Energy Management for Timely Charging a System of Drones

Jiashang Liu, Wenxin Li, Ness B. Shroff and Prasun Sinha

***Abstract*— In recent years, there has been a growing interest in the use of drone for commercial applications. To support these drone systems, autonomous drone charging has received lots of attention. However, control algorithms on how to assign drones to different charging stations have not been well studied. In this work, we consider a drone charging network with charging stations equipped with renewable resources and batteries, and focus on the problem of designing a joint drone-charging-station association and energy control algorithm, with the aim of minimizing the electricity cost of the charging network system. We show that our algorithm achieves asymptotic (as the battery capacity increases) optimality. We then show via simulations that a large cost reduction is achieved by our proposed algorithm under a reasonable battery size.**

I. Introduction

Recently, Unmanned Aerial Vehicles (UAVs), also known as drones, have been widely adopted for commercial, recreational and public use. There is a growing interest in this technology from the industry, especially from oil and gas companies, and utility providers that rely on extensive surveillance, measurements. Moreover, major delivery companies and service providers (e.g. Amazon, Google) are looking into deploying drones for routine delivery operation. Drones are also used for remote monitoring purposes, such as environmental surveying, searching and civil structures health check-up [1]. It is reported that total spending on drones worldwide will be over $100 billion by 2020 [2].

Despite the popularity of drones in commercial applications, the main barrier to wide-spread of drone systems is the short flight time. To address this problem, one option is to equip drones with Energy Harvesting (EH) devices, which increase the lifetime but are not reliable [3]. A more popular option is to deploy autonomous charging stations so that drones can get charged intermittently [1] [4]. Furthermore, equipping drone charging stations with EH devices is considered as an attractive way to keep costs down so that one can store and use renewable energy, but also have the option of drawing from the power grid to ensure stability [5].

Few works have focused on the problem of designing drone-charging-station navigation paths. In both [6] and [7], the authors develop a virtual metric called *Congestion Contribution* and consider the congestion that may occur at the battery charging stations when making drone navigation decisions. However, in their models, the charging station is powered only by the power grid, and electricity price is not considered in their solutions. In commercial areas, the shortest distance from the source to destination is considered in Amazon drones [8], while Google drones only consider the furthest distance they can deliver without battery charging [9]. None of the above works have considered the scenario where charging stations are equipped with EH devices, or

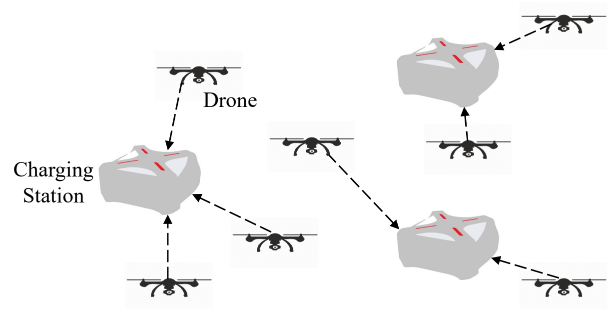

Fig. 1: An example of a drone charging network with one association decision.

when electricity prices are time-varying. To the end, a natural problem that arises is: *to minimize the cost of the whole system, how to design the control algorithm for the drone-charging-station association, and the usage of batteries with renewable resources, when the drone positions, required charging time, deadline as well as the electricity prices are taken into account.*

In this work, we consider a drone charging network, where each charging station in the network is equipped with a battery and can be charged with renewable resources (e.g. wind, solar, etc.) and the power grid with time-varying electricity prices. A drone sends out a charging request when its battery level is below a threshold, and we focus on the problem of deciding which charging station is allocated to the request, when to charge the drone, and how much energy should be purchased from the power grid, by considering drone's position, required charging time, deadline, the batteries state and the electricity price. The objective is to reduce the monetary cost of the network, under the constraint that each request should be completed within its deadline.

The proposed algorithm consists of a drone-charging-station association control component and an energy control component. The association control component tries to navigate the request to the charging station and assign the time-slots with minimum total cost needed to finish that request, by considering both the battery status of the charging station and future electricity prices. Once the association decision is made, the energy control component determines how much energy should be purchased from the power grid of each charging station. Our proposed algorithm is shown

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to achieve asymptotic (as the battery capacity increases) optimality. Finally, we show how to solve the drone-station association problem by reducing it into a minimum weighted r-dimensional matching problem.

The organization of this paper is as follows. We discuss the battery, tasks and energy models in Section II, and formulate the electricity cost minimization problem in Section III. We propose a joint control algorithm and analyze its performance in Section IV. In Section V we conduct and present detailed simulations. Finally, we conclude our paper in Section VI.

II. System Model

We consider a drone charging network system with J charging stations equipped with batteries. Each charging station can provide the same charging capability for all the drones. Time is assumed to be slotted. At each time-slot, those drones that need to be charged will send out a charging request. We focus on the problem of drone-charging-station association, with the aim of minimizing the overall system cost. A graphic illustration of the system is given in Figure 1.

A. Battery Model

Each charging station has a rechargeable battery with maximum battery capacity $B_{\max}$, as depicted in Figure 2. We denote $B_j(t)$ as the battery level of the charging station j at the beginning of time-slot t, with the initial condition

$$B_j(0) = B_{\max}. \tag{1}$$

For charging station j and time-slot t, let $\lambda_j(t)$ denote the amount of harvested energy, and $b_j(t)$ denote the amount of energy drawn from the battery. Note that $b_j(t)$ can either be positive or negative. A positive value of $b_j(t)$ represents the discharge status of the battery, while negative value implies that the battery is charged. Based on the definition above, the battery level evolution of station j can be expressed as

$$B_j(t+1) = \min\{B_j(t) - b_j(t) + \lambda_j(t), B_{\max}\}. \tag{2}$$

For charging station j at every time slot t, we have:

$$b_j(t) \le B_j(t), \tag{3}$$

which indicates that the amount of energy obtained from the battery is no more than the current battery level. In addition, according to the physical limitation of a battery, we assume that the amount of energy charged to the battery in each time-slot is no more than a constant upper bound $b_{\max}$, i.e.,

$$b_j(t) \ge -b_{\max} \tag{4}$$

holds for any charging station j and time-slot t.

B. Tasks Model

Let $N(t)$ represent the set of drone charging requests that arrive at the beginning of time-slot t, with the number of requests being n_t, i.e., $n_t = |N(t)|$, where $|\cdot|$ denotes the cardinality of a set. For each request $i \in N(t)$, let c_t^i denote the required charging time, which is equal to the number of time-slots needed to complete request i. Also, there is a deadline d_t^i associated with each request $i \in N(t)$, which

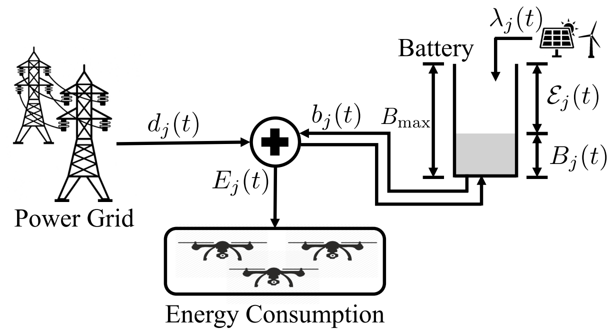

Fig. 2: Model of charging station j.

is the maximum number of time-slots allowed to finish the request after its arrival time t, and let $d_{\max} \triangleq \max_{i,t} d_t^i$. For the request with larger value of d_t^i, it is more delay-tolerant, and we can opportunistically schedule it in order to exploit the fluctuating nature of the electricity price and make use of the renewable energy.

One of the decisions is the request-charging-station allocation state in each time-slot t, which is denoted by a binary matrix $\mathbf{A} \in \{0,1\}^{n_t \times J}$, in which $A_{ij} = 1$ when request i is allocated to charging station j and $A_{ij} = 0$ otherwise. Each charging request can only be assigned to one charging station, which implies the following constraint:

$$\sum_{j=1}^{J} A_{ij} = 1, \text{ for all } i \in N(t). \tag{5}$$

We assume that the drone with a charging request can fly to the assigned charging station with zero extra time and is able to be served immediately in that time-slot. This is because compared to the required charging time and deadline, the time needed to fly to the assigned charging station is small. However, the energy spent during the flight cannot be ignored [10]. To model different amounts of energy spent flying to different charging stations, we introduce an extra charging time l_{ij} if request i is assigned to charging station j. With larger distance between the current drone position and a charging station j, l_{ij} is larger. In such cases, the total required charging time of request $i \in N(t)$ becomes $c_t^i + \sum_{j=1}^{J} A_{ij} l_{ij}$.

Another decision being made for each request is the selection of time-slots to be assigned to the request within its deadline, which is equal to the required charging time for that request. Let binary variables $x_k^i \in \{0,1\}$ denote the selection of time-slot $t+k$ for request i, with $x_k^i = 1$ when time-slot $t+k$ is selected to serve request i, and $x_k^i = 0$ otherwise. We assume that one charging task can be interrupted by another charging task, and those time-slots assigned to one charging request are not necessarily consecutive. Based on the definition of x_k^i, we have the following constraint:

$$\sum_{k=0}^{d_t^i} x_k^i = c_t^i + \sum_{j=1}^{J} A_{ij} l_{ij}. \tag{6}$$

Note that these two decisions are assumed to be made once request $i \in N(t)$ arrives, and they are fixed thereafter.

In the rest of the paper, we call these two decisions together as the *association control decision*.

C. Energy and Price Model

We assume that each charging request needs to consume one unit of energy per time-slot. Let $E_j(t)$ denote the amount of energy consumption of charging station j in time-slot t. Recall that for each charging station j in each time-slot τ, A_{ij} represents whether request $i \in N(\tau)$ is assigned to charging station j, and x_k^i represents whether time-slot $\tau + k$ is assigned to serve request i. Then we must have the following equation:

$$E_j(t) = \sum_{\tau=t-d_{\max}}^{t} \sum_{i=1}^{n_\tau} A_{ij} x_{t-\tau}^i. \tag{7}$$

According to the power consumption constraint of the charging station, at any time-slot, the amount of energy consumption of any charging station is assumed to be upper bounded by a constant $E_{\max}$, i.e.,

$$E_j(t) \leq E_{\max} \tag{8}$$

holds for any charging station j in any time-slot t. For simplicity, $E_{\max}$ is assumed to be an integer.

Let $d_j(t)$ denote the amount of energy purchased from the power grid of charging station j in time-slot t. We claim that $E_j(t) = b_j(t) + d_j(t)$, since $b_j(t)$ represents the amount of energy drawn from the battery, and $E_j(t)$ is the amount of energy consumption of charging station j. In our model, the energy is not sold to the grid, i.e., $d_j(t) \geq 0$, equivalently,

$$b_j(t) \leq E_j(t). \tag{9}$$

For a given value of $E_j(t)$, the value of $b_j(t)$ or $d_j(t)$ can be determined using the information of the other parameter. As consequence, in the control algorithm, it suffices to figure out only one parameter among $b_j(t)$ and $d_j(t)$. We refer to $b_j(t)$ as the *energy control decision* in the rest of this paper.

The price of electricity is assumed to be time-varying according to the specific pricing strategy of energy providers. We assume that the price at time t is $P(t)$ per unit, and $P(\tau)(t \leq \tau \leq d_{\max})$ are known at the beginning of time-slot t. This assumption is valid since we can have a precise prediction from the history information of electricity price [11]. Therefore, the cost of purchasing energy from the power grid for charging station j in time-slot t is:

$$P(t)d_j(t) = P(t)E_j(t) - P(t)b_j(t). \tag{10}$$

In this paper, all the processes in the system are assumed to be ergodic and have bounded values. We let $P_{\max}$ and $\lambda_{\max}$ denote the upper bound of energy price and renewable energy in one time-slot.

III. Problem Formulation

From Eqs (7) and (10), we conclude that in each time-slot, the total cost of the system is determined by (1) the request-charging-station association decision, (2) the selection of time-slots assigned to the requests, and (3) the amount of energy discharged from/charged to the battery. We aim to design online control algorithms with the decisions above in each time-slot, and the objective is to minimize long-term average cost of the whole charging network system. We formally state the problem below as Problem **P1**.

Problem **P1:**

$$\min_{A,b,x} \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{J} \mathbb{E}\left[P(t)E_j(t) - P(t)b_j(t)\right]$$

s. t. For any charging station j and request i in time-slot t: (1)–(9) hold.

The expectation in the objective function for time-slot t is taken over all the randomness of the energy price, arrivals of requests and renewable energy, together with control actions from the beginning to time-slot T.

Note that term $\lim_{T\to\infty} \sum_{t=1}^{T} \sum_{j=1}^{J} P(t)E_j(t)$ represents the total cost demand without drawing energy from the battery in the whole time horizon, which can also be derived by simply adding the cost of all charging requests one by one. Thus, we have the following equation:

$$\begin{aligned} &\lim_{T\to\infty} \sum_{t=1}^{T} \sum_{j=1}^{J} \mathbb{E}[P(t)E_j(t)] \\ &= \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{J} \mathbb{E}\left[\sum_{i=1}^{n_t} \sum_{k=0}^{d_t^i} A_{ij} x_k^i P(t+k)\right], \end{aligned}$$

where $\sum_{k=0}^{d_t^i} \sum_{j=1}^{J} A_{ij} x_k^i P(t+k)$ is the cost demand of request $i \in N(t)$ if no energy is drawn from the battery. Now, we reformulate **P1** as **P2** stated below:

Problem **P2:**

$$\begin{aligned} \min_{A,b,x} \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\Bigg[&\sum_{i=1}^{n_t} \sum_{k=0}^{d_t^i} \sum_{j=1}^{J} A_{ij} x_k^i P(t+k) \\ &- \sum_{j=1}^{J} b_j(t) P(t) \Bigg] \end{aligned}$$

s. t. For any charging station j and request i in time-slot t, (1)–(9) hold.

To proceed with the theoretical analysis, we denote the available space in the battery as $\mathcal{E}_j(t) \triangleq B_{\max} - B_j(t)$. Note that (1) and (2) are equivalent to the following equations:

$$\mathcal{E}_j(0) = 0, \tag{11}$$

$$\mathcal{E}_j(t+1) = \max\{0, \mathcal{E}_j(t) - \lambda_j(t) + b_j(t)\}, \tag{12}$$

$$\mathcal{E}_j(t) \leq B_{\max}. \tag{13}$$

In **P2**, (13) holds in each time-slot, and this upper bound makes it very difficult to find a feasible solution to **P2**. We consider a more tractable problem by considering the following relaxed constraint that can be derived from (12) and (13):

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} b_j(t) \leq \bar{\lambda}_j, \tag{14}$$

where $\bar{\lambda}_j$ is the average incoming rate of renewable energy of charging station j.

We prove this argument by contradiction. Suppose (14) does not hold, then there must exist a $\delta > 0$ such that $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} b_j(t) \geq \bar{\lambda}_j + \delta$, hence $\mathcal{E}_j(t)$ is divergent and approaches infinity, which contradicts with (13).

By replacing (13) with (14), we have a relaxed version of **P2**, which we denote as Problem **P3**, stated formally as the following:

Problem **P3:**

$$\min_{A,b,x} \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[\sum_{i=1}^{n_t}\sum_{k=0}^{d_t^i}\sum_{j=1}^{J} A_{ij}x_k^i P(t+k) - \sum_{j=1}^{J} b_j(t)P(t)\right]$$

s. t. For any charging station j and request i in time-slot t, (4)–(9), (11), (12) and (14) hold.

It can be seen that **P3** is a relaxed version of **P2**, since (14) is a relaxation of (13), while the other constraints remain the same. In fact, in Problem **P3**, an allocation scheme can even be feasible when the process $\{\mathcal{E}_j(t)\}_t$ is unbounded, as long as $\mathcal{E}_j(t)$ is kept to be rate-stable [12] and (14) is satisfied for any charging station j. In other words, in Problem **P3**, the maximum capacity of the batteries is relaxed to be infinity, while the available space in the battery is enforced for guaranteeing stability.

Let C_2^* and C_3^* denote the optimal objective values of **P2** and **P3** respectively. Note that C_2^* is a function of the battery capacity $B_{\max}$, thus we denote it as $C_2^*(B_{\max})$. Given that **P3** is a relaxed version of **P2**, C_3^* is the lower bound of $C_2^*(B_{\max})$, In other words, we must have:

$$C_3^* \leq C_2^*(B_{\max}), \text{ for any finite value of } B_{\max}.$$

In the next section, we propose a joint association and energy control policy under the constraints in **P2**, and show that its average operational cost can get arbitrarily close to C_3^* by increasing the battery capacity $B_{\max}$.

IV. Joint Association and Energy Control Policy

We first introduce a free control parameter $V > 0$, which satisfies the following equation:

$$B_{\max} = VP_{\max} + E_{\max}, \quad (15)$$

where $P_{\max}$ and $E_{\max}$ are defined in Section II. Our proposed algorithm that conforms to the constraints in **P2** naturally breaks into two components. On one hand, to minimize the cost of the system, the association control component tries to allocate those time-slots with small electricity cost of a nearby charging station with sufficient harvested renewable energy to each charging request. On the other hand, the energy control component saves electricity cost by using the battery as a energy storing buffer. It purchases energy from the power grid when the electricity price $P(t)$ is relatively low, while uses the energy in the battery when $P(t)$ is relatively high. In the following we describe our proposed algorithm in detail.

Association control component: Our objective in this component can be characterized by the following Problem **PA**, in which $A_{ij} = 1$ if request i is assigned to charging station j and $x_k^i = 1$ if time-slot $t+k$ is selected to serve request i.

Problem **PA**:

$$\arg\min_{A,x} \sum_{i=1}^{n_t}\sum_{j=1}^{J} A_{ij}\left\{ \min\left(VP(t), B_{\max} - B_j(t)\right) x_0^i + \sum_{k=1}^{d_t^i} VP(t+k)x_k^i \right\}$$

s. t. (5)–(8) hold.

The objective of **PA** is essentially to find out those time-slots that satisfy constraints (5)–(8) with the smallest total cost. According to the objective function of **PA**, we observe that the cost of each time-slot between each request-charging-station pair (i, j) is

$$\underbrace{\min\left(VP(t), B_{\max} - B_j(t)\right)}_{\substack{\text{cost from charging-station } j \\ \text{to request } i \text{ in time-slot } t}} x_0^i + \sum_{k=1}^{d_t^i} \underbrace{VP(t+k)}_{\substack{\text{cost from charging-station } j \\ \text{to request } i \text{ in time-slot } t+k}} x_k^i,$$

with the cost in time-slot t capturing V times the energy price from the power grid capped by the available space in the battery, and the cost in other time-slots capturing the energy prices only. Note that (6) also plays an important role in minimizing the total cost. A charging station with a sufficient amount of energy stored in the battery, i.e., with a low value of the available space in the battery, may not be selected to serve the request because it takes a longer total charging time with higher cost if the charging station is far away from the current drone position. However, the closest charging station may not be selected because its battery level is low compared to a further charging station. We will present a detailed discussion about **PA** in Section IV-B.

Energy control component: The energy control component has a threshold-based structure. The battery at each charging station is either charged with the maximum charging rate, or discharged with the exact amount of energy that needs to be consumed by that charging station, depending on the both the instantaneous energy price and the current battery level:

$$b_j^*(t) = \begin{cases} -b_{\max}, & \text{If } B_j(t) < B_{\max} - VP(t) \\ E_j(t), & \text{If } B_j(t) \geq B_{\max} - VP(t) \end{cases}.$$

The scheme decides whether the battery is charged or not by comparing $B_j(t) + VP(t)$ with the battery capacity $B_{\max}$. If the threshold is not exceeded, it indicates that either the battery level or the electricity price is relatively low. In such cases, energy will be purchased from the power grid to serve the energy demand of the current charging requests, and charge the battery with the maximum charging rate. However, when either the battery level or the electricity price is high, the scheme uses the energy stored in the battery. Here, the parameter V trades-off the monetary cost and the battery level. As the control parameter V increases, the scheme weights more on the electricity price than the battery level, which leads to a larger reduction of electricity cost. However, as shown in (15), the maximum battery capacity $B_{\max}$ is a linear function of V. With a large value of V, the charging station needs to be equipped with a larger battery that incurs a larger one-time installment cost.

A. Performance Analysis

The following theorem shows that the performance of our proposed algorithm can be arbitrarily close to optimal as the battery size $B_{\max}$ increases, with a gap to the optimal average cost shrinking in the order of $O(1/B_{\max})$.

Theorem 1. *The proposed joint association control and energy control policy with battery capacity $B_{\max} = VP_{\max} + E_{\max}$ can achieve long-term average cost*

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[\sum_{i=1}^{n_t} \sum_{k=0}^{d_t^i} \sum_{j=1}^{J} A_{ij} x_k^i P(t+k) - \sum_{j=1}^{J} b_j(t) P(t) \right] \leq C_3^* + C/V,$$

where $C \triangleq J \max(b_{\max}, E_{\max})^2 + J\lambda_{\max}^2$.

Idea of proof: We define Lyapunov function as $L(t) = \frac{1}{2}\sum_{j=1}^{J} \mathcal{E}_j^2(t)$, and use the *drift-plus-penalty* method to obtain a near-optimal algorithm for **P3**. We then find that, with a certain battery capacity, the near-optimal solution also satisfies the constraints of **P2**.

The proof of Theorem 1 is similar to the one of Lemma 1 in [13], thus we omit it here. As we can see from Theorem 1, with increasing value of the battery capacity, the performance of our proposed algorithm approaches the optimal. The reason is that with larger battery capacity, the system can store more renewable energy, and better exploit the fluctuations of electricity prices, by using energy stored in the batteries instead of purchasing energy from the power grid when the price is high, to save the electricity cost.

B. Revisiting Association Control

In each time-slot t, we need to solve Problem **PA** for request-charging-station association and time-slot selection. However, the following theorem indicates that there even exists no polynomial time algorithm which solves the decision problem related to Problem **PA**.

Theorem 2. *Finding a feasible solution that satisfies constraints* (5)–(8) *in Problem* **PA** *is NP-hard.*

The proof is via reducing *Partition Problem* [14] to the decision problem. We first present the formal definition of Partition Problem as follows.

Definition 1 (Partition Problem [14])**.** *Given a collection of positive integers $A = \{a_1, a_2, \dots, a_P\}$, can set A be partitioned into two disjoint subsets, such that the summation of integers in the two sets are identical?*

Proof of Theorem 2. Given any given instance $\{a_1, a_2, \dots, a_P\}$ of the Partition Problem, consider Problem **PA** with input parameters $n_t = P, J = 2, c_t^i = a_i, d_t^i = d_{\max} = \frac{1}{2}\sum_{p=1}^{P} a_p, l_{ij} = 0 (\forall i \in N(t), j \in \{1,2\})$ and $E_{\max} = 1$. We make the observation that any feasible solution to problem **PA** will imply a feasible solution to the Partition Problem. The proof is complete. □

As the decision version of a problem is always easier than or the same as the optimization version, we conclude that in general, one cannot expect the existence of a polynomial time algorithm, which can always find a feasible solution to **PA** with provable performance guarantee. To address this challenge, we show how to convert **PA** into the *minimum weighted* $(K+1)$*-dimensional* n_t*-matching* problem [15], which is known to be NP-hard and can be solved in $O^*(2.851^{Kn_t})$ time[1] according to [15]. In practice, solving the association control problem based on algorithms for $(K+1)$-dimensional n_t-matching problem may suffer from a high computational complexity. However, theoretical speaking, it can be seen that the aforementioned running time is still polynomial in the input size for constant value of K and n_t, where parameter $K \triangleq \max_i(c_t^i + \max_j l_{ij})$ represents the largest possible total charging time needed among all the requests in the context of **PA**. Moreover, we will present a greedy algorithm for practical purpose later.

The formal definition of minimum weighted $(K+1)$-dimensional n_t-matching problem is given as follows.

Definition 2 (Minimum Weighted $(K+1)$-Dimensional n_t-Matching Problem [15][2])**.** *Let $\mathcal{T}_1, \dots, \mathcal{T}_{K+1}$ be finite, pairwise disjoint sets, and $\mathcal{L}$ be a subset of $\mathcal{T}_1 \times \dots \times \mathcal{T}_{K+1}$, i.e., $\mathcal{L}$ consists of tuples $(t_1, \dots, t_{K+1})$ where $t_i \in \mathcal{T}_i (\forall i \in [K+1])$. Let $W : \mathcal{L} \to \mathbb{R}$ be a weight function defining the weight of elements in $\mathcal{L}$, the problem is to find a set $\mathcal{M} \subseteq \mathcal{L}$ with cardinality $|\mathcal{M}| = n_t$ such that any two distinct tuples in $\mathcal{M}$ share no common node in $\mathcal{T}_i (\forall i \in [K+1])$, while the total weight $\sum_{l\in\mathcal{M}} W(l)$ is minimized.*

We first focus on the simple case with $K = c_t^i$, $d_t^i = d_{\max}$, $l_{ij} = 0$ $(\forall i \in N(t), j \in [J])$ and $E_{\max} = 1$. Let $\mathcal{N}, \mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_K$ be finite and disjoint sets, where $\mathcal{N} = \{1, 2, \dots, I\}$ represents the set of requests and $\mathcal{T}_k$ represents the set of time-slots of all the charging stations that can serve a charging request in k^{th} order. Each set $\mathcal{T}_k$ consists of $J(d_{\max} + 1)$ nodes, i.e., $t_{js}^k \in \mathcal{T}_k$ $(j \in [J]$, $s \in \{0, 1, 2, \dots, d_{max}\})$ represents the time-slot $t + s$ of charging station j. We construct set of tuples $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{T}_1 \times$

[1]The O^* notation hides polynomial factors in the running time [15].

[2] [15] considers the problem that maximizes the total weight, which can be easily adapted to the minimization one by converting weight of each tuple to its opposite.

$\mathcal{T}_2 \times \cdots \times \mathcal{T}_K$ as follows:

$$\mathcal{L} = \left\{ (i, t^1_{js_1}, t^2_{js_2}, \ldots, t^K_{js_K}) \,\middle|_{\, 0 \le s_1 \le s_2 \le \cdots \le s_K \le d^i_t}^{\, i \in N(t), j \in [J],} \right\}.$$

Element $l = (i, t^1_{js_1}, t^2_{js_2}, \ldots, t^K_{js_K})$ belonging to $\mathcal{L}$ indicates that request i is served by charging station j in time-slots $t + s_1, t + s_2, \ldots, t + s_K$.

We define the weight function $W : \mathcal{L} \to \mathbb{R}$ as follows:

$$W(l) = \sum_{k=2}^{K} VP(t + s_k) + \begin{cases} \min\left(VP(t), B_{\max} - B_j(t)\right) & \text{If } s_1 = 0 \\ VP(t + s_1) & \text{If } s_1 \ne 0 \end{cases},$$

for $l = (i, t^1_{js_1}, t^2_{js_2}, \ldots, t^K_{js_K}) \in \mathcal{L}$. Now we are able to introduce Problem **PM** shown as below:

Problem **PM**:

$$\min_{\mathcal{M} \subseteq \mathcal{L}} \sum_{l \in \mathcal{M}} W(l)$$

s.t. $\forall t \in \cup_{k=1}^{K} \mathcal{T}_k$, there exists at most one element $l \in \mathcal{M}$ that contains t,

$\forall i \in \mathcal{N}$, there exists exact one element $l \in \mathcal{M}$ that contains i.

It can be verified that **PM** is indeed a minimum weighted $(K + 1)$-dimensional n_t-matching problem. To see how to convert **PA** to **PM**, we first note that (5) indicates that each request i can only be assigned to exact one charging station. By our construction of elements l in $\mathcal{L}$, each request i will be served by those time-slots in the same charging station, and the second constraint of **PM** ensures that each request will be assigned to one charging station. In addition, our construction also guarantees that each request will be served with the amount of c^i_t time-slots, which is implied by (6). Lastly, the first constraint of **PM** claims that each time-slot of a charging station can only be assigned to one request, which satisfies the power constraint shown by (8).

In the general case when $E_{\max} > 1$, we can still convert **PA** into the minimum weighted matching problem by extending each charging station $j \in J$ into a set of $E_{\max}$ identical virtual charging stations $\{j^{(1)}, j^{(2)}, \ldots, j^{(E_{\max})}\}$, and construct the set $\mathcal{T}_k$ with $JE_{\max}(d_{\max} + 1)$ nodes. In the case when requests have different required time c^i_t and $l_{ij} \ne 0$, we can let $K \triangleq \max_i\{c^i_t + \max_j l_{ij}\}$ and introduce dummy nodes in set $\mathcal{T}_k$ for the request i if $k > c^i_t + l_{ij}$.

In practice, due to the short lifetime of drones, a large number of drones are used to support one task, which implies drones can be charged lately and the deadline of charging request is large. For cases where the deadlines of the requests are large enough to ensure that all the requests can be finished within deadlines by only one charging station, we propose an approximate greedy algorithm with low time complexity to solve **PA**. As shown in Algorithm 1, the greedy algorithm essentially selects the request-station pair and time-slots with the minimum total cost for the association decision.

Algorithm 1: Greedy Algorithm

```
Input: P(t+k), c_t^i, d_t^i, l_ij and B_j(t), for
       i ∈ N(t), j ∈ [J], and k ∈ {0, 1, ..., d_max}.
Output: A_ij, x_k^i.
U ← {1, 2, ..., n_t}; T_j ← {0, 1, ..., d_max}, ∀j.
while U ≠ ∅ do
    W* ← ∞; T* ← ∅;
    for i ∈ U, j ∈ [J] do
        /* Find the valid pair (i,j) with
           minimum total weight */
        if |T_j| ≥ d_t^i then
            W_ij ← min { Σ_{k∈T} VP(t+k) 1_{k≠0} +
                   min(VP(t), B_max − B_j(t)) 1_{k=0} | k ≤
                   d_t^i, T ⊆ T_j, |T| = c_t^i };
            if W_ij < W* then
                W* ← W_ij; i* ← i; j ← j*; T* ← T;
    A_{i*j*} ← 1; x_k^i ← 1, ∀k ∈ T*;
    U ← U − i*; T_{j*} ← T_{j*} \ T*
```

TABLE I: Simulation Parameter Settings

Parameters	Value
a time-slot	10 minutes
$\lambda_j(t)$	$U(2, 10)$ Wh/time-slot
$n_t = \lvert N(t) \rvert$	$U(5, 10)$ per 10 time-slots
Required charging time c^i_t	$U(10, 15)$ time-slots
Deadline d^i_t	$U(20, 30)$ time-slots
Maximum charging rate	$-b_{\max} = 10$ Wh/time-slot
Maximum power supply	$E_{\max}$ = 5
Energy price from a power grid	$(1 - 100) \times 10^{-4}$ cents/Wh

V. Numerical Results

A. Simulation Setup

We conduct our simulation on a charging network with $J = 10$ charging stations. Each time-slot is set to 10 minutes. The number of charging requests for every 10 time-slots is uniformly distributed in $[5, 10]$, and the request positions are uniformly distributed on a $2D$ plane. The required charging time and the deadline for each request are uniformly distributed in intervals $[10, 15]$ and $[20, 30]$ respectively. The renewable arrival $\lambda_j(t)$ is uniformly distributed between 2 and 10 Wh/time-slot. For request i and charging station j, the extra required charging time l_{ij} is proportional to the Euclidean distance s_{ij} between the drone position of request i and charging station j, with $l_{ij} = 10s_{ij}$. We use the real market electricity price data obtained from [16], as shown in Figure 3. The system parameter values are summarized in Table I.

Benchmark control policies: We compare our policy with the following benchmark policies:

Closest charging station (CCS) with energy control (EC) policy: For each request, this policy chooses the closest

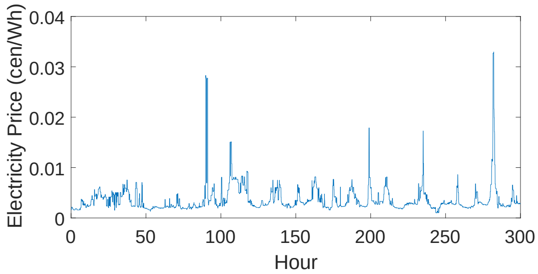

Fig. 3: Energy prices in real electricity market [16].

available charging station and select those time-slots with the lowest electricity prices, without considering the battery status of charging stations. In each time-slot, the charging station chooses to charge/discharge the battery following our proposed energy control component.

CCS without EC policy: This policy allocates the closest charging station for each charging request, and selects the time-slots with lowest prices. In each time-slot, the charging station first uses the energy inside the battery, then purchases energy from the power grid to meet the rest of energy demand.

Baseline policy: This policy allocates the closest charging station for each charging request and starts serving the request right after its arrival. The batteries are only charged with renewable energy, and remaining energy demand is purchased from the power grid.

B. Key Observations

The cost reductions shown in Figure 4 are with respect to the baseline policy. From Figure 4 we observe that:

- Both CCS policy and our proposed policy achieve better performances as the battery capacity increases. This is because with larger battery capacity, there are more chances to use battery as a buffer, and to use the fluctuations of electricity prices. Our proposed algorithm achieves a cost reduction exceeding 50% with a battery size of 5 kWh.
- The performance of CCS without EC policy remains the same under small and large battery capacity. Since the battery is only used to store the renewable energy without EC, increasing the battery size will not bring an additional performance gain, if the the battery size is already large enough to store the renewable energy.
- Compared with CCS with EC, our proposed policy achieves a better performance, which benefits from the utilization of battery status. In our association control component, at the arrivals of requests our proposed policy will prefer charging stations with higher battery level and achieve a better utilization of the renewable energy in those charging stations.

VI. Conclusion

In this work, we focus on the drone charging system with renewable energy and batteries, with the aim of designing joint drone-charging-station association/energy control algorithms to reduce the monetary operational cost. We prove that our proposed algorithm achieves a cost that is

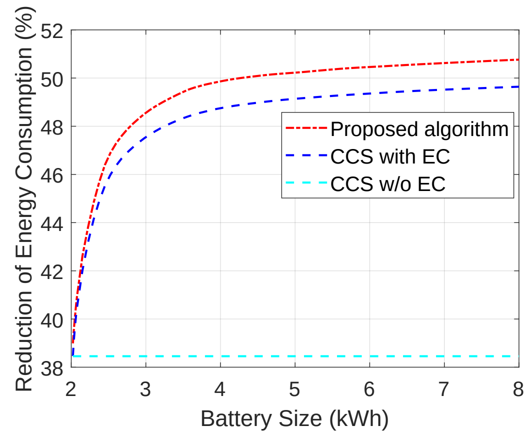

Fig. 4: Reduction in electricity bill under different policies.

arbitrarily close to optimal, as the battery capacity increases. In addition, the simulations show that our proposed algorithm achieves significantly large reduction in electricity cost under reasonable battery capacity.

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