Conditions Under Which Activity-Based Cost Systems Provide Relevant Costs

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Abstract: Activity-Based Cost systems assign costs to products on the basis of multiple "cost drivers," which may or may not be proportional to the volume of output. This is in contrast to most traditional cost systems which use only one allocation basis (usually direct labor or machine hours) that is proportional to volume. Commonly cited reasons for switching to Activity-Based Cost systems are to more accurately estimate product profitability for purposes of making product pricing and drop decisions and to reduce the cost of manufacturing products in the design stage by providing more accurate cost information concerning alternative design specifications. In this paper the conditions under which an Activity-Based Cost system would provide relevant information for just such decisions are derived. These conditions are quite stringent and include, among other things, that all costs must be strictly proportional to their "cost drivers."

INTRODUCTION

Activity-Based Cost (ABC) systems have attracted a great deal of attention. For purposes of costing products, these cost accounting systems assign overhead to products using multiple allocation bases, some of which are related to unit volume and some are not. This is in contrast to the


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typical cost system found in practice in which all overhead is allocated on the basis of direct labor or some other measure of activity that is highly correlated with unit volume. Johnson and Kaplan [1987] and others have argued that the common practice of allocating overhead on the basis of direct labor distorts product costs and overstates the savings that would result from reducing direct labor. ABC systems are intended to reduce these distortions and appear to have been independently invented at about the same time at several firms (Cooper [1989]).

As an important aside, there is potential confusion concerning the distinction between traditional cost systems and ABC systems. Technically, activity-based costing subsumes the more traditional methods of determining product costs. That is, if one were to design generalized software for determining product costs using an activity-based costing system, the same software could be used to determine product costs for a traditional cost system. Traditional costing systems are merely simplified, and perhaps poorly designed, special cases of activity-based costing, just as activity-based costing is a special case of more general costing systems that could be devised. And since traditional systems can be viewed as (primitive) activity-based costing systems, the results in this paper apply to the more traditional costing systems as well as to the more sophisticated forms of activity-based costing. For the rest of this paper, the term ABC system will be understood to include traditional costing systems as special cases. The interest in this paper is in contrasting costs generated by ABC systems with relevant costs rather than contrasting costs generated by ABC systems with costs generated by traditional costing systems.

The term Activity-Based Costing is itself subject to varying interpretation and its definition appears to be evolving over time. For the purposes of this paper, an ABC system is a two-stage allocation process that fully allocates costs to products (or customers or some other ultimate cost object). This describes all of the actual implementations of ABC systems of which I am aware.

The main point in this paper is that if costs generated by a cost system are to be used for decision-making purposes, it would seem prudent to establish the conditions under which those costs are relevant to the decision at hand. In this paper necessary and sufficient conditions under which conventional ABC systems provide relevant cost information are derived. In subsequent research I will investigate whether those conditions are in fact satisfied—even approximately.

**DEFINING ABC SYSTEMS**

ABC systems are based on the notion that products incur costs by giving rise to activities (e.g., preparation of purchase orders, machine setups, run-time labor, loading dock activity, etc.) which generate costs. Suppose the real cost structure of the firm does follow this pattern. Without imposing any particular functional forms, we can quite generally model the real cost relationship linking output to cost as follows:

\[ C = C(a(q)) \]
where \( q \) is the vector of the firm's outputs and \( a \) is the vector of activities that generates costs.\(^2\) Output drives activities, which in turn generates costs. There is nothing that restricts the form of the cost function or the nature of the link between activities and output. For example, the cost function can exhibit increasing returns to scale in the level of a particular activity. However, a conventional ABC system implicitly makes quite strong assumptions about the nature of the links between output and activities and activities and costs. The purpose of this paper is to clearly identify those assumptions.

In an ABC system costs are partitioned into "cost pools" which are in turn allocated to products based upon "cost drivers" or "activity measures" unique to each cost pool.\(^3\) For example, the "purchase order processing" cost pool might be allocated to products based on the number of purchase orders processed for each product. Let \( CP_i \) be the cost assigned to the \( i^{th} \) cost pool. By construction, the sum of the costs assigned to the various cost pools equals the total cost to be allocated.

\[
C = \sum_i CP_i
\]

Each product is assigned an activity measure (possibly zero) for each cost pool. Let \( m_{ij} \) be the activity measure for the \( i^{th} \) cost pool and the \( j^{th} \) product. This activity measure is a function of the volume produced of the \( j^{th} \) product and conventionally has the property that the activity measure is zero if none of product \( j \) is produced and is non-negative otherwise. Symbolically,

\[
m_{ij} = m_{ij}(q_j)
\]

\[
m_{ij}(q_j) = 0 \text{ if } q_j = 0
\]

\[
m_{ij}(q_j) \geq 0 \text{ if } q_j > 0
\]

Note that the activity measure need not be a linear function, or even a continuous function, of output. Cooper [1989], in a paper which also defines the mechanics of ABC systems, identifies three generic levels of activity associated with products: unit level activities, batch level activities, and product sustaining activities. Unit level activities are functions of the volume of a product; batch level activities are functions of the number of batches that are processed; and product sustaining activities are functions of the diversity and complexity of products. Unit level activities would typically be represented by a linear function of the form \( m_{ij}(q_j) = m_{ij} \cdot q_j \). Batch-level activities would be represented by a step function. Product sustaining activities would be represented by a function of the form \( m_{ij}(q_j) = m_{ij} \) if \( q_j > 0 \) and \( m_{ij}(q_j) = 0 \) if \( q_j = 0 \).\(^4\) Activity-based costing thus allows representation

\(^1\)Implicitly, this cost function is for a given state of nature; to be more general, one could explicitly include the random state of nature in the cost function.

\(^2\)This is often referred to as a two-stage allocation process. In the first stage, costs are allocated to cost pools and in the second stage, the costs in the cost pools are allocated to products.

\(^3\)To handle the possibility that product sustaining activities are required even when there is no production, two qs can be defined for each product. One represents the physical output of the product, the other is a 0/1 variable indicating whether the capability to make the product is being maintained.
of more complex cost functions than the typical cost system found in practice in which all allocation bases (e.g., activity measures) are proportional to volume. For example, activity-based costing accommodates costs that are fixed with respect to changes in the batch size—such as machinery set-up costs. This is a significant innovation in cost accounting practice (Shank and Govindarajan [1988]).

The rate for the \( i \)th cost pool, \( r_i \), is determined by dividing a cost pool by its total activity measure.

\[
r_i = \frac{CP_i}{m_i}
\]

where \( m_i = \sum_j m_{ij} \)

The ABC product cost for the \( j \)th product, \( PC_j \), is computed by multiplying the rates times the activity measures and summing across cost pools.

\[
PC_j = \sum_i r_i m_{ij}
\]

An important characteristic of the ABC method, as in all conventional allocations, is that the sum of the product costs equals the total cost that is to be allocated:

\[
\sum_j PC_j = \sum_i \sum_j r_i m_{ij} = \sum_i r_i \sum_j m_{ij} = \sum_i r_i m_i = \sum_i CP_i = C
\]

That is, an activity-based cost system is basically a scheme for allocating costs.

At least in the abstract, the mechanics of ABC systems are not conceptually difficult. Under what conditions, however, do ABC product costs provide relevant cost data for decisions? ABC product costs have been used in pricing and product drop decisions and ABC overhead rates have been used in designing products to minimize cost. In product drop decisions, the only relevant costs are those that would be avoided if in fact the product were dropped. Thus, it is sensible to rely on ABC product cost in a product drop decision only if they represent avoidable product costs. Similarly, using ABC overhead rates at the design stage to minimize costs will provide appropriate signals to engineers if and only if the overhead rates represent incremental costs. Under what conditions will they?

**NECESSARY AND SUFFICIENT CONDITIONS FOR THE RELEVANCE OF ACTIVITY-BASED COSTS**

The objective in this section is to establish necessary and sufficient conditions under which ABC product costs represent avoidable product costs and ABC overhead rates represent incremental activity costs. Before proceeding to prove the main result, it would be prudent to define more carefully what is meant by avoidable product costs, incremental activity costs, and an ABC system.

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5While it is possible to find antecedents to activity-based costing in published academic papers, cases, and in textbooks, the more sophisticated forms of activity-based costing were apparently rarely practiced until the late 1980s.

6The relevant costs may, of course, include opportunity costs.
The avoidable cost of a product is defined to be the change in total cost if the product were dropped, keeping the volumes of all other products the same. Formally,

**Definition** The avoidable cost of product k is \( C(a(q)) - C(a(q_{-k})) \) where \( q = (q_1, q_2, \ldots, q_{k-1}, q_k, q_{k+1}, \ldots, q_j) \) and \( q_{-k} = (q_1, q_2, \ldots, q_{k-1}, 0, q_{k+1}, \ldots, q_j) \).

Thus, ABC product costs are equivalent to avoidable product costs if and only if

\[
PC_k = \sum_i r_i m_{ik} = (C(a(q)) - C(a(q_{-k}))).
\]

Moreover, the incremental cost of changing the level of an activity is the change in total cost that would be incurred. Formally,

**Definition** The incremental cost of changing from activity level \( a \) to \( a' \) is \( C(a') - C(a) \).

Thus, ABC activity costs are equivalent to incremental activity costs if and only if

\[
\sum_i r_i (a_i' - a_i) = C(a') - C(a).
\]

In terms of the mechanics, a well-specified ABC system is simply an allocation scheme in which the cost drivers, or activity measures, are correctly identified and measured on at least the aggregate level.

**Definition** Let \( q = (q_1, q_2, \ldots, q_j) \) be the outputs of a firm, \( a(q) \) be the function relating activities to output, and \( C \) be the total cost to be assigned to products. A well-specified Activity-Based Cost system consists of rates \( r_i(q) \) and activity measures \( m_{ij}(q_j) \) with the following properties:

1. \( m_{ij}(q_j) = 0 \) if \( q_j = 0 \)
2. \( m_{ij}(q_j) \geq 0 \) if \( q_j > 0 \)
3. \( a_i = m_i = \sum_j m_{ij} \)
4. \( \sum_i r_i m_i = C \)

The following theorem establishes that a well-specified ABC system exists in which product costs are avoidable costs and activity costs are incremental costs if and only if: 1) the underlying real cost function \( C(a(q)) \) can be partitioned into cost pools, each of which depends only upon a single activity; 2) the cost in each cost pool is strictly proportional to its activity; and 3) each activity can be divided among products in such a way that the portion attributed to each product depends only upon that product.

**Theorem** There exists a well-specified ABC system that provides valid avoidable product costs and incremental activity costs if and only if

1. \( C(a(q)) = \sum_i C_i \ a_i(q) \)
2. \( C_i \ a_i(q) = p_i \ a_i(q) \)
3. \( a_i(q) = \sum_j a_j(q_j) \) where \( a_j(q_j) = 0 \) if \( q_j = 0 \) and \( a_j(q_j) \geq 0 \) if \( q_j > 0 \).

**Proof** (see the appendix)
DISCUSSION

According to the Theorem, an ABC system can provide relevant data for decision making if and only if the cost system is well-specified and the real underlying cost function $C(a(q))$ satisfies conditions 1) through 3).

The first condition is that total cost can be partitioned into cost pools, each of which depends solely upon one activity. In principle, every cost is the product of a price (or price-surrogate provided by the accounting system) and a quantity of something—which could be regarded as the activity. However, the number of different activities that can be practically accommodated in cost systems, and hence the number of cost pools, is limited. The art in designing an ABC system is in choosing a limited number of activity measures that can satisfactorily proxy for a wide range of the actual activities carried on the firm. An activity measure is a satisfactory proxy if it is highly correlated with an activity. For example, it is likely that the engineering cost pool isn't just a function of the number of engineering change orders. Nevertheless, if the various activities that generate those costs are highly correlated with the number of engineering change orders, then the number of engineering change orders is likely to be a satisfactory proxy for the real cost drivers.

The second condition is that the cost in each cost pool must be strictly proportional to the level of activity in that cost pool. This rules out, at the level of the cost pool, nonlinear cost functions and linear functions in which there are nonzero intercepts. Thus, if ABC systems are to provide relevant cost data in the contexts discussed in this paper, costs that are not strictly variable at the level of the cost pool should be excluded from the allocations and handled in some other manner.

The third condition is that each activity can be partitioned into elements that depend solely upon each product. That is, the activity measures assigned to the individual products can be simply summed to arrive at total activity. This assumption rules out all dependencies between products in the production process. In particular, this assumption rules out joint processes—the classical conundrum of cost accounting. In joint processes, the demands on a resource are determined by the maximum of the demands placed by the individual products, and not by their sum. The assumption that each activity can be partitioned into elements that depend solely upon the volume of each product also rules out more subtle dependencies among products, such as congestion when overall production volume increases (Banker et al. [1988]).

In sum, the conditions required for ABC systems to accurately reflect avoidable product costs and incremental activity cost are quite strong. Only those costs whose behavior more or less conform to the necessary and sufficient conditions should be allocated using ABC methods.

Unfortunately, the typical implementation of an ABC system involves an unquestioned commitment to allocate all costs (or at least all manufacturing costs) to products.7 This is certainly true of traditional costing sys-

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7The only case of which I am aware in which there was apparently an attempt to segregate relevant and irrelevant costs in an implementation of an ABC cost system is reported in Frank et al. [1989].
tems, which are special cases of ABC systems. It is also true of the more recent implementations of more sophisticated ABC systems. Given the second necessary condition above, it does not appear to be prudent to allocate all costs to products. An ABC system will provide relevant data (for the decisions considered here) only if the cost in every cost pool is strictly proportional to its activity measure. Surely there are costs that do not satisfy this condition. Is factory rent, for example, strictly proportional to machine hours? (Some ABC systems allocate factory rent to work center cost pools which are then allocated on the basis of machine hours in the work centers.)

Assuming that the real cost structure for a given cost pool is approximately linear with a positive intercept, the usual result will be ABC system estimates of product and activity costs that are too high. If these costs are interpreted by rational managers as relevant (i.e., avoidable) costs for purposes of making decisions, too few products will be produced in too small a quantity, in too large batches, and with too simple processes. It should be emphasized once again, however, that ABC systems need not be full cost allocation schemes and so these particular problems are not inescapable.

CONCLUSION

Necessary and sufficient conditions under which Activity-Based Cost systems provide relevant costs for product drop decisions and for product design decisions have been identified in this paper. Among other things, these conditions rule out nonlinear cost functions and nonzero fixed costs at the level of the cost pool and they rule out joint processes. If care is not exercised in the design of an ABC system and there do indeed exist fixed costs or joint costs, then the costs generated by the ABC system will not provide reliable signals for the kinds of decisions for which ABC systems have apparently been designed.

There are at least two interesting tracks future research concerning ABC systems may take. First there is the question of the extent to which the conditions derived in this paper hold in practice and hence costs generated by ABC systems are relevant. Second, the widespread and rapid adoption of ABC systems is an interesting phenomenon in and of itself—particularly since it is not obvious that on balance ABC systems as implemented provide greater benefits relative to costs than other possible costing systems. Cooper [1989] has identified situations in which switching to ABC systems from traditional systems is most likely to be beneficial on balance; Cooper's conjectures should be empirically tested.

8In other words, full cost implementations of ABC systems suffer from the inherent and well-known problems associated with such allocations (Thomas [1969, 1974]).

9Indeed, Cooper and Kaplan in their latest papers have been urging that facility-level costs not be allocated down to the product level.

10There may be a sociological/anthropological explanation of this phenomenon of wide-scale abandonment of traditional cost accounting systems. Cleverly's [1973] interpretation of the role of accountants and consultants in business organizations is amazingly prescient with regard to the process and circumstances of this change.
Lemma

Let \( f(q) \geq 0 \) be a function where \( q = (q_1, q_2, \ldots, q_{k-1}, q_k, q_{k+1}, \ldots, q_J) \). Define \( q_i = (q_1, q_2, \ldots, q_{k-1}, 0, q_k, 1, \ldots, q_J) \) and \( f(q_i) = f(0, \ldots, 0, q_i, 0, \ldots, 0) \).

Then \( \sum_k [f(q) - f(q_k)] = f(q) \Leftrightarrow f(q) = \sum_j f_j(q_j) \) and \( f(0) = 0 \) for all \( j \).

Proof

"\( \Rightarrow \)" Suppose \( f(q) = \sum_j f_j(q_j) \) and \( f(0) = 0 \) for all \( j \).

\[
\sum_k [f(q) - f(q_k)] = \sum_k \left[ \sum_j f_j(q_j) - \sum_j f_j(q_j) \right] = \sum_k f_k(q_k) = f(q).
\]

"\( \Leftarrow \)" Suppose \( \sum_k [f(q) - f(q_k)] = f(q) \)

This implies \( f(q) = \frac{1}{J-1} \sum_k f(q_k) \).

Prove the Lemma by induction on \( J \). Show \( f(q) = \sum_j f_j(q_j) \) and \( f(0) = 0 \) for all \( j \) if \( J = 2 \).

\[
f(q_1, q_2) = \left( \frac{1}{2-1} \right) [f(0, q_1) + f(q_2, 0)]
\]

Define \( f_1(q_1) = f(q_1, 0) \) and \( f_2(q_2) = f(0, q_2) \),

\[
f(q_1, q_2) = f_1(q_1) + f_2(q_2)
\]

\[
f_2(q_2) = f(0, q_2) = f_1(0) + f_2(q_2) \Rightarrow f_1(0) = 0
\]

\[
f_1(q_1) = f(q_1, 0) = f_1(q_1) + f_2(0) \Rightarrow f_2(0) = 0
\]

Therefore, the Lemma is true for \( J = 2 \).

Suppose the Lemma is true for \( J = N-1 \). Prove it is true for \( J = N \).

\[
f(q_1, q_2, \ldots, q_N) = \left( \frac{1}{N-1} \right) [f(0, q_2, \ldots, q_N) + f(q_1, 0, \ldots, q_N) + \ldots + f(q_1, q_2, \ldots, 0)]
\]

Since \( f(0, q_2, \ldots, q_N) \) depends only upon the \( N-1 \) variables \( q_2, \ldots, q_N \) and the Lemma is assumed true for \( J = N-1 \), \( f(0, q_2, \ldots, q_N) = \sum_{j=1}^{N-2} f_j(q_j) \) and \( f_j(0) = 0 \) for all \( j \). Similarly, each of the other terms in the right-hand side of the above expression depends upon \( N-1 \) variables and so in general, \( f(q_k) = \sum_{j=1}^{N-1} f_j(q_j) \). Hence,

\[
f(q) = \frac{1}{N-1} \sum_k f(q_k) = \frac{1}{N-1} \sum_k f_j(q_j) = \sum_j f_j(q_j) \quad \Box
\]

Theorem

There exists a well-specified ABC system that provides valid avoidable product costs and incremental activity costs if and only if

1) \( C(a(q)) = \sum_i C_i(a_i(q)) \)

2) \( C_i(a_i(q)) = p_i a_i(q) \)

3) \( a(q) = \sum_j a_j(q_j) \)

4) \( a_j(q_j) = 0 \) if \( q_j = 0 \) and \( a_j(q_j) \geq 0 \) if \( q_j > 0 \).
Proof

Sufficiency. Suppose 1) through 4) are true. The proof consists of demonstrating that there exists a well-specified ABC system that provides avoidable product costs and incremental activity costs.

Let $r_i = p_i$ and $m_{ij}(q_j) = a_{ij}(q_j)$ be the allocation scheme. This allocation scheme is a well-specified ABC system since $m_i = \sum_j m_{ij} = \sum_j a_{ij} = a_i$.

By condition 1): $C(a(q)) - C(a(q_k)) = \sum_i C_i(a_i(q)) - \sum_i C_i(a_i(q_k))$

By condition 2): $\sum_i C_i(a_i(q)) - \sum_i C_i(a_i(q_k)) = \sum_i p_i a_i(q) - \sum_i p_i a_i(q_k)$

By condition 3): $\sum p_i [a_i(q) - a_i(q_k)] = \sum p_i \left[ \sum_j a_{ij}(q_j) - \left( \sum_{j \neq k} a_{ij}(q_j) - a_{ik}(0) \right) \right]$

By condition 4): $\sum p_i \left[ \sum_j a_{ij}(q_j) - \left( \sum_{j \neq k} a_{ij}(q_j) - a_{ik}(0) \right) \right] = \sum p_i a_{ik}(q_k) = d P C_k$

Therefore, ABC product costs provide avoidable costs if conditions 1) - 4) are satisfied for the real underlying cost function.

Moreover, the incremental cost of a change in activity from $a$ to $a'$ is equal to the change in the ABC cost at the two activity levels if conditions 1), 2) and 4) are satisfied:

$$C(a') - C(a) = \sum p_i a_i'(q) - \sum p_i a_i(q) = \sum p_i a_i'(q) - \sum p_i a_i(q) = \sum \eta_i (a_i' - a_i) = \sum \eta_i (m_i' - m_i) \text{ QED}$$

Necessity. Suppose a well-specified ABC system provides valid avoidable product costs and incremental activity costs. Prove that the real cost function must satisfy conditions 1) through 4).

First prove that conditions 3) and 4) of the Theorem must be satisfied:

By property b) of a well-specified ABC system,

$$a_i(q) = m_i(q) = \sum_j m_{ij}(q)$$

Thus, $\sum_k [a_i(q) - a_i(q_k)] = \sum_k [m_i(q) - m_i(q_k)] = \sum \left[ \sum_j m_{ij}(q_j) - \left( \sum_{j \neq k} m_{ij}(q_j) - m_{ik}(0) \right) \right] = \sum m_{ik}(q_k) = m_i(q) = a_i(q)$

Noting from the above that $\sum_k [a_i(q) - a_i(q_k)] = a_i(q)$, by the Lemma there must exist functions $a_{ij}(q) \geq 0$ and $a_{ij}(0) = 0$ such that $a_i(q) = \sum_j a_{ij}(q_j)$. Furthermore, $m_{ij}(q_j) = a_{ij}(q_j)$. Thus, conditions 3) and 4) of the Theorem must be satisfied for a well-specified ABC system.

Now prove that the real cost function must satisfy conditions 1) and 2) of the Theorem. Suppose the ABC system provides valid incremental activity costs. That is, suppose $\sum \eta_i (a_i' - a_i) = C(a') - C(a)$ for all $a'$. Define $C_i(a_i) = C(a_1, a_2, ..., a_{i-1}, a_i, a_{i+1}, ..., a_n)$.

The validity of ABC system activity costs implies
that \( C_i(a_i) = r_i a_i \). This cost function satisfies condition 2), with \( r_i \) equal to \( p_i \). It only remains to be shown that \( C(a) = \sum_i C_i(a_i) \).

By property c) of a well-specified ABC system, \( \sum_i C_i(a_i) = \sum_i r_i a_i = \sum_i r_i m_i = C(a) \).

QED

REFERENCES


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