Private Equity Indices Based on Secondary Market Transactions*

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Abstract

We propose a new approach to evaluating the performance of private equity investments using actual prices paid for LP shares of funds transacted in secondary markets. Our transaction-based indices exhibit substantially higher CAPM betas and lower alphas than NAV-based indices even after adjusting for staleness in NAVs. Our indices load on an additional funding liquidity factor that is uncorrelated with NAV-based index returns. In comparison, a listed PE index exhibits similar loadings on the market and funding liquidity factor as our indices, but significantly lower average returns. Our indices are useful for quarter-to-quarter benchmarking and valuing illiquid stakes in funds.

JEL classification: G11, G23, G24

Key words: Private Equity, Secondary Market for Private Equity Funds, Market Index

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1. Introduction

In recent decades, private equity has become an important asset class for institutional investors. A recent survey of institutional investors finds that 88% are invested in private equity, with nearly a third having an allocation greater than 10% (Whyte (2017)). A typical private equity investment begins with capital commitments at the fund's creation and ends with the final distribution, which is often 12 to 15 years after the initial capital commitment. The return on the fund is determined by the aggregate returns on the individual portfolio companies in which the fund invests, and is therefore only fully observable following the fund's final distribution.[1] The underlying values of these portfolio companies are likely to fluctuate with firm-specific and economy-wide news in a manner similar to that of public equities, but these fluctuations are usually not fully reflected in the valuations that funds report to their investors. Moreover, since returns are measured at such irregular, infrequent intervals, it can be quite challenging to estimate standard performance parameters such as factor alphas and betas. These same challenges also make it difficult to measure how returns in this asset class covary with other known asset pricing factors.

While active markets for trading investments in private equity funds did not exist prior to 2000, in the early 2000s, a secondary market developed in which limited partners (LPs) could transact their stakes in private equity funds. In this paper, we use data obtained from a large intermediary in this market to evaluate the risk and return of private equity funds in a similar manner to the way in which investors regularly use public equity markets to understand the risk and return of publicly traded companies. Using these data, we construct transaction-based indices for buyout funds. By comparing the performance of our indices with that of standard NAV-based indices and an index based on listed private equity companies, we learn new insights about the performance of private equity. In contrast to much of the existing literature,

[1] Funds do report "Net Asset Values (NAVs)" to their investors, which are accounting-based valuations of the fund. These NAVs are adjusted to reflect the fund's actual value, but at any point in time, the gap between the NAV and the value of an investor's stake in a fund can be substantial.

our transaction-based indices suggest that buyout funds do not outperform public markets on a risk-adjusted basis.[2]

A difficulty in constructing an index using our secondary market data for private equity is accounting for the fact that no fund trades in every period (quarter), and many funds do not trade at all. In the subsample of funds that could be matched with cash flow data from the *Preqin* database, there are 955 buyout transactions for 355 unique funds from 2006 through 2018, implying that the average fund trades 2.7 times in our sample, conditional on trading. Because some funds may be more likely to transact than others, we need to account for effects that may arise from sample selection.

We therefore create hedonic indices using the approach of Heckman (1974). On a large sample of buyout funds, we estimate the parameters of an econometric model using observed transaction prices. Using this model we create the inferred price for every fund in every quarter, including those that do not transact. We then construct our transactions-based indices from these inferred prices.[3] We also account for measurement error when estimating performance parameters by applying bias adjustments (e.g. Scholes and Williams (1977), Blume and Stambaugh (1983), Roll (1984), and Dimson (1979)).

Following the above approach, we estimate the betas of our transactions-based buyout indices to be in the range of 1.75, with some variation around that estimate depending on the way in which we address issues of non-synchronous trading and other forms of measurement error. As emphasized by Axelson, Sorensen and Strömberg (2014), the return on a buyout fund is essentially the return on a portfolio of highly levered firms. If the portfolio firms prior to the buyout have unlevered betas around 1, then a fund with a 50 percent debt-to-equity ratio (typical for buyouts during our sample period) should exhibit a portfolio beta around 2.[4]

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[2] For example, current evidence suggests buyout funds outperform on a risk-adjusted basis (Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2016)). It should be emphasized, however, that our study is on a more recent sample period than these other studies, so the results are not comparable and not necessarily inconsistent with these prior studies.

[3] Other indices based on secondary markets have also incorporated some type of interpolation to infer the prices of non-traded assets. Bond indices, for instance, often employ "matrix pricing" to infer the prices of non-traded bonds.

[4] This calculation assumes that the debt beta equals zero. In fact, estimates of buyout debt betas are positive, which would lead the fund-level beta in this example to be less than 2. See Kaplan and Stein (1990).

We benchmark the characteristics of our hedonic transactions-based indices against those of alternative approaches, including indices based on Net Asset Value (NAV) produced by *Burgiss* and *Cambridge*, and the *Listed Private Equity Index* constructed by *Standard and Poor's (S&P)*. Our transactions-based indices are more cyclical and exposed to market-wide risk than indices based on reported NAVs. While others have documented that general partners (GPs) seem to adjust their NAVs slowly (Metrick and Yasuda (2010), Ewens, Jones, and Rhodes-Kropf (2013)), we find that common approaches to adjust for such "value smoothing" in NAVs fail to produce betas comparable to those of our transaction-based indices. Over our sample period, the smoothing-adjusted betas of NAV-based indices are only about *half* the size of the betas of our transaction-based indices.

The higher betas for our indices imply CAPM alphas that are not statistically significantly different from zero. Even though buyout fund average returns are about 17% over the sample period, the relatively high average returns are just enough to compensate investors for their exposure to market-wide risk. This finding is in contrast to positive estimates of CAPM alpha for the NAV-based indices, consistent with the results from the existing literature.

Given the known limitations of NAV-based indices, we also compare the characteristics of our transactions-based indices against the characteristics of the *S&P Listed Private Equity Index*. We estimate the market beta of the *S&P Listed Private Equity Index* to be 1.74, similar to that of our transactions-based indices. However, the average *return* for the *S&P Listed Private Equity Index* over our sample period (9%) differs substantially from that of our transactions-based indices (about 17%). The difference in average returns may arise for a variety of reasons. First, the cash flows of publicly traded securities of private equity firms reflect cash flows of the general partners, whose claim is the present value of future fees and carried interest earned by the fund, rather than the cash flows of the limited partner in a particular fund. Second, large, publicly traded buyout firms such as Blackrock and KKR hold a variety of investments other than private equity, including hedge funds, real estate, advisory services, etc. Third, some of the publicly traded private equity funds are funds of funds that charge an extra layer of fees (that varies with performance) in addition to the fee collected by the managers of the unlisted funds in which funds of

funds invest. Finally, there is potential for sample selection in the types of funds that choose to list public shares. We estimate a negative CAPM alpha for the publicly listed index, which is not surprising given its relatively low average returns.

We also evaluate the degree to which our transactions-based indices covary with known asset pricing factors. The value of a private equity position depends heavily on the GP's ability to obtain debt financing from institutional lenders. To the extent that funding liquidity in private equity is correlated with systematic funding liquidity (Brunnermeier and Pedersen (2009)), a portion of average private equity returns could represent compensation for exposure to funding liquidity risk. We examine this hypothesis using our transactions-based indices and show evidence that the indices positively load on a tradeable funding liquidity factor. The *S&P Listed Private Equity Index* also loads significantly on the funding liquidity factor while NAV-based indices do not. Since the funding liquidity risk premium is positive over our sample period, these results suggest that a portion of the high annual returns generated in private equity is indeed compensation for risk associated with the dependence of GPs on bank financing for leverage.

Besides altering perceptions of private equity performance, the lack of good performance benchmarks could also distort investors' portfolio decisions. For example, during the Financial Crisis of 2008, a number of investors believed that their portfolio weights in private equity had substantially increased, since NAVs of their private equity positions dropped much less in value than the market values of their stock holdings. Our analysis suggests that at the time of the Crisis, private equity fund values had fallen by at least as much as those of public equities. This analysis suggests the portfolio allocation of institutional portfolios to private equity did not increase during the Financial Crisis as much as investors may have thought.

Finally, our indices also allow us to value individual funds at any point in time and to estimate the extent to which a fund's NAV differs from its secondary market value in any particular year. These market-to-book estimates could potentially be used by investors to value stakes of private equity funds in their portfolios. Our results suggest that the values of private equity stakes sometimes differ substantially from their NAVs. For example, our estimates suggest that in 2015 market values were about 14% higher on

average than NAVs for 2007 vintage funds, and about 5% lower for 2006 vintage funds. Consequently, in 2015, investors using NAVs to assess their portfolios were likely to understate the value of their private equity holdings. These understatements could materially affect investors' portfolio decisions as well as their spending decisions, especially if investment organizations set spending levels at a fixed fraction of portfolio values.

While the indices we construct are similar to stock and bond indices that reflect prices achieved by the marginal investor who transacted in secondary markets during our sample, we note that the performance of our transactions-based indices can differ from that of the typical buy-and-hold investor for two reasons. First, the duration of expected cash flows can be held relatively constant in our indices based on secondary market transactions while the duration of cash flows (and hence, systematic risk) for a buy-and-hold investor dramatically shrinks over the life of the investment. Second, the returns of all asset classes in secondary markets are influenced to some degree by transaction costs. In secondary markets for private equity, most transaction costs appear to be borne by sellers. Buyers earn a small premium as market makers if positions are held to the end of a fund's life (Nadauld, Sensoy, Vorkink, and Weisbach, (2019), NSVW hereafter). In contrast, our index returns reflect returns from buying *and* selling at secondary market prices. To the extent that buyers earn a premium for market making, this premium is largely canceled out when the position is sold.

The paper proceeds as follows. Section 2 describes the other approaches that have been used to estimate the risk and return of private equity. Section 3 discusses the secondary market data we use to construct our indices. Section 4 describes the empirical methods we use to construct the indices. Section 5 presents the indices and evaluates their risk and return. Section 6 measures the exposure of the transactions-based indices on well-known asset pricing factors. Section 7 explains how transactions-based indices can be used by limited partners to value their investments. Section 8 concludes. An appendix addresses important institutional features of the secondary market that could influence pricing and discusses further details regarding our secondary market data and econometric methodology.

2. Prior Work Measuring Private Equity Risk and Return

For most asset classes, investors rely on secondary market transaction-based indices to measure risk and return. Because such secondary markets did not exist for many years in private equity, alternative approaches were developed to measure and assess risk and return in this market. In this section, we present these approaches and summarize the estimates that have come from each.

Prior studies about the investment performance of private equity can be broadly classified into three groups, which vary depending on the type of data used. First, a number of studies use fund-level data on cash flows paid to and received by limited partners. Second, other studies use data on private equity funds' investments in their portfolio companies and the distributions they receive from these companies. Last, some studies use other measures of private equity value such as the prices of publicly traded private equity funds.

The first group of papers that use data on cash flows between LPs and funds have relied on the public market equivalent (*PME)* approach, which measures the performance of a fund relative to the public equity market over the same time period.[5] Recent studies that use fund-level cash flow data find the PME for buyout funds to be in the range of 1.19-1.23, suggesting that buyout funds outperform public equity markets even after adjusting for fees (see Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2016)). Other studies use fund-level cash flows to estimate CAPM betas by estimating cross-sectional regressions of fund $IRRs$ on the $IRRs$ of the public equity market measured over the life of each fund (see Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), and Driessen, Lin, and Phalippou (2012)). These papers generally find betas to be in the range of 1.08 to 1.23.[6] Recent work by Ang, Chen, Goetzmann, and Phalippou (2018) uses fund-level cash flow data to extract an unobserved

[5] The *PME* approach was originally developed by Kaplan and Schoar (2005). $PMEs$ are calculated by discounting all cash flows of the fund at a rate equal to the total return on the S&P 500 index, and then dividing the discounted value of cash inflows by the discounted value of cash outflows over the same period. A fund with a PME above 1.0 therefore has outperformed the passive index over the evaluation period. Korteweg and Nagel (2016) provide an extension of the *PME* that allows for risk adjustments.

[6] An exception is Kaplan and Schoar (2005), who find a buyout beta of 0.41. Other papers that investigate fund-level cash flows include Chen, Baierl and Kaplan (2002), Phalippou and Gottschalg (2009), and Phalippou (2012).

performance measure that allows a time series of quarterly PE returns to be decomposed into passive and active components. Their estimates suggest CAPM buyout betas are about 1.25, with multi-factor market betas in a similar range.

The second group of papers that rely on cash flows between private equity funds and their portfolio firms generally estimate cross-sectional regressions of excess IRRs on the excess IRRs of factor portfolios.[7] These papers find CAPM alphas for buyout funds to be in the range of 9.3% to 16.3% with betas in the range of 0.95 to 2.3 (see Frazoni, Nowark, and Philippou (2012) and Axelson, Sorensen, and Strömberg (2014)). In contrast to our work, these studies estimate risk and return *gross of fees*. Carried interest, which is similar to a short call position from the perspective of the investor, causes the net-of-fee beta to be lower than the gross-of-fee beta if the fund itself covaries positively with the market.

Cash flows between private equity funds and LPs and cash flows between private equity funds and portfolio companies are not measured over uniform intervals. Axelson, Sorensen, and Strömberg (2014) argue that the irregular intervals over which returns or IRRs are measured can affect estimates of a fund's risk and return. The IRR of a fund is a function of the IRRs of the individual deals in which the fund invests, which is only observable after the deal exits. The intervals over which the IRRs are calculated vary across deals and, to the extent that funds return capital at different times, across funds as well. Using IRRs or returns compounded over irregular intervals can result in surprisingly large biases when estimating CAPM parameters. Axelson, Sorensen, and Strömberg (2014) simulate deal-level cash flows and estimate the CAPM using cross-sectional variation in IRRs. In some specifications, the beta is underestimated relative to the true beta by 116% while in other specifications, it is overestimated by 123%.[8]

The last group captures other approaches, including the use of public markets. Jegadeesh, Kraussl, and Pollet (2015) develop an approach to estimating private equity performance based on the returns to publicly traded private equity securities. In general, these securities are publicly traded funds of funds and

[7] The use of IRR is necessary since deal-level cash flows sometimes include intermediate cash flows that occur because of interim recapitalizations or equity injections.
[8] One common solution used to account for the irregular intervals over which IRRs are measured is to use log IRRs. This approach, however, requires specific parametric assumptions to correctly estimate alpha.

private equity firms. Their returns are likely to be correlated with those received by limited partner returns in private equity funds but are likely different for four reasons. First, the cash flows of publicly traded securities of private equity firms reflect cash flows of the general partners, whose claim is the present value of future fees and carried interest earned by the fund, rather than the cash flows of the limited partner in a particular fund. Second, large, publicly traded buyout firms such as Blackrock and KKR hold a variety of investments other than private equity, including hedge funds, real estate, advisory services, etc. Third, some of the publicly traded private equity funds are funds of funds that charge an extra layer of fees (that varies with performance) in addition to the fee collected by the managers of the unlisted funds in which funds of funds invest. Finally, there is potential for sample selection in the types of funds that choose to list public shares.

Overall, the literature suggests that buyout funds tend to perform well. The average PME is usually estimated to be greater than 1, which implies that these funds outperform public markets. This outperformance could reflect positive alpha, the greater risk of private equity funds relative to the market, or both. While most studies of private equity performance find evidence of positive alpha, estimates of fund betas are more disperse, with some studies finding betas in the range of 1.0 and other studies finding betas above 2.0.

3. Constructing an Index Measuring Private Equity Returns from Transactions Data

Transactions in the secondary market for private equity are sales of a limited partner's stake in a fund. In a transaction in this market, the buyer pays a price that is stated as a fraction of the NAV of the investments the fund has made to that point. In addition, the buyer assumes the liability for all future capital calls. Therefore, the price will reflect expectations about the performance of the deals that have already been made and also the expectation of the quality of the fund's future deals.

Private equity returns are a function of transaction prices, fund contributions and fund distributions. While we observe quarterly distributions and contributions for a large sample of funds, market prices are available only for transactions intermediated by the firm that provides us with our data. In this sample, no

fund transacts in every quarter and some funds never transact at all.[9] Given this feature of the data, we estimate a hedonic model that uses actual transaction prices of funds that do trade to infer the prices of funds that do not trade. This approach helps avoid potential biases associated with sample selection. We show that if parameter estimates of the hedonic model are unbiased, then measurement error in the inferred prices is zero in expectation and independent of actual (unobserved) prices. The setting is similar to that of Blume and Stambaugh (1983) and Roll (1984).

In addition, the transactions that do occur throughout the quarter are highly nonsynchronous. Nonsynchronous trading causes returns of the individual assets in a portfolio to be observed over different overlapping time intervals as in Scholes and Williams (1977) and Lo and MacKinlay (1990). Our observed index returns therefore contain measurement error coming from two sources: non-trading of some funds, and nonsynchronous trading of funds that do trade.[10] Both sources of measurement error can induce biases in estimated variances and covariances. We next show how we construct our indices and correct for biases in estimated moments resulting from both sources of measurement error.

3.1. Calculating Index Returns

We estimate index returns for buyout funds in the *Preqin* universe. Let $P_{i,t}$ denote the secondary market price of a \$10 million commitment to private equity fund i at the end of quarter t.[11] Suppose at the end of quarter $t-1$, a purchasing LP acquires a $\$10$ million commitment to each of N different private equity funds from a selling LP, where N is the number of funds available in the universe.[12] If the purchasing LP holds for one period and then sells all positions at the end of quarter t, the buy-and-hold return for the portfolio, r_t, is given by:

[9] It is possible that some funds do not transact through the intermediary providing our data, but do transact through one of their competitors.

[10] Non-trading can be modeled as extensive nonsynchronous trading as in Lo and MacKinlay (1990) if all securities trade at some point in time. In our data, however, some funds never transact.

[11] To avoid potential confusion that could arise from our use of notation and language, we note that the term *price* in private secondary markets often gets used in two different ways. Prices in the secondary market are frequently quoted as a percent-of-NAV, notated in our equations as π_t. The term *price* could also reference the actual dollar amount an LP paid in a secondary transaction. We notate the dollar price in our equations as $P_{i,t}$.

[12] The commitment (in our case \$10 million) is arbitrary. In our example we only need all commitments to be of the same size. These results are generalizable to holdings of commitments of different sizes.

$$r_t = \frac{P_{N,t} + D_{N,t} - C_{N,t}}{P_{N,t-1}} - 1, \tag{1}$$

where $P_{N,t} = \sum_{i=1}^{N} P_{i,t}$ denotes the end-of-quarter value of the fund portfolio, while $D_{N,t} = \sum_{i=1}^{N} D_{i,t}$, and $C_{N,t} = \sum_{i=1}^{N} C_{i,t}$ represent total distributions and calls associated with the positions held during quarter t, respectively. Equation (1) defines the arithmetic return on a price-weighted portfolio of *N* funds and can be extended to other weighting methods by appropriately scaling prices and cash flows. We consider the universe of buyout funds in *Preqin* to be a fixed population for which we only observe a random sample of prices each quarter. We use observed prices and data on fund characteristics for all funds in this population to estimate the return on a size-weighted private equity portfolio (or index) of all funds each quarter, where size is total capital committed to the fund.

Fund transactions are non-synchronous. Assume that all funds in the population are assigned a transaction time each quarter (of which some transaction prices are unobserved), and let $Z_{i,t}$ denote the non-synchronous price of fund i as of its assigned transaction time. Let $\pi_{i,t} = Z_{i,t}/NAV_{i,t}$ be the price scaled by the fund's net asset value reported at the end of the quarter, and let $\mathbf{x_{i,t}}$ be a $k \times 1$ vector of chosen characteristics associated with fund i in quarter t. The population of non-synchronous prices and characteristics can be characterized by the linear panel data model,

$$\pi_{it} = \mathbf{x}_{\mathbf{i,t}}'\boldsymbol{\theta} + e_{i,t}\ , \tag{2}$$

where $\boldsymbol{\theta}$ is the non-stochastic $k \times 1$ vector of population parameters, and $e_{i,t}$ is the disturbance term perfectly uncorrelated with $\mathbf{x_{i,t}}$.

The sample of prices we observe is drawn from the fixed *Preqin* population where some funds may have a greater probability of being selected than others. Using data on the random sample of funds that transact, we obtain an estimate of $\boldsymbol{\theta}$, defined as $\widehat{\boldsymbol{\theta}}$. While we observe $\pi_{i,t}$ only for funds in the random sample, we observe the k characteristics for all funds in the population. We therefore use these characteristics and the vector of estimated parameters, $\widehat{\boldsymbol{\theta}}$, to estimate fitted scaled prices for all funds in the population as

$$\hat{\pi}_{i,t} = x'_{i,t}\widehat{\boldsymbol{\theta}}, \tag{3}$$

and obtain estimates of fund prices, $H_{i,t}$, as

$$H_{i,t} = \hat{\pi}_{i,t} \times NAV_{i,t}, \tag{4}$$

where $NAV_{i,t}$ is the net asset value of fund i in quarter t.[13] The estimated index value based on these fitted prices is then given by $H_{N,t} = \sum_{i=1}^{N} H_{i,t}$, where N is the number of funds in the population in quarter t.[14] The observable index return in quarter t is

$$r_{H,t} = \frac{H_{N,t} + D_{N.t} - C_{N,t}}{H_{N,t-1}} - 1. \tag{5}$$

The OLS estimate of $\widehat{\boldsymbol{\theta}}$ is unbiased only if we can assume that population residuals from the linear panel data model given in (2) are independent of $\mathbf{x_{i,t}}$ conditional on selection (see, for example, Hall (2000)). Because unknown factors are likely to impact both fund prices and their probability of being selected for transaction, we estimate the parameters of a standard Heckman (1974) sample selection model by maximum likelihood, by which we jointly estimate the parameters of the linear panel data model given in (2) and the parameters of a linear latent variable model of the fund selection process. As is standard, we assume the error terms of the pricing and selection processes are jointly normally distributed and potentially correlated, which among other things, enables the parameters of the latent selection process to be estimated as a probit model. Presuming that the normality assumption holds, the Heckman sample selection model delivers an unbiased estimate of $\widehat{\boldsymbol{\theta}}$. In the special case that the correlation between the population residuals of the pricing and selection models is zero, then the Heckman and OLS estimates of $\widehat{\boldsymbol{\theta}}$ are identical in expectation. We discuss estimates of the selection model in Section 5.1.

[13] While it is common to use hedonic models to estimate price changes for a basket of goods with fixed characteristics over time (see, for example Pakes (2003)), our objective is to understand the price changes of a portfolio when some transaction prices are not observed, and fund characteristics evolve over time. Doing so enables us to estimate the return on the set of funds accessible to investors.

[14] In computing $r_{H,t}$ we use only those funds with observed characteristics in the population both at time t and at time $t-1$. Funds virtually always remain in the population with observed characteristics until they exit based on specified criteria. Funds effectively never exit and re-enter the population.

Before further illustrating the properties of $r_{H,t}$, we first make two comments regarding the linear panel data model given in (2). First, we use scaled prices as the dependent variable in these regressions instead of raw prices since raw prices are not stationary. Second, components of $\mathbf{x_{i,t}}$ may be endogenous and this model may suffer from omitted variables. For our present purposes, however, we are not interested in identifying causal forces behind π_{it}, nor in estimating the parameters of some "true" model. Instead, our current aim is to use observable fund characteristics to explain variation in $\pi_{i,t}$ that will help us infer prices for funds that do not transact in the fixed population.

3.2. Estimating Index Parameters

We use the observable return on our transaction-based index defined in (5) to estimate parameters such as beta, alpha, and volatility for r_t, the index return based on the *full* population of actual (unobserved) end-of-quarter prices. We compare these estimates against estimates from NAV-based indices and an index based on listed private equity companies. In this section we illustrate how we estimate these parameters and account for biases arising from measurement error.

Given that we can obtain an unbiased estimate of $\widehat{\boldsymbol{\theta}}$, Appendix C shows that to an approximation,

$$r_{H,t} = r_t + \Delta\delta_t - \Delta\tilde{r}_t, \tag{6}$$

where δ_t is mean zero and independent of r_t and $\tilde{r}_t$ represents the portfolio return from buying all funds at their non-synchronous prices during quarter t and selling at the end of the quarter. The parameter $\Delta\delta_t$ arises from measurement error in computing fitted hedonic prices to represent true prices and leads to spurious negative autocorrelation in $r_{H,t}$ as in Roll (1984). The parameter $\Delta\tilde{r}_t$ arises from non-synchronicity in the prices we observe and leads to spurious positive autocorrelation and positive cross-autocorrelation with lagged market returns as in Scholes and Williams (1977). Empirically we find $r_{H,t}$ to be positively autocorrelated, suggesting that measurement error from non-synchronous trading dominates measurement error arising from our hedonic model.

If we make the simplifying assumption that r_t, δ_t and transaction times are i.i.d., it follows that:

$$
\begin{aligned}
E[r_t] &= E\left[r_{H,t}\right] \\
Var[r_t] &= Var\left[r_{H,t}\right] + 2Cov\left[r_{H,t}, r_{H,t-1}\right] \\
Cov\left[r_t, r_{m,t}\right] &= Cov\left[r_{H,t}, r_{m,t}\right] + Cov\left[r_{H,t}, r_{m,t-1}\right],
\end{aligned}
\tag{7}
$$

where $r_{m,t}$ represents the quarterly public market return. The first line of (7) suggests that average estimated portfolio returns ($\bar{r}_{H,t}$) are unbiased estimates of the expected return based on end-of-quarter prices. The second and third lines of (7), however, indicate that estimated variances and covariances using $r_{H,t}$ require some adjustments to be unbiased. The bias-adjusted moments given in (7) can also be used to derive other bias-adjusted parameters, such as alpha, beta, Sharpe ratios, and correlations.

NAV-based indices are also considered to contain measurement error since GPs smooth their NAVs over time and respond slowly to market information about future cash flows (see, for example, Metrick and Yasuda (2010) and Ewens, Jones, and Rhodes-Kropf (2013)). These actions generate autocorrelation in NAVs as well as cross-autocorrelation with lagged market returns. For this reason, we apply bias adjustments based on the model of Dimson (1979) when estimating betas using NAV-based indices. We estimate Dimson-adjusted betas by regressing NAV-based returns on current and lagged market returns, specified as:

$$
r_{NAV,t=}\alpha_t + \beta_0 r_{mt} + \beta_1 r_{m,t-1} + \cdots + \beta_k r_{m,t-k} + \varepsilon_t \tag{8}
$$

where $r_{NAV,t}$ is the return on a NAV-based index calculated using quarterly NAVs and cash flows, and $r_{m,t}$ measures contemporaneous and lagged market returns. The return on a NAV-based index is specified as in (5) with $NAV_{N,t} = \sum_{i=1}^{N} NAV_{i,t}$ in place of $H_{N,t}$. Adjusted estimates of beta are achieved by summing the significant beta coefficients from the estimation. According to the Dimson (1979) model, average observed returns are also unbiased, similar to the framework we develop based on Scholes and Williams (1977). Appendix D describes our method to bias-adjust volatilities based on the Dimson (1979) model. To produce results that are consistent across bias-adjustment methodologies we also report Dimson-

adjusted betas and volatilities for the transaction-based hedonic index, $r_{H,t}$. In this context, the Dimson-adjusted betas and volatilities help account for non-synchronous trading, but make no explicit adjustment for measurement error in the fitted Hedonic prices.

4. Data on Transactions in the Private Equity Secondary Market

Our transaction data come from a large intermediary in the private equity secondary market and are similar to the data used in NSVW (2019).[15] These data identify the fund being sold, the vintage, the total capital committed by the seller, the amount unfunded by the seller, the purchase price, and the transaction date for all transactions consummated through this intermediary from January of 2006 through December of 2018. We clean the data as detailed in Appendix A and pull the most recent transaction for each fund each calendar quarter.

LPs transact in the secondary market for many reasons, most commonly, according to practitioners, for portfolio rebalancing purposes. Transactions on funds that are 10 years or older, commonly referred to as "tail-end transactions", are essentially transactions in which the buyer is speculating about the value of the one or two particular portfolio firms left in the fund and the seller is motivated to clean up their books and close out a fund. For this reason, the prices of tail-end transactions do not reflect the underlying economics of private equity as an asset class, since in these transactions, most of the fund is liquidated prior to the transaction. Similarly, when young funds, those less than four years old, transact, the majority of the fund's portfolio firms often have yet to be acquired. Sales of young funds are typically motivated because a seller wishes to be relieved of the liability associated with future draw downs. Prices for young funds therefore often reflect the market value for liquidity rather than the expected value of private equity funds' portfolio firms.[16]

[15] The data used in this paper extend the data from NSVW (2019) through the end of 2018.

[16] An important feature of these markets is that transactions often occur for portfolios of funds rather than individual funds. In Appendix B, we discuss the way in which the portfolio nature of some of our transactions affects the construction of the indices.

For these reasons, transactions of funds in the 4 to 9 year range are most likely to reflect the value of funds' investments in portfolio firms. To the extent that common factors affect all private equity funds, these prices are likely to be informative about general PE values as well and are a metric through which private equity prices and returns can be measured. Consequently, we construct our indices using funds that are 4 to 9 years old.

We obtain data on other fund characteristics, such as calls, distributions, NAV, fund LP type, and size, from *Preqin*, and clean these data as detailed in Appendix A. Within each calendar quarter we sum all contributions and distributions for a given fund. We then merge transactions data with the *Preqin* sample, some of which is done by hand (see Appendix A for details). After this process, we end up with a sample of 703 funds for which we have fund-level data. We observe 955 transactions on 355 of these funds between the first quarter of 2006 and the second quarter of 2018.

The first two columns of Table 1 report summary statistics for funds that transact and funds that do not. The third column of Table 1 reports statistics for all funds in our sample that are four to nine years old, whether or not they transact. The first three rows report the mean, first quartile (Q1), and third quartile (Q3) for scaled-transaction prices as a fraction of NAV, defined previously as $\pi_{i,t}$. Consistent with prior findings, funds on average transact at a discount relative to NAV (see NSVW (2019)). The overall average $\pi_{i,t}$ is 0.86, and for funds four to nine years old the average is 0.92.

The deviation between a fund's NAV and its market price will depend, in part, on the market's assessment of the fund's performance since making its investments. A fund that has made poorly performing investments may have a market price less than NAV, so a trade for less than NAV is not necessarily reflective of a liquidity discount. Similarly, a fund trading at a premium to NAV likely reflects good performing investments. The economic discount or premium at which a transaction occurs should be measured relative to the (unobservable) underlying value of the fund's assets, not the NAV.[17]

[17] NSVW (2019) develop this issue further. They argue that a second measure of a trade's discount relative to fundamental value is the difference in returns between buyers and sellers. If transactions always occurred at fundamental values and expected returns do not change over time, then buyer and seller expected returns should be

Funds that transact in the secondary market tend to be larger than the average fund in our sample. The average fund size for funds that transact is about $4.8 billion, compared to an average fund size for funds that do not transact of about $2.7 billion. The average age of all transacting funds is 8.8 years, and 6.3 years for non-transacting funds. Buyout funds that transact display slightly higher average $PMEs$, in the range of 1.15 compared to 1.12 for non-transacting funds.[18] The sample of funds that are four to nine years old exhibits a similar average size, age and *PME* as the full sample.

Figure 1 reports the number of transactions per quarter for the full sample, of which there are 955 transactions. The figure highlights the rapid growth in the secondary market. The years 2017 and 2018 report almost 5 times the number of transactions as occurred in 2006 and 2007.

5. Estimates of Secondary Market Based Private Equity Indices

5.1. Estimation Details

As discussed in section 3.2, we specify our hedonic model as a Heckman (1974) sample selection model and estimate parameters by maximum likelihood, using the full panel of data from the *Preqin* population.[19] For comparison, we also estimate the pricing equation from the sample selection model by OLS. Table 2 describes the explanatory variables we use in these estimations. The first seven rows of Table 2 list a set of macroeconomic or "state" variables that are constant across funds and vary only over time. The final six rows of Table 2 list the "fund-specific" variables that vary both across funds and over time.

It is common in selection models to include an exclusion restriction, a variable in the selection equation that is not correlated with the main dependent variable of interest: in our case, prices scaled by

approximately the same. Instead, these authors find that buyers of LP stakes outperform sellers, suggesting that, on average, transaction prices tend to be lower than fundamental values.

[18] We calculate the *PME* for each fund using all cash flows up to the most recent date for which we have cash flow data in *Preqin*, using NAV as the terminal value for funds that have not liquidated.

[19] Monte Carlo experiments indicate that MLE is often more efficient than the two-step approach originally proposed by Heckman (1979) (see Puhani (2000)). In addition, MLE allows for straightforward computation of robust asymptotic standard errors, is convenient for conducting standard model diagnostics, and imposes the natural restriction that the absolute correlation between the residuals of the pricing model and the selection model is less than or equal to one. Regardless, our results are virtually unchanged using either approach to estimate the parameters of the model.

NAV. Estimates of the standard parametric Heckman (1974) model that do not include exclusion restrictions are still unbiased and consistent.[20] Adding an exclusion restriction, however, causes the model to be over-identified, and therefore potentially more efficient and less sensitive to the normality assumption.

We include as an exclusion restriction in our model the fraction of LPs invested in a fund that are pension funds. Pension funds are typically buy-and-hold investors with the main investment objective of matching investments with the duration of their liabilities. This argument suggests that pension funds should be less likely to trade in the secondary market than other PE investors. Consistent with this idea, NSVW (2019) document that pension funds are less likely than other investors to sell their private equity stakes. Moreover, the characteristics of LPs in general are unlikely to be correlated with changes in transaction prices because buyers do not know the identity of the seller (or the type of seller) during the price discovery process of the transaction. The intermediary providing our transaction data keeps the names of clients confidential. Competition among bidders should move prices to be independent of the seller's identity.

Evidence on the investment performance of pension funds is somewhat mixed. Lerner, Schoar, and Wongsunwai (2007) find evidence that pension funds outperform other LPs (other than endowments) in their PE reinvestment decisions. Others find that pension funds make poor investment decisions relative to other institutions (see Hochberg and Rauh (2012), Sensoy, Wang, and Weisbach (2014), and Andonov, Hochberg, and Rauh (2017)). Regardless, results are very similar with or without this exclusion restriction in our sample selection model. We also find evidence that the residuals of the pricing and selection models in our sample selection model are uncorrelated. This suggests that even OLS estimates of $\widehat{\boldsymbol{\theta}}$ (with no exclusion restriction), which are very similar to those obtained from the Heckman sample selection model and also reported below, are unbiased.

The indices we create are size-weighted, where size is defined as the total capital commitment to a given fund. We use these size-weighted indices to estimate index parameters such as expected return, alpha, beta, and volatility using the bias adjustments defined in (7).

[20] See Wooldridge (2010) pp 803-808.

We estimate the standard errors of the sample selection model using the quasi-maximum-likelihood approach of White (1982), which accounts for heteroscedasticity and any cross-sectional or time-series dependence. Index parameters, such as beta, are estimated from index returns that depend on estimates of the sample selection model. To account for this two-pass approach when estimating the standard errors of index parameters, we use the subsampling approach of Politis and Romano (1994). We subsample consecutive overlapping blocks of data used to estimate the hedonic model, where a block is defined as all data in the cross section for dates within a given range. The number of dates that define a block (the block size), is determined using the "minimum volatility method" (see Politis, Romano, and Wolf (1999), pp. 197-200.) By sampling blocks we preserve the time-series dependence structure. For a given block we estimate the parameters of the hedonic model and use these parameters to create index returns and corresponding index parameters for the given block. The standard error for an index parameter is the standard deviation of the parameter across all blocks.[21]

5.2. Pricing Parameters

Table 3 presents our estimates of the parameters for the sample selection model. Panel A reports estimates of the selection equation, the latent variable (probit) model for the selection process. Panel B reports the parameters of the pricing equation, the parameters we use to infer the prices of funds that do not transact, using both OLS and the Heckman sample selection model.

The estimates presented in Panel A suggest that a number of variables are associated with fund selection. Among the economy-wide state variables, funds are more likely to transact when public equity markets are valued highly. The coefficient on the aggregate market-to-book ratio for equities, MTB is 0.63, with a t-statistic of 6.9. Funds are also more likely to transact when the Valuation Confidence Index is high, and when the Crash Confidence Index is low, that is, when institutional investors believe the public market

[21] Alternatively, we estimate standard errors for index parameters by GMM assuming we observe index returns without estimation error. For index parameters of the transactions-based index, subsampled standard errors are about 40% larger than GMM standard errors that ignore estimation error from the hedonic model. For index parameters of the NAV-based indices and the *S&P Listed Private Equity Index* which have no estimation error from fitted prices, GMM standard errors are similar to those we create using the subsampling method.

is not overvalued but the likelihood of a crash is also relatively high. Funds are also more likely to sell when average market NAVs are low.

Among the fund-specific variables, not surprisingly, larger funds are much more likely to transact than smaller funds. In addition, fund age also affects the likelihood of a transaction. The coefficient on $AGE < 4$ (dummy equal to one if fund is less than 4 years old) is negative and statistically significant, while $AGE > 9$ (dummy equal to one if fund is older than 9 years) is positive and significant. Funds 4 to 9 years old (the group excluded in the age-dummy-variable classification) are the most common type of transaction in our data in terms of absolute number, but the percentage of four-to-nine-year-old funds that transact is smaller than the percentage of older funds that transact. Finally, funds that transact have lower average *PME*s at the time of the transaction after controlling for other variables in our model.

In Model 2, we add an additional set of covariates to the selection and pricing equations. Many of the transactions that occurred during the Financial Crisis were driven by LPs' immediate liquidity demands. For this reason, it is possible that fund-specific attributes influenced selling decisions in a different manner during the Crisis than outside the Crisis. To account for this possibility, we introduce a Crisis (2008-2009) fixed effect and interact the Crisis fixed effect with each of the fund-specific characteristics. The inclusion of these effects alters the coefficients on some of the fund-specific estimates, but none substantially.

Panel A of Table 3 also reports estimates of the contemporaneous correlation between the population residuals of the pricing and selection models, ρ. As mentioned in section 3.2, if $\rho = 0$, then the Heckman and OLS estimates of $\widehat{\boldsymbol{\theta}}$ are identical in expectation. For both Models 1 and 2 we find ρ to be insignificant. For Model 1 the estimate of ρ is -0.14 and for Model 2 the estimate of ρ is -0.18, both with a t-statistic of about -0.8. For both models we also test the null hypothesis that $\rho = 0$ using Wald, Likelihood Ratio, and Lagrange Multiplier tests. All three tests fail to reject the null for both models. These results indicate that after controlling for our chosen set characteristics, innovations in price and selection are uncorrelated, and even OLS estimates of the parameters of the pricing equation, $\widehat{\boldsymbol{\theta}}$**,** are unbiased.

We present estimates of $\widehat{\boldsymbol{\theta}}$ in Panel B of Table 3 from both the sample selection model and using OLS. These results indicate that scaled prices tend to be higher for funds with higher valuations as measured by $NAV_{i,t}$, and for transactions aged between 4 and 9 years (the omitted age category). We observe higher valuations for funds that are 4 to 9 years old, in part, because such funds are likely to have more capital invested than younger or older funds. Since portfolio firms increase in value on average and NAVs, in part, reflect historical cost, NAVs of funds with more invested capital tend to have relatively high deviations from the underlying value of their portfolio firms. Prices are also larger for funds with higher *PME*s at the time of the transaction, as we might expect if higher current *PME*s signal promising performance in the future.

The results in Panel B of Table 3 are nearly identical for both the Heckman sample selection model and using OLS. This is to be expected given that $\rho = 0$ as suggested by Panel A. The results indicate that after controlling for our chosen set of characteristics, there is no further correlation between selection and scaled prices. In this case, simple OLS estimates of the pricing equation are unbiased, and estimating the full sample selection model only results in loss of power. For our main results below, however, we continue to rely on the sample selection model to estimate $\widehat{\boldsymbol{\theta}}$.

5.3. Private Equity Indices over Time

Figure 2 graphs the size-weighted hedonic index using Model 2 over the 2006-2018 sample period. For comparison we also present the performance of the public equity market, as reported on Ken French's website, the index based on *Preqin* reported NAVs, the *Burgiss* and *Cambridge* indices, and the *S&P Listed Private Equity Index*. We create the *Preqin* index using the same population of funds that we use to create the transaction-based index. We observe NAVs for all funds in the population and create the *Preqin* index return as in (5) with $NAV_{N,t} = \sum_{i=1}^{N} NAV_{i,t}$ in place of $H_{N,t}$.

Figure 2 illustrates that the transactions-based index is more volatile than the three NAV-based indices (*Preqin*, *Burgiss*, and *Cambridge*). NAVs adjust slowly to changes in the value of the private equity funds' assets and they do not immediately adjust to information likely to affect future cash flows in the

manner of public equity markets. Consequently, the indices based on NAVs tend to be "smoother" than the transactions-based private equity indices or public equity markets. For example, the only sharp decline in equity markets during our sample period occurred during the 2008 Financial Crisis. During this period, the NAV indices declined somewhat because assets were written down, but did not decline nearly as much as public equity markets. In contrast, the decline in the transactions-based index over 2008 is similar to that in public equity markets. Over 2008 the public equity market index declined 37% , the *S&P Listed Private Equity Index* declined 64%, while the transactions-based buyout index declined 53%. Our *Preqin* NAV-based index, on the other hand, only fell by 25% during 2008. Even though private equity NAVs were written down by nearly one quarter of their value during the 2008 Crisis, their actual value most likely declined similar to public equity markets at that time.

5.4. Risk and Return of Private Equity Indices

In this section we report estimates of expected returns, beta, alpha, volatility, and Sharpe ratios for transactions-based indices of buyout funds. As first discussed in section 3.2, the Dimson (1979) adjustment for measurement error assumes index returns are correlated with lagged market returns. We report regressions of index returns on lagged market returns in Table 4. The first column reports results for the transactions-based hedonic index based on Model2, while the remaining columns contain estimates on the *Preqin*, *Burgiss*, and *Cambridge* NAV-based indices. The final column includes results for the *S&P Listed Private Equity Index*.

We find that the hedonic transactions-based index loads significantly on lagged market returns through only one lag. This finding supports the assumptions of our framework based on Scholes and Williams (1977) which imply that our measured fund returns from the hedonic model should be uncorrelated with the market beyond one lag.[22] In contrast, each of the NAV-based indices loads significantly on lagged market returns through three lags. The *S&P Listed Private Equity Index* appears to be uncorrelated with lagged market returns, with potentially spurious correlation at lag two. The results of

[22] See discussion in section 3.2, the bias adjustments given in (7), and Appendix C.

Table 4 suggest that when applying adjustments based on Dimson (1979), we should account for cross-autocorrelation with market returns at one lag for the transactions-based index, at three lags for the NAV-based indices, and zero lags for the *S&P Listed Private Equity Index*.

Panel A of Table 5 reports index parameter estimates for transactions-based hedonic indices. Results in the first two columns of Panel A are based on Model 1 from Table 3 as the hedonic pricing model, while results in the next two columns are based on Model 2, noting that Model 2 adds a crisis fixed effect and crisis-fixed-effect interaction terms to Model 1. We report estimates using bias adjustments based on Dimson (1979) and Scholes-Williams (1979) as discussed in section 3.2. In the table, "Dimson(x)" implies the use of a Dimson bias adjustment using x lags. Panel B reports index parameter estimates for the NAV-based indices and the *S&P Listed Private Equity Index*. Panel C reports differences in some parameters estimates between the transaction-based indices of Panel A, and the *Preqin* NAV-based index, which is based on the same underlying funds. Panel D reports differences for some index parameters between the transaction-based indices of Panel A, and the *S&P Listed Private Equity Index*.

Consistent with Figure 2, the estimates in Panel A of Table 5 indicate that buyout funds have performed very well over our sample period. Average returns for our transaction-based indices vary from 17% to 19% as reported in the first row of Panel A of Table 5. In contrast, NAV-based index returns have averaged from 12-14%, while the *S&P Listed PE Index* averaged a return of only 9% over our sample period.

Model 1 produces estimates of beta that vary from 1.77 to 2.14, while Model 2 produces estimates of beta from 1.42 to 1.72. The Dimson (1979) adjustment (with one lag for the transaction-based indices) produces estimates of beta that are somewhat lower than the Scholes-Williams (1979) adjustment. These estimates are larger than most betas reported in prior literature (an exception is Axelson, Sorensen, and Strömberg (2014) who estimate betas gross of fees). To understand the magnitude of these estimates, recall that private equity funds are portfolios of equity positions in leveraged buyouts. Since buyouts tend to be much more highly levered than public firms, Modigliani-Miller Proposition 2 implies that buyouts should have substantially higher betas than public firms. For example, Axelson, Jenkinson, Strömberg, and

Weisbach (2013) report a mean debt-to-total-capital ratio of 70% in their sample of 1,157 LBOs, with mean leverage closer to 50% during our 2006-2018 sample period. In contrast, typical large publicly traded firms have approximately a 20-25% debt-to-total-capital ratio. If the firms experiencing buyouts have asset betas equal to 1 and debt betas are positive but relatively small, the equity portion of the LBO should nonetheless have a beta close to 1.60-1.80, which is consistent with our estimates.[23]

The betas of our indices could be higher than betas of the underlying assets because of unfilled capital commitments. When an LP buys a PE stake from another LP, she buys the holdings in the underlying portfolio companies and also agrees to pay on any unfilled capital commitments. To the extent that GP's time their capital calls to buy additional portfolio companies when prices are cheap, the timing of capital calls may be negatively correlated with market returns, increasing the market beta of the LP return. One method of assessing this possibility is to re-estimate returns and betas using funds for which all the committed funds have already been called. This is the case for about 25% of our sample. These sub-sample estimates result in estimated expected returns of 14% and betas in the range of 1.57 to 1.90 range (unreported), and are not substantially different from our baseline results. These results suggest that the betas of our indices are not heavily influenced by unfilled capital commitments.

The S&P Listed Private Equity Index produces a beta similar to that of our transaction-based indices as reported in Panel B at 1.74. In contrast, the estimated betas for the NAV indices are between 0.76 and 0.85. These results suggest that NAV-based indices, even after Dimson adjusting as is common in the literature, fail to capture important dynamics in private equity that is related to public equity markets. Further, adding more lags to the Dimson adjustment does cause the betas of NAV-based indices to become more similar those of the transaction-based indices or the *S&P Listed Private Equity Index*. Even if we include seven lags in the Dimson adjustment (e,g. Metrick and Yasuda (2010)), we find that NAV-based betas over our sample period are only 0.73 to 0.85.

[23] See Axelson, Sorensen, and Strömberg (2014) for more discussion.

In addition, there is a large difference in volatility between the transactions-based indices (about 40%) and the NAV-based indices (ranging from 17 to 20%). The volatility of the *S&P Listed Private Equity Index* (30%) is also larger than the NAV-based indices. Panel C indicates that betas and volatilities are statistically significantly higher in a number of cases for the transaction-based indices than the *Preqin* NAV-based index.

Because estimates of beta for the transaction-based indices are relatively high, the estimated alphas are not statistically significantly different from zero even though average returns are also relatively high (17-19% depending on the model). The high returns earned by buyout funds appear to reflect standard market risk coming from leverage in the buyouts rather than an abnormal return. In contrast, estimated alphas for the NAV-based indices are positive (6% for *Preqin* and 4% for both *Burgiss* and *Cambridge*) and significantly different from zero.

While the betas of our transaction-based indices are similar to that of the *S&P Listed Index*, average returns and alphas are quite different. *The S&P Listed PE index* earned an average a return of 9% (t-statistic of 1.3) and an alpha of -7% over our sample period (t-statistic of -3.2). Panel D indicates that average returns and alphas are significantly lower for the *S&P PE Listed Index* than most of our transaction-based indices. The difference in average returns and alphas may arise for a variety of reasons, as described in detail in Section 2.

We also note in Table 5 that autocorrelations tend to be positive. Autocorrelations in this table are unadjusted for all indices, to better understand the nature of measurement error. For the bias adjustments we develop based on Scholes-Williams (1977) given in (7), measurement error arising from the hedonic model produces negative autocorrelation in returns as in Roll (1984), while non-synchronous trading produces positive autocorrelation. For the transaction-based indices, positive autocorrelations indicate that measurement error arising from non-synchronous trading dominates measurement error arising from our hedonic model.

5.5. The Impact of the 2008 Financial Crisis

An important issue in interpreting our results is the role of the 2008 Financial Crisis. Some may view the Financial Crisis was a one-time event that will not be repeated, and as such, some studies exclude this period in the hope of having a sample that will be more representative of the future. A fund's beta, however, is relevant to an investor because it measures the sensitivity of fund returns to market declines, and the Financial Crisis was the only major market-wide decline in equity markets during our sample period. In addition, because the cash flows produced by private equity funds for investors come through the highly pro-cyclical exits of portfolio companies, investors in private equity funds such as endowments who rely on cash flows to fund operations should be particularly concerned about the way their value changes during market downturns.

For these reasons, we focus our discussion on the estimates from Table 5 that includes the financial crisis, but for completeness, we present estimates of private equity risk and return omitting the Financial Crisis in Table 6. To calculate the hedonic index without the Financial Crisis, we use the same parameter estimates for the pricing equation as in Tables 3 and then omit years prior to 2010 when estimating index returns and parameters. Not surprisingly, the estimated betas on the transactions-based index are much lower when the crisis years are excluded, an estimated 0.97-1.16. Excluding the Financial Crisis, the alphas for the transaction-based index are higher, at 4% to 6%, but are still not statistically significantly different from zero.

6. Factor Exposures

The hedonic transactions-based indices can also be used to evaluate the exposure of private equity as an asset class to known, tradeable asset pricing factors capturing priced risks in addition to market risk. Given that buyout funds usually purchase companies that are smaller and faster growing than average publicly traded companies, we evaluate whether returns from the transactions-based indices covary with the well-established SMB and HML asset pricing factors. Liquidity potentially also plays an important role in the buyout space along two dimensions: aggregate *trading* liquidity and aggregate *funding* liquidity. While NSVW (2019) quantify idiosyncratic liquidity discounts associated with trading in the secondary market, they do not explore whether some portion of liquidity discounts can be explained by systematic

liquidity factors. It is possible that aggregate trading liquidity, influenced by systematic fluctuations in asymmetric information or uncertainty, may positively covary with aggregate buyout returns. We evaluate this possibility using the aggregate trading liquidity factor of Pastor and Stambaugh (2003).

A second measure of liquidity particularly pertinent to buyouts is the concept of funding liquidity, first introduced by Brunnermeier and Pederson (2009). General partners rely heavily on financial institutions' willingness and ability to provide the funding for leveraged buyouts. When lending conditions tighten, GPs will have limited purchasing capacity, driving buyout multiples lower.[24] We evaluate whether PE returns positively covary with a tradeable measure of funding liquidity created by Chen and Lu (2018). The measure of aggregate funding liquidity rests on the intuition that when funding conditions are tight, investors' ability to obtain high returns through leveraged positions in public securities diminishes, leading borrowing-constrained investors to prefer high beta stocks to low beta stocks on account of the embedded leverage in high beta stocks. Chen and Lu (2018) argue that this effect should be especially prevalent among stocks with high margin requirements for which funding liquidity constraints are more likely to bind. To separate aggregate funding liquidity from other factors that may drive demand for high beta stocks, these authors first sort on portfolios of high and low margin stocks. Within each of these portfolios, stocks are then sorted by high and low beta. The funding liquidity factor is the return on low beta stocks minus high beta stocks for stocks with high margin requirements, minus the return on low beta stocks minus high beta stocks for stocks with low margin requirements. When funding constraints bind, returns on high beta stocks increase, especially for stocks with high margin requirements, leading to a low realization of the funding liquidity factor.

Panel A of Table 7 reports regression results of returns from the transactions-based indices regressed on market returns, SMB, HML, the Pastor and Stambaugh (2003) trading liquidity factor, and the

[24] While a limited partner's commitment is binding, limited partners sometimes put informal pressure on general partners to curtail investments during periods of cash shortfalls such as during the Financial Crisis. If a general partner insists on drawing down capital when limited partners do not wish to provide it, limited partners can refuse to reinvest in the general partner's next fund. For this reason, GPs usually do their best to comply with these requests even though they are not legally obligated to.

Chen and Lu (2018) funding liquidity factor. As in Table 5, the first two columns of Panel A are based on Model 1 from Table 3 as the hedonic pricing model, while results in the next two columns are based on Model 2. To conserve power for the results of this table, we estimate the hedonic models using OLS rather than the sample selection model. Point estimates are nearly identical using either approach. We also report estimates using bias adjustments based on Dimson (1979) and Scholes-Williams (1979) as discussed in section 3.2.

When controlling for all five factors, index returns covary significantly only with the market. Market betas however, are consistently smaller than those reported previously when all five factors are included, and range between 1.05 to 1.42. We next estimate a series of regressions that include market returns and each of the four tradeable factors separately along with the market factor. In these regressions we find evidence that the transaction-based indices load significantly only on the funding liquidity factor when combined with the market factor. Funding-liquidity betas for the transaction-based indices range from 0.39 to 0.84 as reported in Panel A of Table 7.

In Panel B of Table 7 we report results of the factor regressions using each of the three NAV-based indices and the *S&P Listed Private Equity Index*. NAV-based indices do not covary significantly with the funding liquidity factor, with point estimates very close to zero or slightly negative. In contrast, the *S&P Listed Private Equity Index* loads significantly on the measure of funding liquidity, as reported in Column (4) of Table 7, Panel B, with a point estimate of 0.49, similar to those of the transaction-based indices.

Taken together these results suggest that the large market betas estimated earlier in the paper are partially explained by private equity's exposure to funding liquidity risk. Since the funding liquidity premium over our sample is about 8% (the average value of the tradable funding liquidity factor), our results suggest that a portion of the high annual returns generated in private equity is compensation for exposure to funding liquidity risk.

7. An Application: "Matrix Pricing" of Private Equity Funds

Unlike some securities, private equity funds rarely trade, so for most funds, no recent transaction prices exist to help value the fund. For this reason, most funds are valued by limited partners at the NAV, which can deviate substantially from the best available estimate of the fund's underlying value. These valuations are used for a number of purposes by investors in funds, including portfolio allocation decisions across asset classes, and spending decisions, which are usually set by investors as a fixed percentage of a portfolio's assessed value. Examples of this policy approach include universities and foundations. More accurate pricing of limited partner stakes in private equity funds may improve the ability of investors to make investment decisions since it may lead to portfolio allocations and spending rules corresponding to better estimates of the underlying values of an institution's private equity investments.

One possible approach to value stakes in private equity funds more accurately is to follow a procedure similar to "Matrix Pricing," commonly used to price bonds, by which the prices of bonds that do not trade are determined based on the prices of bonds that do. The idea is that the same fundamentals affect similar bonds in the same manner, so prices of bonds that do not trade likely move approximately the same amount as prices of similar bonds that do trade. Since private equity funds that invest in one type of asset are likely affected by a number of the same shocks to their fundamentals, they can be priced using comparable methods with transactions-based indices.

For any fund, the fund's history of quarterly cash inflows and outflows can be combined with the quarterly returns of the hedonic indices to calculate market values. Beginning at the end of some chosen quarter t we set fund value for fund i, $V_{i,t}$, equal to NAV. Then for all subsequent quarters we estimate fund value as

$$V_{i,t} = V_{i,t-1}(1 + r_{H,t}) + C_{i,t} - D_{i,t}, \tag{9}$$

where C_t and D_t denote capital calls and distributions between times t and $t + 1$, and $r_{H,t}$ represents the return on our hedonic transaction-based index based on Model 2. Equation (9) is based on a simple inversion of equation (5) for a single fund.

We perform this calculation for each fund in our sample for every quarter in our sample period (2006 to 2018). Because our hedonic indices are formed using transactions from funds that are between four and nine years old, we only calculate market values for four-to-nine-year-old funds. Following the procedure described above, we set market value equal to NAV at the end of the fourth year following each fund's vintage year and iterate forward to identify market values at the end of subsequent quarters.[25] We report year-end aggregate market-to-book ratios by vintage in Table 8 by summing year-end market values for each fund for a given vintage and dividing by the sum of year-end NAVs for the same set of funds.[26]

Average market-to-book ratios for each vintage across time are reported in the bottom of Table 8. The average ranges from a low of 0.90 for the 2002 vintage of funds to high of 1.16 for 2004 vintage. Market-to-book ratios are considerably lower during the Financial Crisis, ranging between 0.67 and 0.79 in 2008 for the 3 vintages that were old enough for our hedonic estimation. Funds that invested out of 2007 and 2008 vintage funds did so at lower valuations, and therefore have high average market-to-book ratios in subsequent years when markets recovered. For example, these estimates indicate that by the end of 2015, a 2007 buyout fund has a NAV that is understated by 14% relative to its market value.

Individual funds could mark their values to market using the hedonic approach in one of two ways. The most accurate approach would be to generate fund-specific market values using the hedonic-estimated coefficients applied to the fund's attributes. A simpler approach, but one that nonetheless represents a substantial improvement over using NAVs, is to multiply the NAV of each of LP's investments by the average market-to-book ratio of the industry. For example, an LP would multiply the NAV of each four-to-nine-year-old fund in his or her portfolio by the appropriate ratio from Table 8. For younger funds, the deviation between NAVs and market value is likely to be smaller but could be estimated using the coefficients from the hedonic regressions. Tail-end funds will have only a few portfolio companies left and

[25] For the 2005 vintage, we set NAV equal to market value at the end of year three due to irregularities in reported NAVs associated with the financial crisis.
[26] The 2018 market-to-book ratio is reported as of Q2 because of data availability.

their values will vary depending on the fortunes of these particular investments. As such, this approach is likely to be less useful for valuing these funds.

8. Conclusion

Measuring the performance of private equity investments has historically only been possible over long horizons because a fund's return is only observable following the fund's final distribution. However, in recent years, a secondary market has developed in which investors in private equity funds can trade their stakes. Prices from this market provide a source of data useful for measuring the risk and return of private equity funds in a similar manner to that commonly used to measure returns for other securities.

We construct indices of buyout performance using a proprietary database of secondary market prices of private equity stakes between 2006 and 2018. The indices provide a new approach to measuring the risk and return of private equity investments. In addition, the indices can be used by investors for a number of purposes, including using the estimates of private equity risk and return to make better portfolio decisions. In addition, it could be used to value investors' private equity investments by a method similar to the "matrix pricing" approach commonly applied to thinly traded bonds.

Our estimates indicate that the transactions-based indices of buyout funds track public equity much more closely than NAV-based indices we consider. The hedonic buyout indices we construct have market betas in the 1.75 range, consistent with the notion that buyouts with increased leverage have higher betas, in marked contrast to the NAV-based indices, whose Dimson-adjusted betas are estimated to be around 0.8. Even after adjusting for known staleness in NAV's, estimated betas from NAV-based indices likely understate the extent to which private equity returns depend on market-wide factors.

The high betas on the transactions-based indices have important implications for our understanding of private equity performance. Because of their high betas, the buyout indices have estimated alphas close to zero, even though the indices have average returns close to 17% during our sample period. Our estimates of alpha are not significantly different from zero. This finding is counter to our estimates of alpha for NAV-

based indices as well as estimates of alpha in the literature, which are positive and statistically significantly different from zero.

While the market betas of our transaction-based indices are similar to that of the *S&P Listed Private Equity Index*, we find that the average returns of the transactions-based indices are significantly higher. Over our sample period the transactions-based indices average a 17% return while the *S&P Listed Private Equity Index* averages only a 9% return. The difference in average returns may arise for a variety of reasons. First, the cash-flows of publicly traded securities of private equity firms tend to reflect the cash flows of the general partners rather than those of the limited partner in a particular fund. Second, the publicly traded index is based on the performance of public firms which hold assets other than private equity. Third, some of the publicly traded private equity funds are funds of funds that charge an extra layer of fees in addition to the fee collected by the managers of the unlisted funds in which funds of funds invest. Finally, the difference may also reflect sample selection in the companies that choose to list public shares. In contrast to the near-zero alphas of the transaction-based indices, we estimate a negative CAPM alpha for the publicly listed index.

Private equity returns appear to covary with a tradable measure of funding liquidity risk, which is plausible given the importance to buyout firms of having access to debt financing. Our estimated market betas shrink to the 1.05-1.42 range when controlling for funding liquidity risk. These estimates indicate that the high expected returns in buyouts reflect compensation for exposure to both market risk and the risk that debt financing, crucial to buyouts, varies in availability and costliness through economic cycles.

The buyout indices we construct have a number of potential uses for investors. Better estimates of private equity risk and return should affect the optimal portfolio decisions of investors when deciding on the allocation to private equity in their portfolios. In addition, the indices can be used to provide more accurate valuations of stakes in private equity funds that investors hold in a manner similar to the "matrix pricing" approach commonly used to price illiquid bonds. Our estimates suggest that the use of NAV for valuation as is done by most LPs can be misleading, and NAVs often substantially misstate the value of an

investor's private equity holdings. Improving these valuations is likely to affect investors' decisions about both the portfolio allocations and the amount they spend from their invested assets.

Undoubtedly, there are uses for the indices we have not discussed in this paper. For example, one could design derivative contracts based on an index of private equity returns. These derivatives could potentially be useful to investors or GPs who wish to hedge risks in their portfolios, or to speculate on the performance of the buyout sector. Better indices of private equity performance such as the ones presented here clearly have much to offer the private equity community.

References

Andanov, Aleksandar, Yael V.Hochberg, and Joshua D. Rauh (2017), Political Representation and Governance: Evidence from the Investment Decisions of Public Pension Funds, *The Journal of Finance*, forthcoming.

Ang, Andrew, Bingxu Chen, William N. Goetzmann, and Ludovic Phalippou (2018), Estimating Private Equity Returns from Limited Partner Cash Flows, *The Journal of Finance*, 78(4), 1751-1783

Axelson, Ulf, Tim Jenkinson, Per Strömberg, and Michael S. Weisbach (2013) Borrow Cheap, Buy High? The Determinants of Leverage and Pricing in Buyouts. *The Journal of Finance*, 68 (6), 2223-2267.

Axelson, Ulf, Morten Sorensen, and Per Strömberg (2014), Alpha and Beta of Buyout Deals: A Jump CAPM for Long-term Illiquid Investments, Working paper, London School of Economics.

Blume, Marshall E., and Robert F. Stambaugh (1983), Biases in Computed Returns, *Journal of Financial Economics*, 12(3), 387-404.

Brunnermaier, Markus K., and Lasse H. Pedersen (2009), Market Liquidity and Funding Liquidity, *Review of Financial Studies*, 22, 2201-2238.

Chen, Peng, Gary Tl Baierl, and Paul D. Kaplan (2002), Venture Capital and its Role in Strategic Asset Allocation, *Journal of Portfolio Management*, Winter 2002, 83-89.

Chen, Zhuo, and Andrea Lu (2018), A Market-Based Funding Liquidity Measure, Working Paper.

Dimson, Elroy (1979), Risk Measurement When Shares are Subject to Infrequent Trading, *Journal of Financial Economics*, 7, 197-226.

Driessen, Joost, Tse-Chun Lin, and Ludovic Phalippou (2012), A New Method to Estimate Risk and Return of Nontraded Assets from Cash Flows: The Case of Private Equity Funds, *Journal of Financial and Quantitative Analysis*, 47(3), 511-535.

Ewens, Michael, Charles M. Jones, and Matthew Rhodes-Kropf (2013), The Price of Diversifiable Risk in Venture Capital and Private Equity, *Review of Financial Studies*, 26(8), 2013.

Frazoni, Francesco, Eric Nowak, and Ludovic Phalippou (2012), Private Equity Performance and Liquidity Risk, *Journal of Finance*, 67(6), 2341-2373.

Gatzlaff, Dean H., and Donald R. Haurin (1998), Sample Selection and Biases in Local House Value Indices, *Journal of Urban Economics*, 43(2), 199-222.

Gornall, Will, and Ilya A. Strebulaev (2018), Squaring Venture Capital Valuations with Reality, *Journal of Financial Economics*, forthcoming.

Hall, Notes on Sample Selection Models (2000), Working Paper, University of Oxford.

Harris, Robert S., Tim Jenkinson, and Steven N. Kaplan (2014), Private Equity Performance: What do We Know? *Journal of Finance*, 69(5), 1851-1882.

Heckman, James J. (1974) Shadow Prices, Market Wages, and Labor Supply, *Econometrica*, 42(4), 679-694.

Heckman, James J. (1979), Sample Selection Bias as a Specification Error, *Econometrica*, 47(1), 153-161.

Higson, Chris and Rudiger Stucke (2012), The Performance of Private Equity, Working Paper.

Hochberg, Yael V. and Joshua D. Rauh (2012), Local Overweighting and Underperformance: Evidence from Limited Partner Private Equity Investments, *Review of Financial Studies*, 26, 403-451.

Hwang, Min., John M. Quigley, and Susan E. Woodward (2005), An Index for Venture Capital, *The B.E. Journal of Economic Analysis and Policy*, 4(1), 1-45.

Jegadeesh, Narasimhan, Roman Kraussl, and Joshua M. Pollet (2015), Risk and Expected Returns of Private Equity Investments: Evidence Based on Market Prices, *Review of Financial Studies*, 28(12), 3269-3302.

Kaplan, Steven N., and Antoinette Schoar (2005), Private Equity Performance: Returns, Persistence, and Capital Flows, *Journal of Finance*, 60(4), 1791-1823.

Kaplan, Steven N., and Jeremy C. Stein (1990), How Risky is the Debt in Highly Leveraged Transactions? *Journal of Financial Economics*, 27, 215-245.

Korteweg, Arthur and Stefan Nagel (2016), Risk Adjusting the Returns to Venture Capital, *The Journal of Finance*, forthcoming.

Lerner, Josh, Antoinette Schoar, and Wan Wongsunwai (2007), Smart Institutions, Foolish Choices: The Limited Partner Performance Puzzle, *Journal of Finance*, 62, 731-764.

Ljungqvist, Alexander and Matthew Richardson (2003), The Cash Flow, Return, and Risk Characteristics of Private Equity, NBER Working Paper 9454.

Lo, Andrew and Craig McKinlay (1990) An Econometric Analysis of Nonsynchronous Trading, *Journal of Econometrics*, 45, 181-211.

Metrick, Andrew and Ayako Yasuda, (2010) *Venture Capital and the Finance of Innovation*, John Wiley and Sons.

Nadauld, Taylor D., Berk A. Sensoy, Keith Vorkink, and Michael S. Weisbach (2019), The Liquidity Cost of Private Equity Investments: Evidence from Secondary Market Transactions, *Journal of Financial Economics*, 132, 158-181.

Pakes, Ariel (2003), A Reconsideration of Hedonic Price Indexes with an Application to PC's, *American Economic Review*, 93(5), 1578-1596,

Peng, Liang (2001), Building a Venture Capital Index, Yale ICF Working Paper No. 00-51.

Phalippou, L., (2012), Performance of Buyout Funds Revisited? *Working Paper*.

Philaippou, Ludovic and Oliver Gottschalg (2009), The Performance of Private Equity Funds, *Review of Financial Studies*, 22(4), 1747-1776.

Politis, Dimitris N., and Joseph P. Romano (1994), Large Sample Confidence Regions Based on Subsamples under Minimal Assumptions, *Annals of Statistics*, 22 2031-2050.

Politis, Dimitris N., Joseph P. Romano, and Michael Wolf (1999), *Subsampling*, Springer, New York.

Robinson, David T., and Berk A. Sensoy (2013), Do Private Equity Fund Managers Earn Their Fees? Compensation, Ownership, and Cash Flow Performance, *Review of Financial Studies*, 11(1), 2760-2797.

Roll, Richard (1984), A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market, *The Journal of Finance*, 39(4), 1127-1139.

Scholes, Myron and Joseph Williams (1979), Estimating Betas from Nonsynchronous Data, *Journal of Financial Economics*, 5(3) 309-327.

Sensoy, Berk A., Yingdi Wang, and Michael S. Weisbach (2014), Limited Partner Performance and the Maturing of the Private Equity Industry, *Journal of Financial Economics*, 112, 320-343.

White, Halbert (1982), Maximum Likelihood Estimation of Misspecified Models, *Econometrica*, 50(1) 1-25.

Whyte, Amy (2017), Survey: Endowments and Foundations Unfazed by Private Equity Valuations, *Institutional Investor*, www.institutionalinvestor.com.

Wooldridge, Jeffrey M. (2010) *Econometric Analysis of Cross-Section and Panel Data*, MIT Press.

Appendix A: Data Details

In this appendix we describe how we clean our data. From the transactions data we first pull all records for which their "detailed strategy" is classified as "Buyout". We then identify unique funds by their fund names, hand checking fund names that appear similar. We omit funds labeled as parallel funds, feeder funds, annex funds, sub funds, top-up funds, duplicate funds, co-investment funds, supplemental funds, and side-cars. We then clean the transactions data as follows:

1) Eliminate all funds with a total commitment less than $500M.
2) Eliminate transactions with a price less than zero.
3) Eliminate transactions with a NAV less than zero.
4) Eliminate transactions that have the same price for every fund in the portfolio transaction.
6) Eliminate transactions for which the total amount committed by the seller minus the unfunded commitment is less than zero.
7) Eliminate transactions for which the total capital committed is less than or equal to zero.
8) Eliminate transactions for which the fund name is missing.
9) If multiple transactions occur on the most recent transaction date for a given fund/quarter, use only the transaction based on the highest total commitment.
10) If multiple transaction records exist with the same fund name and commitment on the most recent transaction date for a given fund/quarter, choose one of these transactions at random as the transaction that represents the end-of-quarter transaction price.
12) Eliminate all remaining transactions for which the price, as a percent of NAV, is greater than 3 standard deviations away from the mean price across funds for a given quarter.

After pulling data from *Preqin* for funds with a "category_type" equal to "Buyout", we clean the data as follows:

1) Eliminate any fund-quarters for which $NAV_{i,t-1} = 0$, or the NAV-based return is otherwise missing. Note that we retain records for which $NAV_{i,t} = 0$. Once NAV hits zero, however, we no longer include the fund in the sample.
2) Eliminate stale NAVs, those with a report date prior to 30 days before the end of each quarter.
3) Identify fund-quarters for which the NAV-based return is greater than 3 standard deviations from the mean across all funds for a given quarter. These returns appear to be inconsistent with reported IRRs from *Preqin*.

We then merge our *Preqin* data with the explanatory variables described in Table 2, and then merge these data with the cleaned transactions dataset. To merge the transaction and cash-flow data, we first identify funds with identical fund names in the two databases and designate these as a match. We then identify fund names in the transaction and *Preqin* data that are "similar" and that also have the same vintage. Fund names A and B are considered similar if fund name A contains the first 5 characters of fund name B anywhere in the fund name string or vice-versa. We then hand check this list to determine which funds match. After merging we have data on 703 unique funds, 355 of which account for 955 transactions during our sample period from March of 2006 through June of 2018.

Appendix B. Portfolio Transactions in the Secondary Market

A unique feature of the PE secondary market is the fact that individual funds are frequently sold as part of a larger portfolio transaction. For example, a seller might offer to sell their ownership stake in five unique funds, hoping to sell a portfolio of holdings in one large transaction. In a portfolio transaction, the buyer submits an offer price for the entire portfolio of funds, and the buyer and seller enter into a contract to eventually transfer ownership based on the portfolio offer price. Given that the construction of our price index relies on the market prices paid for individual funds, rather than one price for a portfolio of funds, it is important to consider the economics that govern how prices get assigned to individual funds in a portfolio transaction to determine whether and how portfolio transactions could influence the index.

Once a buyer and seller are in contract to transfer ownership of the portfolio of funds, the process moves to a second phase. During the diligence process, buyers assign prices to each of the individual funds in the portfolio, subject to the constraint that the size-weighted average of the individual prices equals the winning offer price. The price allocation process can be nuanced because although the buyer bids on the full portfolio, they may in reality only have strong demand for certain funds in the offered portfolio. Conversations with industry experts indicate that there are times when prices allocated to individual funds result in particular funds being excluded from the final transaction. Thus, the prices assigned to individual funds are a reflection of demand for the funds, albeit filtered through the portfolio purchasing process. High prices are assigned to funds the buyer most demands and lower prices are assigned to funds they demand less, again, subject to the constraint that size-weighted average prices equal the full portfolio bid. The assignment of prices to individual funds is, therefore, ultimately a reflection of demand.

Appendix C. Estimating Index Parameters Using Observed Returns

In this appendix we motivate the bias adjustments given in (7). Consider a fixed population of funds, prices, and characteristics over T quarters. Let $Z_{N,t} = \sum_{i=1}^{N} Z_{i,t}$ and $H_{N,t} = \sum_{i=1}^{N} H_{i,t}$ respectively denote the value of the index based on non-synchronous and hedonic fitted prices for quarter t. Results C.R1 and C.R2 below show that

$$H_{N,t} = Z_{N,t}(1 + \delta_t) \tag{C.1}$$

where δ_t is mean zero and independent of $Z_{N,t}$ conditional on $\mathbf{X}$. The key assumption we need is that $E\left[\widehat{\boldsymbol{\theta}}\middle|\mathbf{X}\right] = \boldsymbol{\theta}$. Equation (C.1) implies that to an approximation, we can express $r_{H,t}$ as

$$\begin{aligned} r_{H,t} &\approx \log\left(1 + r_{H,t}\right) = \log\left(1 + r_{Z,t}\right) + [\log(1 + \delta_t) - \log(1 + \delta_{t-1})] \\ &\approx r_{Z,t} + \Delta\delta_t \end{aligned} \tag{C.2}$$

where we use the first-order approximation $\Delta \log(1 + \delta_t) = \Delta\delta_t$ and we ignore cash flows. Accounting for cash flows amounts to adding a small adjustment term.[27]

To account for non-synchronous trading, we follow Scholes and Williams (1977) and write the non-synchronous return approximately as

$$\begin{aligned} r_{Z,t} &\approx \log\left(1 + r_{Z,t}\right) = \log(1 + r_t) - [\log(1 + \tilde{r}_t) - \log(1 + \tilde{r}_{t-1})] \\ &\approx r_t - \Delta\tilde{r}_t \end{aligned} \tag{C.3}$$

where r_t represents the portfolio return based on actual end of quarter values as in equation (1) in section 3.1, and $\tilde{r}_t$ is the return from buying funds at their non-synchronous times during quarter t and selling at the end of the quarter. Here we again ignore cash flows in the approximation. Combining (C.2) and (C.3) gives

$$r_{H,t} \approx r_t + \Delta\delta_t - \Delta\tilde{r}_t. \tag{C.4}$$

The bias adjustments given by (7) follow from (C.4) under the following additional set of assumptions:

1) Fund returns, r_t, are independent and identically distributed across time.
2) Each fund is assigned a random transaction time relative to the end of the quarter, $t - s_i(t)$, where t defines the end of the quarter. Similar to the framework of Scholes and Williams (1977) we

[27]The adjustment term is given by $\phi = \log[(1 + \delta_t)Z_t + D_t - C_t] - \log[(1 + \delta_t)(Z_t + D_t - C_t)]$.

assume $s_i(t)$ is independent and identically distributed across time (though potentially correlated across funds), and $0 < s_i(t) < 1$. This implies that $\tilde{r}_t$ can be considered i.i.d. across quarters.

3) The parameter δ_t is identically distributed and independent across time.[28]

We now develop two results useful for the discussion above. These two results rely on estimating a panel regression with quarter fixed-effects and (quarter-fixed-effect)×NAV interaction terms. In our application we use $\ln(\text{NAV})$ instead of NAV. Using either $\ln(\text{NAV})$ or NAV has virtually no impact on our empirical results. In addition, we are missing transactions entirely for two quarters in our sample (see Figure 1), making it impossible to include quarterly fixed effects. In our application we instead estimate a panel regression that includes state variables (such as the market-to-book ratio of public securities) in $\mathbf{x_{i,t}}$ that are the same across all funds in a given quarter. These state variables capture the influence of quarter fixed effects. Moreover, in one of our specifications we include a fixed effect associated with the Financial Crisis, and Crisis fixed-effect interaction terms with all fund-level characteristics, including NAVs. This specification should approximate a panel regression with quarter fixed effects and quarter fixed-effect interaction terms if the relationship between prices and characteristics is stable outside of the financial crisis.

Result C.R1: If $\mathbf{X}$ contains appropriate controls, then $(\mathbf{X_{N,t}\theta})'\mathbf{NAV_{N,t}} = Z_{N,t}$ for every quarter t, where $\boldsymbol{\theta}$ represents the non-stochastic $k \times 1$ vector of population parameters, regardless of other variables included (or not included) in $\mathbf{X}$.

Proof:

$$(\mathbf{X_{N,t}\theta})'\mathbf{NAV_{N,t}} = \boldsymbol{\pi}_{\mathbf{N,t}}{}'\mathbf{NAV_{N,t}} - \mathbf{e}'_{\mathbf{N,t}}\mathbf{NAV_{N,t}}, \tag{C.10}$$

[28] Using notation from Result C.R2, $Cov(\delta_t, \delta_{t-1}|\mathbf{X}) = E\left[N\mathbf{AV}'_{\mathbf{N,t}}\mathbf{X_{N,t}}\mathbf{vv}'\mathbf{X}'_{\mathbf{N,t-1}}\mathbf{NAV_{N,t-1}}\middle|\mathbf{X}\right]$. This autocovariance can be zero if, for example, the panel data model includes time fixed effects and time-fixed-effect interaction terms along with appropriate restrictions on $E[\mathbf{vv}']$.

where $\boldsymbol{\pi}_{\mathbf{N,t}}$ is the $N \times 1$ vector of scaled prices for all funds in quarter t, and $\mathbf{X}_{\mathbf{N,t}}$, $\mathbf{NAV}_{\mathbf{N,t}}$, and $\mathbf{e}_{\mathbf{N,t}}$ respectively represent the set of characteristics, NAVs, and population residuals (as defined in equation (2)) for the population of funds in quarter t. If $\mathbf{X}$ contains fund NAVs, quarter fixed-effects, and quarter-fixed-effect ×NAV interaction terms the inner product between each element of $\mathbf{e}_{\mathbf{N}}$ and $\mathbf{NAV}_{\mathbf{N}}$ is equal to zero within each quarter, since these represent residuals and NAVs for the entire fixed population. Hence,

$$(\mathbf{X}_{\mathbf{N,t}}\boldsymbol{\theta})'\mathbf{NAV}_{\mathbf{N,t}} = \boldsymbol{\pi}_{\mathbf{N,t}}'\mathbf{NAV}_{\mathbf{N,t}} = Z_{N,t}. \tag{C.11}$$

Result C.R2: Assume $E[\widehat{\boldsymbol{\theta}}|\mathbf{X}] = \boldsymbol{\theta}$. If $\mathbf{X}$ contains appropriate controls, then $H_{N,t} = Z_{N,t}(1 + \delta_t)$ where $E[\delta_t|\mathbf{X}] = 0$ and $E[\delta_t Z_{N,t}|\boldsymbol{X}] = 0$.

Proof:

$$\begin{aligned} H_{N,t} &= (\mathbf{X}_{\mathbf{N,t}}\widehat{\boldsymbol{\theta}})'\mathbf{NAV}_{\mathbf{N,t}} \\ &= [\mathbf{X}_{\mathbf{N,t}}(\boldsymbol{\theta} + \mathbf{v})]'\mathbf{NAV}_{\mathbf{N,t}}, \end{aligned} \tag{C.12}$$

where $\mathbf{X}_{\mathbf{N,t}}$ and $\mathbf{NAV}_{\mathbf{N,t}}$ respectively represent the set of characteristic and NAVs for the population of funds in quarter t, and $\mathbf{v}$ denotes the $k \times 1$ error vector that differentiates $\widehat{\boldsymbol{\theta}}$ from $\boldsymbol{\theta}$,

$$\mathbf{v} = \widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}. \tag{C.15}$$

Randomness in $\mathbf{v}$ is driven by the randomness in the set of funds selected for transaction. If $\mathbf{X}$ contains fund NAVs, quarter fixed-effects, and quarter-fixed-effect × NAV interaction terms, then $(\mathbf{X}_{\mathbf{N,t}}\boldsymbol{\theta})'\mathbf{NAV}_{\mathbf{N,t}} = Z_{N,t}$ as shown by Result 1. We can then write

$$\begin{aligned} \boldsymbol{H}_{N,t} &= Z_{N,t} + (\mathbf{X}_{\mathbf{N,t}}\mathbf{v})'\mathbf{NAV}_{\mathbf{N,t}} \\ &= Z_{N,t} + w_t \end{aligned} \tag{C.13}$$

where $w_t = (\mathbf{X}_{\mathbf{N,t}}\mathbf{v})'\mathbf{NAV}_{\mathbf{N,t}}$. If $E[\widehat{\boldsymbol{\theta}}|\mathbf{X}] = \boldsymbol{\theta}$ then $E[\mathbf{v}|\mathbf{X}] = \mathbf{0}$, and if $\mathbf{X}$ contains fund NAVs then $E[w_t|\mathbf{X}] = \mathbf{0}$ implying that $H_{N,t}$ is an unbiased estimator of $Z_{N,t}$. In addition $E[w_t Z_{N,t}|\mathbf{X}] = 0$ since $Z_{N,t}$ is merely a function of $\mathbf{X}_{\mathbf{N,t}}$ and the (non-stochastic) population parameters, $\boldsymbol{\theta}$, as shown by Result 1. Population residuals vanish in calculating $Z_{N,t}$. We can also write $H_{N,t}$ as

$$H_{N,t} = Z_{N,t}(1 + \delta_t) \tag{C.14}$$

where $\delta_t = w_t / Z_{N,t}$. Since w_t is mean zero and independent of $Z_{N,t}$ conditional on $\mathbf{X}$, it follows that $E[\delta_t | \mathbf{X}] = E\left[\delta_t Z_{N,t} \middle| \mathbf{X}\right] = 0$. That is, δ_t is mean zero and independent of $Z_{N,t}$ conditional on $\mathbf{X}$.

Appendix D. Dimson Adjusted Volatility

In this Appendix we present a method to bias-adjust volatility when observed returns can be characterized by the model of Dimson (1979), who presumes that securities trade intermittently at the ends of specified periods. Similar to Dimson (1979), assume that (NAV-based) index returns may be written as

$$r_{NAV,t} = \sum_{j=0}^{l} \lambda_j r_{t-j} + u_t \tag{D.1}$$

where l characterizes the nature of ``price smoothing'', r_t represents the portfolio return based on end of quarter values, λ_i represents the probability that trading occurs at the end of quarter t, and u_t is a mean zero i.i.d. error term. We further assume that r_t is i.i.d. Note that since $\sum_{j=1}^{l} \lambda_j = 1$, $E\left[r_{NAV,t}\right] = r_t$. Given that $r_t = a + \beta r_{mt} + \epsilon_t$ where β denotes the true beta, we can then write

$$r_{NAV,t} = c + \sum_{j=0}^{l} \psi_i r_{m,t-j} + \varepsilon_t \tag{D.2}$$

where c is a constant, $\psi_i = \lambda_i \beta$ and $\varepsilon_t = \sum_{j=0}^{l} \epsilon_{t-j} + u_t$ is i.i.d. and mean zero.

From (D.1) it follows that

$$Cov\left(r_{NAV,t}, r_{NAV,t-1}\right) = \sum_{j=1}^{l} \lambda_j \lambda_{j-1} Var(r_t) \tag{D.3}$$

or rather,

$$Var(r_t) = \frac{Cov\left(r_{NAV,t}, r_{NAV,t-1}\right)}{\sum_{j=1}^{l} \lambda_j \lambda_{j-1}}. \tag{D.4}$$

We first estimate $\psi_0, \dots, \psi_m$ from (D.2) by regressing observed NAV returns on lagged market returns. Since by assumption $\sum_{j=0}^{l} \psi_j = \beta$ and $\psi_j = \lambda_j \beta$ we estimate λ_j as $\lambda_j = \psi_j / \sum_{j=0}^{l} \psi_j$. We then compute the variance by scaling the auto-covariance as in (D.4).

Figure 1. Transactions per Quarter

This figure illustrates the number of transactions we observe per quarter in the sample of transactions merged with Preqin. In total, this sample contains 955 buyout transactions.

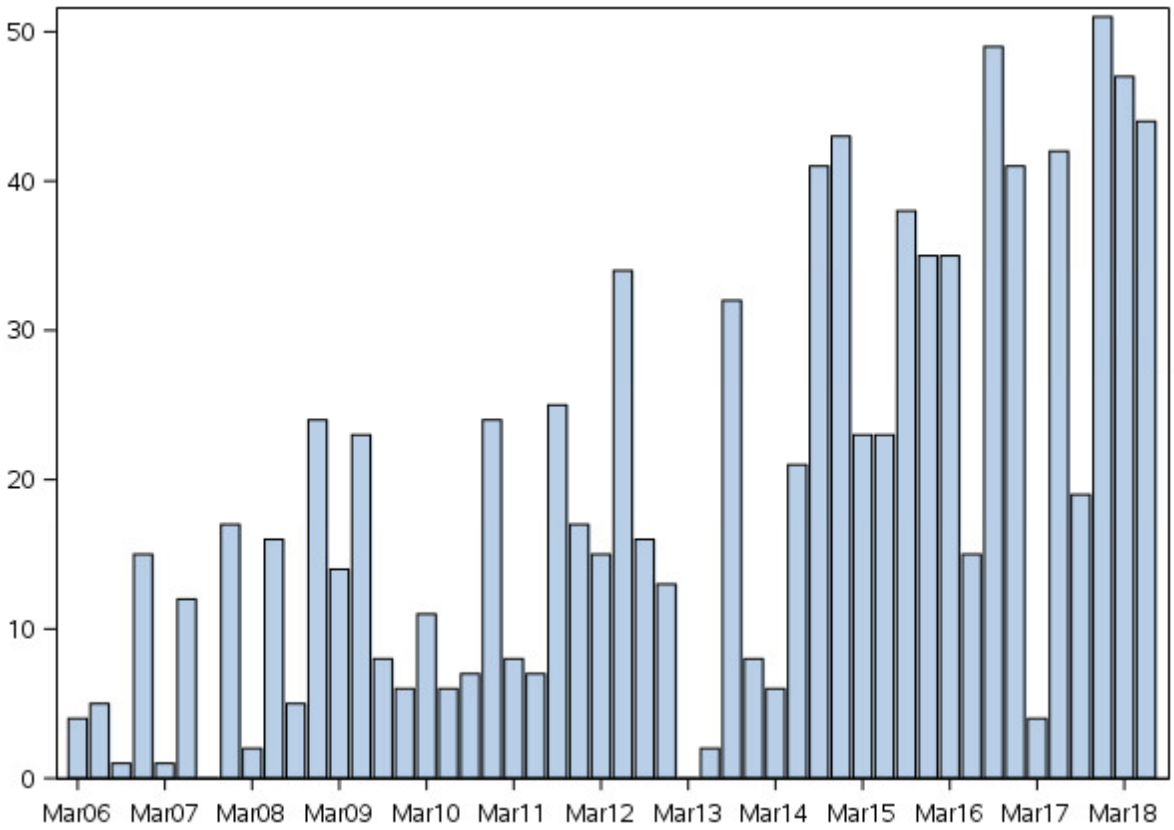

Figure 2. Hedonic Transactions-Based Buyout Index Over Time

This figure illustrates the value of investing $1 in an index at the beginning of 2006 in each buyout index as labeled. The market index is based on the public market return as posted on Ken French's website. We build the *Preqin* indices based on NAVs reported in Preqin using funds that are four to nine years old. The transactions indices are the size-weighted hedonic indices. The *Burgiss* and *Cambridge* indexes are NAV-based buyout indexes. The *S&P Listed Private Equity Index* is an index comprised of publicly traded private equity funds. The chart uses a log scale for the vertical axis.

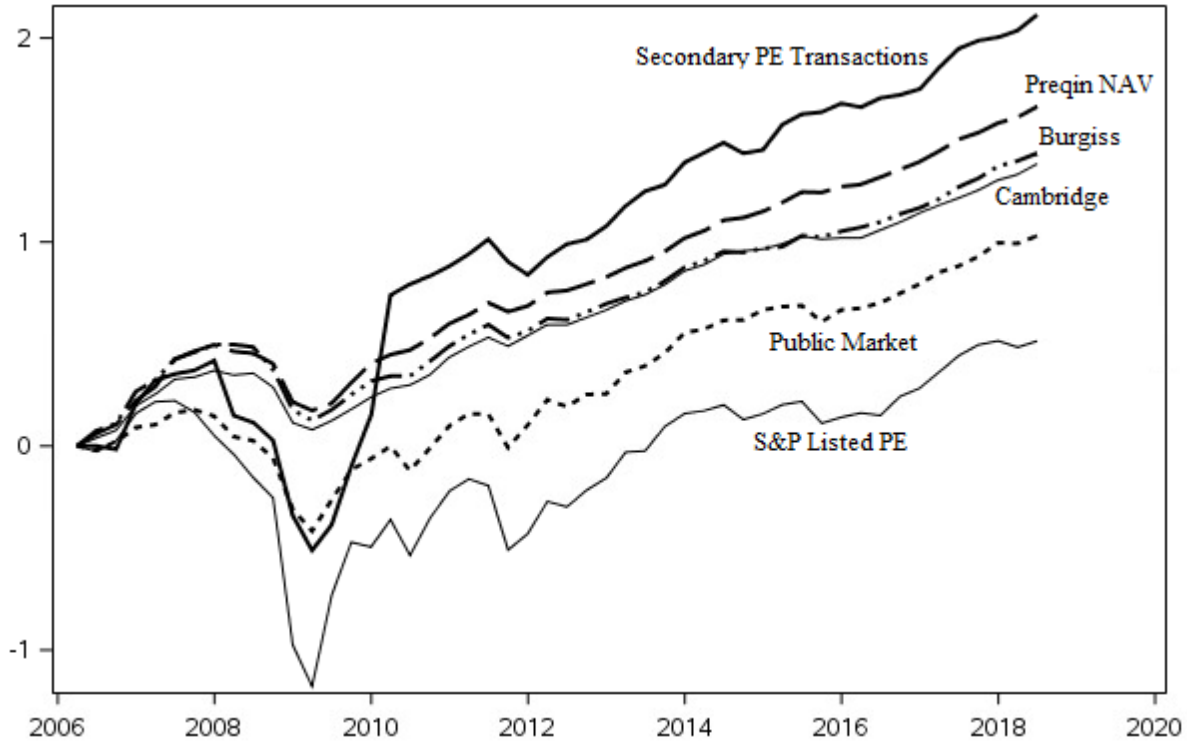

Table 1. Summary Statistics

This table reports summary statistics for the data samples we use. The first two columns report statistics for the sample of transactions merged with *Preqin*. This is the data we use to estimate the hedonic model. The last column reports statistics for funds that are four to nine years old. This is the sample used to construct the transaction-based indices based on estimated parameters of the hedonic model. In the table, $\pi_{i,t}$ is the fund price as a fraction of NAV. $Size(\$MM)$ represents total commitments in $US million. Age is fund age in years. $Trans\ per\ Qtr$ is the number of transactions per quarter. PME is the Kaplan Schoar (2005) PME using NAV as terminal value for funds that have not yet liquidated. N is the total number of observations in the sample. "Mean" is the average across funds and across time, "Q1" is the 25th percentile, and "Q3" is the 75th percentile.

		All Fund/Quarters		*All Fund/Quarters*
		Transactions	*No Transaction*	*Funds 4-9 Yrs Old*
$\pi_{i,t}$	Mean	0.86		0.92
	Q1	0.74		0.81
	Q3	1.00		1.05
Size ($MM)	Mean	4,802	2,656	2,874
	Q1	1,273	806	830
	Q3	5,800	3,333	3,500
Age (years)	Mean	8.8	6.3	6.4
	Q1	6.0	3.0	5.0
	Q3	11.0	9.0	8.0
PME	Mean	1.15	1.12	1.14
	Q1	0.92	0.89	0.92
	Q3	1.34	1.29	1.29
Funds per Qtr	Mean	19.1	313.6	147.7
	Q1	6	255	100
	Q3	32	377	196
N		955	15,678	7,384

Table 2. Explanatory Variable Descriptions

This table describes the explanatory variables used in our hedonic models. The first seven variables are "state variables" that are the same across all funds and vary only across time. The last six variables are "fund specific variables" that vary across funds and also (potentially) across time.

State Variables	MTB	The average end-of-month market-to-book ratio over the transaction quarter, calculated using all stocks with share code 10 or 11 in CRSP.
	Volatility	The annualized standard deviation of the value-weighted portfolio of all stocks in CRSP with share code 10 or 11 using daily data over the transaction quarter.
	Value Confidence Index	The average end-of-month value of the Valuation Confidence Index over the transaction quarter from the International Center for Finance at Yale. Institutional Investors are asked to report their assement of stock market value relative to fundamental value. The Valuation Confindence Index is one minus the percentage of respondants who think that the market is overvalued. We scale this variable by 100.
	Crash Confidence Index	The average end-of-month value of the Crash Confidence Index over the transaction quarter from the International Center for Finance at Yale. Institutional Investors are asked to report the probability of a catostrophic market crash in the next six months. The Crash Confidence Index is the percentage of respondants who think that the probability is less than 10%. We scale this variable by 100.
	Average Market NAV	The average log NAV reported at the end of the transaction quarter across funds in our Preqin Universe.
	Average Market PME	The average Kaplan Schoar (2005) PME at the end of the transaction quarter across funds.
	Crisis Fixed Effect	Dummy that equals 1 if the transaction occurred in 2008 or 2009.
Fund SpecificVriables	Log Size	The log size of the transacting fund (total commitments of all limited partners).
	Log NAV	The log NAV for the transacting fund reported as of the end of the transaction quarter.
	PME	The Kaplan Schoar (2005) PME of the transacting fund as of the end of the transaction quarter, calculated using data from Preqin.
	Age Fixed Effects	Two age dummies: the first equals 1 if the age of the fund is less than four years old, the second equals 1 if the age of the fund is greater than 9 years old. We define age as the year of the transaction quarter minus the vintage year for the transacting fund.
	Crisis Interaction Effects	Five explanatory variables given by the product of the crisis dummy and five fund-specific explanatory variables (Log Size, Log NAV, PME, and the two age fixed effects).
	Pension	The fraction of the transacting fund's limited partners that are pension funds. We use this variable as an exclusion restriction in the Heckman model.

Table 3. Sample Selection Model Parameters

This table reports the estimates of the Heckman (1979) sample selection model. Panel A reports estimates of parameters for the selection equation while Panel B reports estimates of $\boldsymbol{\theta}$ for the pricing model. "Heckman" refers to the sample selection model, while "OLS" indicates the pricing model is estimated by simple OLS with no selection equation. Variables are described in Table 2. We estimate the standard errors of model parameters using the quasi-maximum-likelihood approach of White (1982). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A. Selection Equation						
		Heckman				
		Model 1			Model 2	
		estimate	(t-stat)		estimate	(t-stat)	
	Intercept	7.98	(2.2)	**	15.45	(3.4)	***
State Variables	MTB	0.63	(6.9)	***	0.59	(6.3)	***
	Volatility	0.33	(1.0)		0.19	(0.6)	
	Value Confidence Indx	0.76	(2.9)	***	1.16	(4.2)	***
	Crash Confidence Indx	-2.37	-(5.5)	***	-2.34	-(5.5)	***
	Average Market NAV	-0.90	-(3.6)	***	-1.44	-(4.7)	***
	Average Market PME	0.70	(1.2)		1.28	(2.2)	**
Fund Specific	Log Size	0.29	(14.2)	***	0.28	(13.2)	***
	Log NAV	0.02	(1.1)		0.00	(0.1)	
	PME	-0.18	-(3.9)	***	-0.23	-(4.4)	***
	Age < 4	-0.56	-(10.9)	***	-0.66	-(10.7)	***
	Age > 9	0.39	(8.0)	***	0.39	(7.6)	***
	Pension	-0.48	-(5.6)	***	-0.43	-(4.9)	***
	ρ	-0.14	-(0.8)		-0.18	-(0.8)	
	Crisis Fixed Effect	No			Yes		
	Crisis Interaction Effects	No			Yes		
	Ho: ρ=0	$\chi^2(1)$	p-value		$\chi^2(1)$	p-value	
	Wald	0.59	(0.44)		0.61	(0.43)	
	Likelihood Ratio	0.25	(0.62)		0.32	(0.57)	
	Lagrange Multiplier	0.10	(0.75)		0.13	(0.71)	

Table 3. *Continued*

	Panel B. Pricing Equation											
	Model 1						Model 2					
	Heckman			OLS			Heckman			OLS		
	estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)	
Intercept	-13.99	-(6.8)	***	-12.96	-(6.4)	***	-4.81	-(2.0)	*	-3.63	-(1.6)	
State Variables												
MTB	0.04	(0.8)		0.04	(0.8)		0.01	(0.2)		0.01	(0.2)	
Volatility	0.39	(2.9)	***	0.43	(3.3)	***	0.21	(1.5)		0.25	(1.9)	*
Value Confidence Indx	-0.87	-(5.4)	***	-0.90	-(5.8)	***	-0.39	-(2.2)	**	-0.41	-(2.6)	**
Crash Confidence Indx	0.72	(3.4)	***	0.81	(4.0)	***	0.82	(3.9)	***	0.88	(4.6)	***
Average Market NAV	1.05	(7.3)	***	0.98	(7.1)	***	0.38	(2.1)	**	0.29	(1.8)	*
Average Market PME	-0.83	-(2.9)	***	-0.87	-(3.1)	***	-0.26	-(0.9)		-0.24	-(0.8)	
Fund Specific												
Log Size	0.00	(0.2)		0.01	(1.4)		0.01	(0.7)		0.02	(2.7)	***
Log NAV	0.02	(2.6)	**	0.02	(2.6)	**	0.02	(2.2)	**	0.02	(2.2)	**
PME	0.08	(3.3)	***	0.07	(3.2)	***	0.06	(2.4)	**	0.05	(2.3)	**
Age < 4	-0.04	-(1.2)		-0.06	-(2.3)	**	0.03	(0.7)		0.01	(0.4)	
Age > 9	-0.09	-(3.4)	***	-0.09	-(4.1)	***	-0.09	-(3.1)	***	-0.08	-(3.8)	
Crisis Fixed Effect	No			No			Yes			Yes		
Crisis Interaction Effects	No			No			Yes			Yes		
R-square	25%			25%			30%			30%		

Table 4. Predicting Index Returns Using Lagged Market Returns

This table reports estimates from regressions of the hedonic transactions-based index returns on contemporaneous and lagged market returns, as specified in equation 8 of the paper. The Hedonic index returns are generated using Model 2 of Table 3 and are size-weighted. We build the *Preqin* indices based on NAVs reported in *Preqin* using funds in the *Preqin* universe that are four to nine years old. The *Burgiss* and *Cambridge* indexes are NAV-based buyout indexes. The *S&P Listed Private Equity Index* is an index comprised of publicly traded private equity funds.

	Hedonic Buyout			Preqin			Burgiss			Cambridge			S&P Listed		
a	0.01	(0.24)		0.02	(5.04)	***	0.01	(4.26)	***	0.01	(2.30)	**	-0.01	-(0.81)	
b_0	0.93	(4.01)	***	0.46	(7.92)	***	0.44	(11.03)	***	0.50	(7.51)	***	1.70	(8.40)	***
b_1	0.47	(1.88)	*	0.18	(6.90)	***	0.11	(6.59)	***	0.14	(7.51)	***	0.02	(0.26)	
b_2	0.26	(0.63)		0.09	(3.58)	***	0.08	(2.34)	**	0.08	(2.78)	***	-0.32	-(2.13)	**
b_3	0.28	(0.45)		0.09	(3.38)	***	0.12	(4.68)	***	0.12	(5.16)	***	0.02	(0.24)	
b_4	-0.42	-(1.58)		-0.03	-(1.42)		0.02	(0.72)		0.01	(0.20)		-0.05	-(0.83)	

Table 5. Index Parameters

This table reports index parameters for buyout indices using data from 2006-2018. Panel A is for our transactions-based indices, Panel B is for NAV-based indices. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. *Dimson(x)* implies a Dimson adjustment using *x* lags. We estimate standard errors for index parameters using the subsampling methods of Politis and Romano (1994). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A: Transactions-Based Indices												
	Model 1						Model 2					
	Dimson(1)			Scholes-Williams			Dimson(1)			Scholes-Williams		
	estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)	
E[r]	0.19	(3.3)	***	0.19	(3.3)	***	0.17	(2.1)	**	0.17	(2.1)	**
β	1.77	(4.2)	***	2.14	(3.5)	***	1.42	(3.2)	***	1.72	(2.7)	***
α	0.02	(0.6)		-0.01	-(0.2)		0.04	(0.8)		0.01	(0.2)	
σ	0.39	(2.3)	**	0.38	(2.4)	**	0.34	(1.6)		0.35	(1.1)	
Sharpe	0.46	(0.8)		0.47	(1.3)		0.47	(0.9)		0.46	(0.7)	
Corr Mkt	0.71	(2.0)	**	0.87	(13.9)	***	0.64	(1.8)	*	0.75	(3.2)	***
Autocorr	0.43	(2.9)	***	0.43	(2.9)	***	0.41	(3.2)	***	0.41	(3.2)	***

Panel B: Other Indices												
	Preqin			Burgiss			Cambridge			S&P Listed		
	Dimson(3)			Dimson(3)			Dimson(3)			No Adjustment		
	estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)	
E[r]	0.14	(7.4)	***	0.12	(5.8)	***	0.12	(6.1)	***	0.09	(1.8)	*
β	0.81	(7.7)	***	0.85	(8.2)	***	0.76	(13.5)	***	1.74	(8.1)	***
α	0.06	(4.3)	***	0.04	(2.5)	**	0.04	(3.9)	***	-0.07	-(4.4)	***
σ	0.17	(2.4)	**	0.20	(2.2)	**	0.17	(2.5)	**	0.30	(3.3)	***
Sharpe	0.80	(0.3)		0.58	(0.3)		0.64	(0.5)		0.26	(0.8)	
Corr Mkt	0.76	(1.2)		0.66	(0.6)		0.69	(1.0)		0.89	(17.5)	***
Autocorr	0.48	(2.5)	**	0.46	(2.3)	**	0.46	(2.9)	***	0.22	(2.4)	**

Panel C: Differences Between Transaction-Based Indices and Preqin NAV-Based Index												
	Dimson(1)			Scholes-Williams			Dimson(1)			Scholes-Williams		
	estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)	
E[r]	0.04	(0.8)		0.04	(0.8)		0.03	(0.3)		0.03	(0.3)	
β	0.95	(2.5)	**	1.32	(2.4)	**	0.60	(1.5)		0.90	(1.6)	
α	-0.04	-(1.1)		-0.07	-(2.3)	**	-0.03	-(0.5)		-0.05	-(1.2)	
σ	0.22	(5.3)	***	0.22	(6.4)	***	0.18	(1.5)		0.19	(0.7)	

Panel D: Differences Between Transaction-Based Indices and S&P Listed Index												
	Dimson(1)			Scholes-Williams			Dimson(1)			Scholes-Williams		
	estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)	
E[r]	0.10	(2.0)	*	0.10	(2.0)	*	0.08	(1.0)		0.08	(1.0)	
α	0.10	(3.1)	***	0.06	(2.3)	**	0.11	(2.2)	**	0.08	(1.9)	*

Table 6. Index Parameters Excluding the Financial Crisis

This table reports index parameters for buyout indices using data from 2010-2018. Panel A is for our transactions-based indices, Panel B is for *Preqin* NAV-based indices. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the baseline merged samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation. *Dimson(x)* implies a Dimson adjustment using *x* lags. We estimate standard errors for index parameters using the subsampling methods of Politis and Romano (1994). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A: Transactions-Based Indices

	Model 1					
	Dimson(1)			Scholes-Williams		
	estimate	(t-stat)		estimate	(t-stat)	
E[r]	0.20	(2.7)	***	0.20	(2.7)	***
β	1.16	(4.1)	***	0.97	(4.4)	***
α	0.04	(0.7)		0.06	(1.1)	

Panel B: Prequin NAV Indices

	Preqin NAV			Burgiss			Cambridge			Listed		
	Dimson(3)			Dimson(3)			Dimson(3)			No Adjustment		
	estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)		estimate	(t-stat)	
E[r]	0.15	(13.0)	***	0.14	(8.1)	***	0.14	(7.7)	***	0.14	(2.9)	***
β	0.54	(3.7)	***	0.57	(2.8)	***	0.59	(4.2)	***	1.35	(10.4)	***
α	0.08	(2.4)	**	0.06	(1.4)		0.06	(1.6)	*	-0.05	-(1.5)	

Table 7. Factor Loadings

Panel A of this table reports regression estimates of transactions-based index returns on a set of asset pricing factors. Panel B reports the same results for NAV-based indices, and the *S&P Listed Private Equity Index* S&P. The sample period runs from 2006-2018. For Panel A we use OLS to estimate the hedonic model. Returns representing the SMB and HML factors are taken from Ken French's website. LIQ is the Pastor Stambaugh (2003) trading liquidity factor. FLIQ is a funding liquidity factor constructed in Chen and Lu (2018), and returns for this factor are available on Zhuo Chen's website. We estimate standard errors using the subsampling methods of Politis and Romano (1994). Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A. Transaction-Based Indices

	Model 1				Model 2			
	Dimson(1)		Scholes-Williams		Dimson(1)		Scholes-Williams	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
MKT	1.24 (4.4) ***	1.45 (5.0) ***	1.42 (4.2) ***	1.40 (4.0) ***	1.05 (1.8) *	1.17 (4.0) ***	1.18 (3.1) ***	1.19 (3.7) ***
SMB	-0.11 -(0.1)		-0.65 -(0.5)		0.14 (0.2)		-0.23 -(0.2)	
HML	0.36 (0.7)		0.40 (0.7)		0.16 (0.2)		0.13 (0.3)	
LIQ	0.32 (1.1)		0.35 (0.9)		0.02 (0.1)		0.06 (0.2)	
FLIQ	0.49 (1.5)	0.40 (1.6)	0.91 (2.1) **	0.84 (2.2) **	0.43 (1.3)	0.39 (1.1)	0.77 (1.8) *	0.76 (1.9) *

Panel B. Other Indices

	Preqin Nav		Burgiss		Cambridge		Listed	
	Dimson(3)		Dimson(3)		Dimson(3)		No Adjustment	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
MKT	0.71 (0.0)	0.75 (3.4) ***	0.92 (0.0)	0.81 (3.0) ***	0.81 (0.0)	0.75 (5.1) ***	1.19 (11.8) ***	1.33 (20.2) ***
SMB	-0.19 -(0.1)		-0.17 (0.0)		0.08 (0.0)		0.58 (3.7) ***	
HML	0.41 (0.0)		0.31 (0.0)		0.21 (0.0)		0.22 (1.4)	
LIQ	0.40 (0.0)		0.47 (0.0)		0.29 (0.0)		-0.2 -(1.7) *	
FLIQ	0.02 (0.0)	0.06 (0.4)	-0.08 (0.0)	0.04 (0.3)	-0.09 (0.0)	0.01 (0.1)	0.49 (2.1) **	0.5 (2.2) **

Table 8. Market-to-Book Ratios of Private Equity Investments

This table reports year-end average market-to-book ratios. Market values for each fund are calculated using the following procedure. We begin by assuming that the market value of the fund is equal to NAV in years one through four of the fund's life. We then calculate the market value each quarter from years 5-9 for fund *i* using the following formula:

$$market\ value_{i,t} = market\ value_{i,t-1} * (1 + r_t) + Calls_t - Distributions_t\ .$$

For the first quarter in year five, we use NAV as the preceding quarter's market value. The aggregate market-to-book ratio reported in this table is calculated as the sum of the individual fund's market value within each quarter divided by the sum of the individual fund's NAV in each quarter. We report the resultant market-to-book ratio for Q4 of each year, with the exception of 2018, where we report values as of Q2 due to data limitations. Panel A reports results for buyout funds. Panel B reports results for venture funds.

	Vintage Year												
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
2006	1.05	--	--	--	--	--	--	--	--	--	--	--	--
2007	0.95	0.98	--	--	--	--	--	--	--	--	--	--	--
2008	0.75	0.67	0.79	--	--	--	--	--	--	--	--	--	--
2009	0.77	0.86	0.93	0.75	--	--	--	--	--	--	--	--	--
2010	1.01	1.08	1.24	0.91	0.99	--	--	--	--	--	--	--	--
2011	0.87	0.92	1.11	1.22	0.87	0.91	--	--	--	--	--	--	--
2012	--	1.05	1.30	1.09	0.94	1.03	1.03	--	--	--	--	--	--
2013	--	--	1.59	1.20	1.02	1.13	1.08	1.05	--	--	--	--	--
2014	--	--	--	1.41	0.91	1.03	0.96	0.93	0.97	--	--	--	--
2015	--	--	--	--	0.95	1.14	1.04	1.01	1.04	1.01	--	--	--
2016	--	--	--	--	--	1.05	0.98	0.92	0.96	0.96	0.97	--	--
2017	--	--	--	--	--	--	1.01	0.89	0.94	0.98	0.99	0.97	--
2018	--	--	--	--	--	--	--	0.88	0.94	1.01	1.01	1.02	1.03
Average	0.90	0.93	1.16	1.09	0.95	1.05	1.02	0.95	0.97	0.99	0.99	0.99	1.03