen that aggregation can be beneficial in minimizing expected compensation because it relaxes the conditions necessary for the owner to motivate the employee. However, we have also seen that aggregation can be costly, because it reduces the amount of information available to the owner for contracting purposes. We now pose the question: Is there a way for the owner to reap the benefits of aggregation without incurring its costs? Perhaps surprisingly, the answer is yes. The method for doing so is to control the point at which the performance measure is revealed. In particular, there can be benefits to designing an accounting system that delays information. These benefits are available if the accounting system were designed to report to the owner and manager the disaggregated outcomes of the two tasks, but only after the second task has been completed.

The rationale follows from the analysis in the previous section. As in the aggregate case, a benefit accrues to the owner because the manager does not know the first task.

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16 The solution presented in Table 2, Panel A for the disaggregate case is not unique. Alternative optimal solutions pay more for outcomes (100, 200) and (100, 0) and less for outcome (100, 100). For example, paying the manager 12.8, 136.09, and 12.8 for outcomes (100, 200), (100, 100), and (100, 0), respectively, while leaving the other payments as in Table 2 is an optimal solution. We again suggest that if students do not arrive at the same values for the payments as those in Table 2, they enter the Table 2 solution in their spreadsheet to verify that it also satisfies all the constraints and provides the same objective function value as the one they found.

17 Exercise 1.d in Appendix C illustrates the effect of the probability distribution on outcome informativeness.
outcome when he undertakes the second task. However, unlike the aggregate case, if the information arrives late, then the owner would still have all the information available in the disaggregate case. Hence, delayed information not only relaxes the contracting constraints in the same manner as aggregation, but also allows the owner to fully use all possible information in the outcomes available in disaggregation when evaluating the manager.

An important implication of these results is that management control concerns, in and of themselves, can explain why firms might prefer to issue reports less frequently. By delaying feedback from their earlier decisions to managers, the firm can more efficiently motivate managers. As a practical example, suppose a store manager receives real-time data on sales. Late in the month he realizes that he cannot meet the sales threshold needed to receive a bonus. He may be tempted to work less diligently or to delay sales to the future. Upper management would not desire either of these behaviors. Delaying feedback in this situation, so that the manager was unaware of his bonus status until after month-end, could induce more desirable behavior. This management control motivation for delaying feedback is especially interesting, when electronic recording and transmission of data have reduced the cost of record keeping and report preparation. One might be tempted to overlook the incentive costs associated with making information available on a real-time basis.

To demonstrate more concretely the benefits of delaying information, we now introduce the delayed disaggregation case. Delayed disaggregation refers to the situation where information arrives in disaggregate form, and is not available until after the manager has chosen his action on the second task. This means the owner does not have to provide incentives to work hard on the second task, conditional on each outcome that can occur from the first task. Continuing with the three-outcome example, the owner’s optimal contracting program under delayed disaggregation can be derived from the disaggregate program by simply eliminating the (IC-200), (IC-100), and (IC-0) incentive compatibility constraints.

Table 2, Panel B continues with the same parameters as in Table 2, Panel A. It illustrates the benefit of delayed disaggregate information relative to a timely disaggregate system. The solution under delayed disaggregation exploits the fact that the owner need not provide incentives for high effort on the second task for every potential outcome on the first task. For example, if the outcome is 0 on the first task, then the manager will receive zero in compensation, regardless of the second task outcome. Clearly, if the manager could observe the outcome of 0 on the first task before his effort choice on the second task, then he would not provide high effort on the second task. However, under delayed disaggregation, the manager cannot make that observation at the time he chooses his second action. Further, by eventually observing individual outcomes from each task, the owner can use the disaggregated information to design a more efficient contract. The example illustrates the general property that within our simplified setting the owner always prefers delaying detailed manager performance measures to providing timely detailed performance measures.

The example also illustrates the benefit of delayed disaggregate information relative to an aggregate system. In the delayed disaggregation case, the owner provides positive compensation, 125, for an equivalent of an aggregated outcome of 200. However, she has

---

18 To be precise, the owner is unconcerned whether she receives information on the first task before or after the manager completes the second task. What is of concern to the owner is whether the manager receives the information before taking an action on the second task.

19 This situation is further explored in Exercise 4 in Appendix C.

20 The solution presented in Table 2, Panel B for the delayed disaggregate case is not unique. For example, paying the manager 59.84 for (100, 0), 113.43 for (100, 100), and 32.75 for (100, 200) is an optimal solution.
sufficient information to enable her to do so only when the outcome of 100 is observed on the first task. In the aggregate case, the owner would compensate the manager only for an aggregated outcome of 200, which as discussed previously does not arise only from an outcome of 100 on the first task. We see that if 200 obtains under aggregation, then: (1) it includes only one outcome for which positive compensation would have been chosen in the more owner-favorable delayed disaggregation case, and (2) it includes others that would have led to zero payment in the delayed disaggregate case. The example illustrates the general property that within our simplified setting the owner always prefers delaying detailed manager performance to providing aggregate performance measures.

Analysis of the delayed disaggregate case allows us to quantify the costs and benefits of aggregation. The cost of aggregation results from a loss of information. Both the aggregate and delayed disaggregate cases assume the manager does not know the first task outcome before he chooses his second action, but in the aggregate case the owner has less information. The cost of aggregation is equal to the difference in the manager’s expected compensation between the aggregate and delayed disaggregate cases. In the example above, the cost of aggregation is $64.29 - 50 = 14.29$. The benefit of aggregation arises due to the relaxed conditions for motivating manager effort. Both the disaggregate and delayed disaggregate cases have the same information available to the owner, but the delayed disaggregate case involves fewer incentive compatibility constraints. The benefit of aggregation is equal to the difference in the manager’s expected compensation between the disaggregate and delayed disaggregate cases. In the example above, the benefit of aggregation is $61.43 - 50 = 11.43$. In this example, the cost of aggregation is 14.29 and the benefit is 11.43. Hence, in this example a disaggregate system is preferred to an aggregate system.

As noted above, in order to reap the contracting benefits of aggregation without its costs, the owner need only delay the disclosure of the first task outcome to the manager. However, in this note we have only considered settings in which the owner and manager receive information concurrently. We make this assumption because if the owner attempts to collect and record the outcome from the first task early, but delays its disclosure to the manager, important issues arise over information security. The manager now would have an incentive to circumvent the controls in order to obtain outcome information on the first task before taking his second task action (Antle and Fellingham 1995). Rather than explicitly incorporating a noncontracting benefit to obtaining information early, giving the owner a reason to avoid delaying information’s arrival, as well as a cost of information security, we merely assume that access to information is gained concurrently for the owner and manager.

**SUMMARY**

The learning objective of this teaching note is to illustrate the effects of aggregation and delay, two fundamental characteristics of accounting data, in a management control context. We employ simple settings and eschew the presentation of formal proofs and theorems. Using this approach, we demonstrate that the reduction of information attendant with aggregation can reduce the expected compensation necessary to motivate a manager. We further demonstrate that if the loss of information due to aggregation is severe enough, then a disaggregate accounting system is optimal from the owner’s perspective. Finally, we demonstrate that delayed arrival of accounting data can serve as means for the owner to obtain the benefits of aggregation (relaxed conditions necessary to motivate the manager) without incurring its cost (less information available to infer the action taken by the manager).
Employing a simplified linear model has provided a straightforward platform to illustrate the effects of aggregation and timeliness and further allows students to solve the examples contained herein in a relatively short amount of time. However, as in any modeling exercise, simplifying assumptions can limit the generality of the results. There are two salient limitations arising from our assumptions. First, we assumed a restricted outcome space. An increase from a two-outcome task to a three-outcome task induces a cost to aggregation. Further increases in potential outcomes would increase the cost of aggregation. The effect on the cost of aggregation is intuitive, because aggregation would destroy more information when there are more potential task outcomes. Second, we assumed the owner can costlessly withhold information from the manager in the aggregate and delayed disaggregate systems. In practice, information security costs can be considerable. In fact, the manager may receive real-time (although perhaps imperfect) feedback about performance, regardless of whether the formal accounting system tracks it. In the extreme situation, only a disaggregate accounting system may be feasible.

Although our model’s assumptions are limiting, they should not be viewed as overly restrictive. With respect to the limited outcome space, it should be noted that many real-world tasks do result in a binary outcome, (for example, whether a bid has been accepted). Further, the benefit of aggregation is not reduced by an expansion of the outcome space; this complexity simply increases the cost of aggregation. With respect to information security, in many situations feedback can be withheld at a small cost. For example, if success can only be determined with independent, difficult-to-perform measurements, as with many engineering tasks, then employees may not be aware of whether they have succeeded for a significant period of time. Further, even if employees gained some imperfect information on an initial task outcome, aggregation would still provide benefits, providing perfect feedback was not given before subsequent tasks are undertaken. In general, we believe the model we present provides an appropriate balance between ease of use and external validity.

CLASSROOM IMPLEMENTATION

We have typically used these exercises, and others like them, in advanced managerial accounting classes at the undergraduate and master’s level, in a master’s-level accounting theory class, and in introductory doctoral seminars. These are entire courses or sections of courses where management control is the general topic. Before we introduce the materials to the class, we present examples to illustrate expected value and the value of information in a single-person setting. Then we present a simple agency model with numerical examples. Several chapters from Kreps (1990) are useful as a general supplement. We follow with readings such as Demski (1997, Chapters 18–20) and articles such as Antle and Demski (1988) and Thornton (1984, 1985) to motivate the relevance of the agency model to accounting. Finally, we introduce specific topics, such as the material contained in this note, by assigning a reading and also providing a case or exercise of the type in Appendix C.

In class, we often pair up students and ask them to jointly present the case. The readings suffice to enable the students to successfully present the material, although sometimes students seek help from the instructor outside of class before they present. The instructor

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21 Texts that have been used in the past for these courses are Brickley et al. (2001), Christensen and Demski (2002), Demski (1997), Milgrom and Roberts (1992), and Zimmerman (2003).
22 Examples of other readings we have used in this way include Arya et al. (1998), Arya et al. (1996), Fellingham and Young (1990), and Lambert (1986).
normally follows up on the presentation by taking questions from the students, raising some issues to encourage discussion and, when necessary, adding some clarifying points.

Recently this teaching note was introduced in a master’s level accounting theory class. After the students read the note and completed assignments, but before the instructor critiqued their work, a survey instrument was administered. The results of the survey indicate that over 98 percent of the students found the material either not difficult or difficult but understandable with sufficient work. Further, over 70 percent of the students had their perceptions of the usefulness of aggregation changed through exposure to this note. These survey results are consistent with our typical experience: most students can meaningfully participate given sufficient time is devoted to laying the foundation to study management control. For those students who are particularly interested in the effects of aggregating and delaying information in a managerial control setting or who intend to pursue an academic career, we suggest two follow-up readings, Arya et al. (2004) and Gigler and Hemmer (2002).

APPENDIX A

Linear Program for Two Outcomes and Two Identical Tasks

**Notation**

- $\bar{U}$ = expected utility obtained by manager if he works for another firm;
- $x$ = performance measure (outcome) on each task, $\{x = 0, X\}$;
- $e$ = manager effort, where $e = H$ or $L$;
- $p_H$ = probability $x = X$ given $e = H$;
- $p_L$ = probability $x = X$ given $e = L$;
- $c_H$ = disutility of effort when $e = H$ (disutility of effort when $e = L$ is assumed to be 0);
- $s_{ij}$ = payment from owner to manager for outcome $iX$ in the first task and $jX$ in the second task, $i, j = 0, 1$; and
- $EC|HH$ = expected compensation to the manager if he chooses $e = H$ on both tasks, equal to: $p_H^2s_{11} + p_H(1 - p_H)s_{10} + (1 - p_H)p_Hs_{01} + (1 - p_H)^2s_{00}$.

**Disaggregation Contracting Program**

Minimize $p_H^2s_{11} + p_H(1 - p_H)s_{10} + p_H(1 - p_H)s_{01} + (1 - p_H)^2s_{00}$

Subject to:

1. $EC|HH - 2c_H \geq \bar{U}$
2. $EC|HH - 2c_H \geq p_Hp_Ls_{11} + p_H(1 - p_L)s_{10} + p_L(1 - p_H)s_{01}$
3. $\quad + (1 - p_H)(1 - p_L)s_{00} - c_H$ (IC-HL)
4. $EC|HH - 2c_H \geq p_Hp_Ls_{11} + p_L(1 - p_H)s_{10} + p_H(1 - p_L)s_{01}$
5. $\quad + (1 - p_H)(1 - p_L)s_{00} - c_H$ (IC-LH)
6. $EC|HH - 2c_H \geq p_L^2s_{11} + p_L(1 - p_L)s_{10} + p_L(1 - p_L)s_{01} + (1 - p_L)^2s_{00}$ (IC-LL)

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23 By administering the survey instrument at this point in time, we hoped to capture the students’ reaction to the written material rather than to the instructor’s presentation.
The aggregation contracting program is formed from the disaggregation contracting program by eliminating (IC-0) and (IC-1) and appending the constraint \( s_{10} = s_{01} \). The delayed disaggregation contracting program is formed from the disaggregation contracting program by eliminating (IC-0) and (IC-1).

**Explanation of Constraints**

\( P \) ensures that the manager prefers participating in the contract than accepting the next best offer elsewhere;

\( \text{IC-}HL \) ensures that the manager would prefer to provide effort \( H \) on both tasks rather than to provide effort \( H \) on the first task and effort \( L \) on the second task;

\( \text{IC-}LH \) ensures that the manager would prefer to provide effort \( H \) on both tasks rather than to provide effort \( L \) on the first task and effort \( H \) on the second task;

\( \text{IC-}LL \) ensures that the manager would prefer to provide effort \( H \) on both tasks rather than to provide effort \( L \) on the first task and effort \( H \) on the second task;

\( \text{IC-}0 \) ensures that, given an outcome of 0 on the first task, the manager would prefer to provide effort \( H \) on the second task rather than to provide effort \( L \) on the second task (disaggregate case only); and

\( \text{IC-}1 \) ensures that, given an outcome of \( X \) on the first task, the manager would prefer to provide effort \( H \) on the second task rather than to provide effort \( L \) on the second task (disaggregate case only).

**APPENDIX B**

*Linear Program for Three Outcomes and Two Different Tasks*

**Notation**

\( U \) = expected utility obtained by manager if he works for another firm;

\( x \) = performance measure (outcome) on each task, \( \{x = 0, X, 2X\} \);

\( e \) = manager effort, where \( e = H \) or \( L \);

\( p_{iH} \) = probability of \( x = iX \) on task 1 given effort \( = H \), \( i = 0, 1, 2 \);

\( p_{iL} \) = probability of \( x = iX \) on task 1 given effort \( = L \), \( i = 0, 1, 2 \);

\( q_{iH} \) = probability of \( x = iX \) on task 2 given effort \( = H \), \( i = 0, 1, 2 \);

\( q_{iL} \) = probability of \( x = iX \) on task 2 given effort \( = L \), \( i = 0, 1, 2 \);

\( c_H \) = disutility of effort when \( e = H \) (disutility of effort when \( e = L \) is assumed to be 0);

\( s_{ij} \) = payment from owner to manager for outcome \( iX \) in the first task and \( jX \) in the second task; \( i, j = 0, 1, 2 \); and

\( \text{ECH}HH \) = expected compensation to the manager if he chooses \( e = H \) on both tasks, equal to:

\[ p_{2H}q_{2H}s_{22} + p_{2H}q_{0H}s_{20} + p_{1H}q_{2H}s_{12} + p_{1H}q_{0H}s_{10} + p_{0H}q_{2H}s_{02} + p_{0H}q_{0H}s_{00} \]

**Disaggregation Contracting Program**

Minimize

\[ p_{2H}q_{2H}s_{22} + p_{2H}q_{0H}s_{20} + p_{1H}q_{2H}s_{12} + p_{1H}q_{0H}s_{10} + p_{0H}q_{2H}s_{02} + p_{0H}q_{0H}s_{00} \]
Subject to:

$$\begin{align*}
EU|HH & \geq U \\
EU|HH - 2c_H & \geq p_{2H}q_{2L}s_{22} + p_{2L}q_{1L}s_{21} + p_{2L}q_{0L}s_{20} + p_{1H}q_{2L}s_{12} \\
& + p_{1H}q_{1L}s_{11} + p_{1H}q_{0L}s_{10} + p_{0H}q_{2L}s_{02} + p_{0H}q_{1L}s_{01} + p_{0H}q_{0L}s_{00} - c_H \quad \text{(P)} \\
EU|HH - 2c_H & \geq p_{2L}q_{2H}s_{22} + p_{2L}q_{1H}s_{21} + p_{2L}q_{0H}s_{20} + p_{1L}q_{2H}s_{12} \\
& + p_{1L}q_{1H}s_{11} + p_{1L}q_{0H}s_{10} + p_{0L}q_{2H}s_{02} + p_{0L}q_{1H}s_{01} \\
& + p_{0L}q_{0H}s_{00} - c_H \quad \text{(IC-HL)} \\
EU|HH - 2c_H & \geq p_{2L}q_{2H}s_{22} + p_{2L}q_{1H}s_{21} + p_{2L}q_{0H}s_{20} + p_{1L}q_{2H}s_{12} \\
& + p_{1L}q_{1L}s_{11} + p_{1L}q_{0L}s_{10} + p_{0L}q_{2L}s_{02} + p_{0L}q_{1L}s_{01} + p_{0L}q_{0L}s_{00} \quad \text{(IC-LH)} \\
q_{2H}s_{22} + q_{1H}s_{21} + q_{0H}s_{20} - c_H & \geq q_{2L}s_{22} + q_{1L}s_{21} + q_{0L}s_{20} \quad \text{(IC-2)} \\
q_{2H}s_{12} + q_{1H}s_{11} + q_{0H}s_{10} - c_H & \geq q_{2L}s_{12} + q_{1L}s_{11} + q_{0L}s_{10} \quad \text{(IC-1)} \\
q_{2H}s_{02} + q_{1H}s_{01} + q_{0H}s_{00} - c_H & \geq q_{2L}s_{02} + q_{1L}s_{01} + q_{0L}s_{00} \quad \text{(IC-0)} \\
s_{22}, s_{21}, s_{20}, s_{12}, s_{11}, s_{10}, s_{02}, s_{01}, s_{00} & \geq 0 \quad \text{(LIM)}
\end{align*}$$

The aggregation contracting program is formed from the disaggregation program by eliminating (IC-0), (IC-1), and (IC-2) and appending the constraints $s_{10} = s_{01}$, $s_{11} = s_{20} = s_{02}$ and $s_{12} = s_{21}$. The delayed disaggregation contracting program is formed from the disaggregation contracting program by eliminating (IC-0), (IC-1), and (IC-2).

**Explanation of Constraints**

- **P** = ensures that the manager prefers participating in the contract than accepting the next best offer elsewhere;
- **IC-HL** = ensures that the manager would prefer to provide effort = $H$ on both tasks rather than to provide effort = $H$ on the first task and effort = $L$ on the second task;
- **IC-LH** = ensures that the manager would prefer to provide effort = $H$ on both tasks rather than to provide effort = $L$ on the first task and effort = $H$ on the second task;
- **IC-LL** = ensures that the manager would prefer to provide effort = $H$ on both tasks rather than to provide effort = $L$ on both tasks;
- **IC-0** = ensures that, given an outcome of 0 on the first task, the manager would prefer to provide effort = $H$ on the second task rather than to provide effort = $L$ on the second task (disaggregate case only);
- **IC-1** = ensures that, given an outcome of 1 on the first task, the manager would prefer to provide effort = $H$ on the second task rather than to provide effort = $L$ on the second task (disaggregate case only); and
- **IC-2** = ensures that, given an outcome of 2 on the first task, the manager would prefer to provide effort = $H$ on the second task rather than to provide effort = $L$ on the second task (disaggregate case only).
1. Cost and benefits of aggregation (identical tasks, two outcomes). Consider the example in the text with two identical tasks and two outcomes. The disutility of high effort = 10 and the disutility of low effort = 0.

Parameters for Each Task (original from text):

<table>
<thead>
<tr>
<th>Potential outcomes</th>
<th>100</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.80</td>
<td>.20</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.48</td>
<td>.52</td>
</tr>
</tbody>
</table>

a. Verify that the contracts described in Table 1 for the disaggregate and aggregate cases minimize the manager’s expected compensation.

b. Solve for the contract in the delayed disaggregate case that minimizes the manager’s expected compensation. Calculate the benefits of aggregation. Calculate the costs of aggregation.

c. Modify the probability distribution on the outcome as follows, but retain the assumptions that the owner wishes to motivate high effort on both tasks and the disutility of high effort = 10, the disutility of low effort = 0.

Parameters for Each Task (new):

<table>
<thead>
<tr>
<th>Potential outcomes</th>
<th>100</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.80</td>
<td>.20</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.64</td>
<td>.36</td>
</tr>
</tbody>
</table>

Find the contract that minimizes the manager’s expected compensation in the disaggregate, aggregate, and delayed disaggregate cases. What are the benefits of aggregation? What are the costs of aggregation?

d. Why did the expected compensation to the manager increase relative to the original example in the text?

2. Cost and benefits of aggregation (different tasks, two outcomes). Consider the case of two different tasks and two outcomes and an owner who wishes to motivate high effort on both tasks, with the parameters shown below. Retain the assumptions that the disutility of high effort = 10, the disutility of low effort = 0 and the manager’s next best alternative produces an expected utility of 0.

Parameters for Task One:

<table>
<thead>
<tr>
<th>Potential outcomes</th>
<th>100</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.70</td>
<td>.30</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.56</td>
<td>.44</td>
</tr>
</tbody>
</table>

24 Solutions are available from the authors upon request.
Parameters for Task Two:

<table>
<thead>
<tr>
<th>Potential outcomes</th>
<th>100</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.50</td>
<td>.50</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.20</td>
<td>.80</td>
</tr>
</tbody>
</table>

a. Solve for the contract that minimizes the manager’s expected compensation in the disaggregate, aggregate and delayed disaggregate cases.
b. Show that the costs of aggregation are still zero in the two-outcome setting, even though the tasks are not identical.

3. **Cost and benefits of aggregation (different tasks, three outcomes).** Consider the case of two different tasks and three outcomes and an owner who wishes to motivate high effort on both tasks, with parameters shown below. Retain the assumptions that the disutility of high effort $H_1 = 10$, the disutility of low effort $H_1 = 0$ and the manager’s next best alternative produces an expected utility of 0.

Parameters for Task One:

<table>
<thead>
<tr>
<th>Potential outcomes</th>
<th>200</th>
<th>100</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.60</td>
<td>.20</td>
<td>.20</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.20</td>
<td>.60</td>
<td>.20</td>
</tr>
</tbody>
</table>

Parameters for Task Two:

<table>
<thead>
<tr>
<th>Potential outcomes</th>
<th>200</th>
<th>100</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability if manager chooses high effort</td>
<td>.50</td>
<td>.20</td>
<td>.30</td>
</tr>
<tr>
<td>Probability if manager chooses low effort</td>
<td>.20</td>
<td>.40</td>
<td>.40</td>
</tr>
</tbody>
</table>

a. Find the contract that minimizes the manager’s expected compensation for the aggregate and delayed disaggregate cases.
b. Treat the two tasks separately and write a contract for each of the tasks that minimizes the manager’s expected compensation. Then construct a contract for the disaggregate case that minimizes the manager’s expected compensation and verify that the expected compensation to the manager under the single disaggregate accounting system is equal to the total expected compensation under two separate contracts.
c. Why is there a cost to aggregation for the three-outcome example in the text but not in the above three-outcome example? Hint: Compare the optimal contracts in the delayed disaggregate and aggregate cases. Is there information missing in the aggregate case that the owner wishes to use?

4. **Sales bonus contracts.** Suppose that toward the end of the year a salesperson has only two potential customers left with whom to close a sale. Given the amount of sales made prior to this point, the salesperson is aware that if he makes both sales, then he will receive a year-end bonus of $40,850. However, if he only makes one sale or no sales, then his bonus for the year will be $40,000. The bonus is the only way in which these two potential sales will affect his compensation. All salespeople have the same
contract and it would not be possible for the owner to revise the contract at this late date. The salesperson can either work hard or take it easy in trying to close these sales. If he works hard, then the probability of closing the sale is .8. If he takes it easy, then the probability is .4. This is true for both sales. For each potential sale, the monetary equivalent of the disutility of working hard is $200, while the monetary equivalent of the disutility for taking it easy is $0. The gross value to the owner of each sale is $10,000. The following table summarizes the probabilities of each individual outcome contingent upon effort.

Parameters for Potential Sale:

<table>
<thead>
<tr>
<th>Potential outcomes</th>
<th>No Sale</th>
<th>Sale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value to Owner</td>
<td>$0</td>
<td>$10,000</td>
</tr>
<tr>
<td>Probability if salesperson works hard</td>
<td>.20</td>
<td>.80</td>
</tr>
<tr>
<td>Probability if salesperson takes it easy</td>
<td>.60</td>
<td>.40</td>
</tr>
<tr>
<td>Disutility of high effort (monetary equivalent)</td>
<td>$200</td>
<td></td>
</tr>
<tr>
<td>Disutility of low effort (monetary equivalent)</td>
<td>$0</td>
<td></td>
</tr>
</tbody>
</table>

a. If the salesperson does not know the outcome of the first sale before choosing his effort level for the second sale, will he work hard on both sales?
b. If the salesperson learns that he has not made the first sale before choosing his effort level on the second sale, will he work hard on the second sale?
c. From the owner’s perspective, taking into consideration compensation to the salesperson and the value of each sale, would she rather the salesperson learned of the outcome of the first sale before making his effort choice on the second sale or would she rather he learned of both outcomes after both effort choices have been made?
d. What would your answer to part (c) be if the gross value of each sale to the owner was $400?

REFERENCES