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A DEFENCE OF ARBITRARY OBJECTS

Kit Fine and Neil Tennant

II—Neil Tennant

§0. Fine's theory of arbitrary objects is both intriguing and perplexing. I shall explore the logical structure of difficulties facing it and solutions Fine proposes. In §1 I shall outline the structure of his argument for accepting arbitrary objects, indicating various rejoinders to Fine on certain points. Then I shall take up some of these points in greater detail: in §2, the problematic status of the principle of generic attribution, and in §3 the nature of the commitment Fine would have us make to arbitrary objects.

§1. Fine arrives at his theory by successively refining his ideas to take care of certain objections. These are as follows.

Objection 1 There are no arbitrary objects
Fine's reply There are. They are abstractions. But they are not on an ontological par with individuals.

Objection 2 The theory of arbitrary objects is logically incoherent
Fine's reply The arguments behind this objection depend upon the failure to distinguish between two basically different formulations of the principle of generic attribution: one in the material mode; the other in the formal mode. Once the distinction is made, the arguments are seen to be without foundation.

Rejoinder In a semantically closed language, the principle of generic attribution, if itself generic, leads to incoherence in a way that makes the principle itself suspect, rather than the fact of semantic closure. I shall develop this point in §2 below.

Objection 2a Semantical rules fail for complex predications on arbitrary objects.
Fine's reply The statement Q(a), regardless of its inner complexity, simply has the same truth conditions as ∨ iQ(i).
Rejoinder. All sorts of differences now emerge between predications on arbitrary objects and predications on individual objects. Moreover they emerge even when the object language is logically perfect, with no vague predicates etc. Fine admits that his proposal for evaluating disjunctions by resorting to lambda abstraction means that the lambda conversion principle fails. He concedes that 'it is impossible to achieve complete logical parity between individual and arbitrary objects'. But this is to play down the importance of the difference: to offer a picture of a progressively diminishing but never disappearing difference in logical behaviour upon successive theoretical adjustments and manoeuvres. It seems to me, however, that the gap between arbitrary and individual objects yawns just as wide as we shunt the difference around from evaluation of complex predications to principles of property abstraction etc; and that one hardly need be an 'adamant logical purist' to be disturbed by this persisting difficulty.

Objection 2b. Absurdity results from taking the principle of generic attribution within its own scope.

Fine's reply. Distinguish generic from non-generic (classical) conditions.

Rejoinder. How? Does the fault lie only in such predicates as 'being an individual number' or 'being in the range of'? Might it be generated also by certain logical operations, such as unrestricted quantification (especially in the case of set theory)? How do we know when a given condition is generic? Fine nowhere answers this question. On p. 94 he acknowledges the problem, but offers a circular answer:

...it is not as if the principle [of generic attribution] had no application. Call a language generic if all of the conditions obtainable by its use are themselves generic. Then many languages, of natural and independent interest, will be generic; and so the principle (G3) [for any generic condition Q(x), Q(a) is true iff \( \forall i Q(i) \) is true] will have wide application to all such languages.

'Being generic' ought to be a decidable property of conditions expressible in the language. Only then will the principle of generic attribution have application of sure axiomatic status.

We cannot wait to see whether a given condition, within the scope of the principle, will lead to undecidability of first order logic, and if so, this matter, would appear to prevent one from proceeding.

§2. Fine distinguishes formulations of the material and in the formal sense. The reason for this is that if the principle is taken in the formal mode (as in his (G1): a Q's iff every individual\( \forall i Q(i) \), we cannot correctly infer Q(a).

But the fallacy involved in the material mode must be taken to be that of applying the principle in the formal context in which the name of an arbitrary object appears. For although we may ask whether the arbitrary number \( a \) (Ex \( \equiv \forall i Ei \)) of being even iff all individual numbers are even. What the intuitive principle seems to tell us is that an arbitrary number is even if and only if the condition is true of all individual numbers.

What is this, if not applying the principle in the biconditional, rather than 'internally' of its left hand side?

Fine's distinction between material and formal principle of generic attribution can be disregarded. The principle merely prohibits semantic closure. He offers something like a type restriction on application, especially on the satisfaction and truth in its formulations in the formal mode, to emerge more clearly when formal proofs are discussed. Tarski's reason for resorting similarly to the theory of truth was to avoid the semantic paradox of the Liar. Now with the semantical parallel...
differences now emerge between predicates and predications on individual objects and predicators on individual objects even when the object language is a non vague predicates etc. Fine admits that stating disjunctions by resorting to lambda the lambda conversion principle fails. It is impossible to achieve complete logical dual and arbitrary objects. But this is to miss the point of the difference: to offer a picture of a thing but never disappearing difference in the successive theoretical adjustments and which to me, however, that the gap between actual objects yawns just as wide as we shunt from evaluation of complex predicators by the abstraction etc and that one hardly "logical purist" to be disturbed by this

...y results from taking the principle of... within its own scope.

A generic from non-generic (classical) containment the fault lie only in such predicates as number or 'being in the range of'? Might it by certain logical operations, such as abstraction (especially in the case of set theory)? When a given condition is generic? Fine question. On p. 64 he acknowledges the circular answer:

the principle [of generic attribution] had... Call a language generic if all of the unactable by its use are themselves generic. Languages, of natural and independent generic; and so the principle (G3) [for any Q(x), Q(a) is true iff \( \forall x Q(x) \) is true] will situation to all such languages.

...to be a decidable property of conditions language. Only then will the principle of have application of sure axiomatic status.

We cannot wait to see whether a given condition, when taken within the scope of the principle, will lead to absurdity. Yet the undecidability of first order logic, and Fine's silence on the matter, would appear to prevent one from doing any better.

§2. Fine distinguishes formulations of the principle of generic attribution in the material and in the formal mode. The reason for this is that if the principle is taken in the material mode then Berkeley's example of odd or even numbers precipitates absurdity. But it is not exactly clear how the distinction is to be drawn. According to Fine, taking the principle in the material mode (as in his (G1): a Q's iff every individual Q's)

rests on the fallacy of applying the principle internally to only a part of the context in which the name of an arbitrary object appears. For although we may affirm \( \forall i Q(i) \equiv \forall i Q(i) \), we cannot correctly infer Q(a) \( \equiv \forall i Q(i) \).

But the fallacy involved in the material mode might just as well be taken to be that of applying the principle externally to the whole context in which the name of an arbitrary object appears. If we recall Fine's own account of the matter, we

ask whether the arbitrary number a satisfies the condition (Ex \( \equiv \forall i E(i) \)) of being even iff all individual numbers are even. What the intuitive principle of generic attribution seems to tell us is that an arbitrary number a should satisfy the condition iff all individual numbers do . . .

What is this, if not applying the principle to the wider context of the biconditional, rather than 'internally' to the part consisting of its left hand side?

Fine's distinction between material and formal modes for his principle of generic attribution can be discarded. He can instead merely prohibit semantic closure. He ought to be calling for something like a type restriction on applications of the principle, especially on the satisfaction and truth predicates occurring in its formulations in the formal mode. (This point will emerge more clearly when formal proofs are examined below.) Tarski's reason for resorting similarly to language levels in the theory of truth was to avoid the semantical paradoxes such as the Liar. Now with the semantical paradoxes, the problem lies
in the natural rules of inference for the truth (or satisfaction) predicate, coupled with inferential moves licenced by the facts of self-reference afforded by semantic closure. Elsewhere I have analyzed the proofs of absurdity associated with the paradoxes. They all appear to share the feature of not being normalizable. That is, the reduction sequence of a paradoxical proof does not terminate after finitely many steps. Instead it enters a loop whose periodicity depends on the logical structure of the paradoxical statement(s) in question. In the light of this, it seems reasonable to conjecture that the test of looping reduction sequences, applied to an enumeration of proofs in the semantically closed language, would yield an axiomatization of paradox.

The relevance of these remarks on paradox is this. If we take Fine’s formulation of his principle in the formal mode, but allow the language to be semantically closed, absurdity results from a proof virtually identical to the one he gave for the case of the material mode. Thus semantic closure short-circuits the distinction between modes. In what follows I use these abbreviations:

\[
\begin{align*}
gQ & \quad \text{Q is generic} \\
a/\varphi & \quad \text{a satisfies } \varphi \\
\top & \quad \varphi \text{ is true} \\
\text{PGA}(x) & \quad \forall gQ(x/\varphi \equiv T \forall iQj) \\
\text{Ex} & \quad x \text{ is even} \\
\text{Ox} & \quad x \text{ is odd}
\end{align*}
\]

PGA (x) is the principle of generic attribution for the case where x is an independent arbitrary object. In this case we can simply put \( T \forall iQj \) on the right hand side of the biconditional, instead of resorting to talk of satisfaction by each individual in the range of the arbitrary object concerned. For an arbitrary object a, PGA (a) is axiomatic. Let us further assume that we are concerned only with natural numbers. n will range over genuine individual numbers. As an axiom schema we have \( \text{En} \lor \text{On} \). In the absence of explicit criteria allowing us to determine otherwise, let us assume that PGA is a generic property. We have the following proof P of \( \forall gQ(n/Q \equiv T \forall jQj) \) from assumptions PGA (a) and gPGA:

\[
\begin{align*}
\text{PGA(a)} & \quad \text{gPGA} \\
a/\text{PGA} & \quad T \forall i\text{PGA}_i \\
\forall i\text{PGA}_i & \quad \forall gQ(n/Q \equiv T \forall jQj) \\
\end{align*}
\]

We continue now as follows:

\[
\begin{array}{c}
\text{p} \\
\begin{align*}
(1) & \quad gE \\
\forall gQ(n/Q \equiv T \forall jQj) & \quad (1) & \quad gO \\
\forall gQ(n/Q \equiv T \forall jQj) & \\
\end{align*}
\end{array}
\]

\[
\begin{array}{c}
\text{Ej} \\
\begin{align*}
\text{Ej} & \\
\text{En} & \quad \forall \text{Ej} \\
\text{En} \lor \text{O} & \quad \text{Ej} \\
\text{En} \lor \text{O} & \quad \Lambda
\end{align*}
\end{array}
\]

Our proof of \( \Lambda \) is in normal form, by construction. For a semantical paradox, as mention above, we strongly to suspect the principle of generic attribution, rather than the fact of semantic closure, as the source of absurdity. The full list of assumptions on which our proof above is

\[
\begin{align*}
\text{PGA(a), gPGA, gE, gO, En} \lor \text{On}
\end{align*}
\]

Of these, only PGA (a) could be disputed, and it is clear from the way he introduced the distinction that he would dispute it. But the others would do so here not have been made clear and justify the reductio provided by our proof.

Another would-be reductio is the following:
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We continue now as follows:

\[
\begin{array}{c}
\text{P} \\
\begin{array}{c}
gE \\
\forall gQ(n/Q = T \forall jQj)
\end{array} & \begin{array}{c}
gO \\
\forall gQ(n/Q = T \forall jQj)
\end{array}
\end{array}
\begin{array}{c}
\frac{n/E}{T \forall jEj} & \frac{n/O}{T \forall jOj}
\end{array}
\]

\[
\begin{array}{c}
\text{En} \lor \text{On} \\
\frac{n/E \lor n/O}{\Lambda}
\end{array}
\begin{array}{c}
\text{El} \\
\frac{0}{\Lambda}
\end{array}
\]

Our proof of \( \Lambda \) is in normal form, by contrast with proofs of semantic paradox, as mentioned above. This leads me strongly to suspect the principle of generic attribution itself, rather than the fact of semantic closure, to be the source of absurdity. The full list of assumptions on which \( \Lambda \) depends in the proof above is

\[
\text{PGA (a), gPGA, gE, gO, En} \lor \text{On}
\]

Of these, only PGA (a) could be disputed by Fine. Indeed it is clear from the way he introduced the generic/classical distinction that he would dispute it. But the grounds on which he would do so have not been made clear and justified independently of the reductio provided by our proof.

Another would-be reductio is the following:
The suspect move here is of course (*). From the fact that an arbitrary object satisfies a disjunction, it does not follow that it satisfies one or other of the disjuncts. Fine notes as much, observing that the semantical rule for evaluating disjunctions fails for statements about arbitrary objects. But I think the point comes out more vividly in a proof theoretic context. The last proof shows just how abruptly ordinary reasoning about objects can be stopped dead in its tracks by the indeterminacy of the arbitrary. Far from being a theory about arbitrary objects, it is rather a theory of arbitrary obstacles. We lose the distributive laws for satisfaction by objects across logical operators, thus arguably being deprived of a notion of objecthood at all.

We have seen from the proofs above that if the language is semantically closed then the principle of generic attribution (in the formal mode) leads to absurdity unless one denies both that the principle itself expresses a generic property and that arbitrary objects behave like individuals with respect to satisfaction of predicates. Let us concur with the latter denial for the time being, thereby refusing to admit the last ‘proof’ of absurdity. I want now to make a constructive suggestion which might enable Fine to avoid having to specify what generic properties are, and indeed enable him to formulate the principle of attribution without restriction to generic properties. It appears that merely indexing levels is all that is needed to avoid the first proof of absurdity given above. Let us assign predicates to levels in the obvious way. Let \( \text{PA}_n \) be the generically unrestricted but type restricted principle

\[
\forall Q_n \forall a (a/\equiv \forall Q_{n+1})
\]

However we now try to write down a version of \( P \) observing type restrictions we fail. The type restrictions can be grammatical or inferential. That is, we can either typed formulae as ill-formed, or incorrect, or fallacious. To illustrate the effect of either we try writing down a version of \( P \) starting with

\[
\text{PA}_1 (a)
\]

i.e. \( \forall \forall Q_0 \forall a/\equiv \text{PA}_1 \)

\[
\forall a/\equiv \text{PA}_1
\]

\[
\forall Q_0 \forall a/\equiv \text{PA}_1
\]

\[
\forall i \text{PA}_i
\]

\[
\forall i \text{PA}_i
\]

\[
\forall i \text{PA}_i
\]

\[
\forall i \text{PA}_i
\]

Type restrictions are violated repeated generalization over zero level predicate, predicate. (II) does not raise the level of predicate from 1 to 2 as it should, giving argument a predicate of level 1. (III) has the wrong formula already of level 1. We thus thoroughly of passage to \( \Lambda \) by this route; and this with the notion of generic properties and with the principle of attribution to generic properties

§3. On a Carnapian distinction between questions about existence, one might doubt there are arbitrary objects. This Fine does about the applications of his theory makes professed willingness to go along with a proposal that would put his idea of way into positive to the external question was, after all, the utility application framework involved. Yet Fine describes negatively what he calls the ‘ontologically about existence. He likens himself to the not the ultimate necessity of number talk for one. But if Fine wishes to deny the ultimate nature arbitrary objects for our scientific purposes.
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The principle of generic attribution (inference to absurdity) unless one denies both that expressions a generic property and that arbitrary individuals with respect to satisfaction of the latter denial for the time requiring admission of the last 'proof' of absurdity. I consider the notion of generic properties and how to formulate the principle of attribution to generic properties. It appears that merely is needed to avoid the first proof of absurdity. Let us assign predicates to levels in the hierarchy, be the generically unrestricted but type restricted.

\[ \forall Q \forall x \phi \]

Type restrictions are violated repeatedly. (I) instantiates a generalization over zero level predicates with a first level predicate. (II) does not raise the level of the satisfaction predicate from 1 to 2 as it should, given that it has as an argument a predicate of level 1. (III) has \( T_1 \) applying to a formula already of level 1. We thus thoroughly forfeit our rights of passage to \( \Lambda \) by this route; and this without having to invoke the notion of generic properties and without having to restrict the principle of attribution to generic properties.

§3. On a Carnapian distinction between external and internal questions about existence, one might deny (externally) that there are arbitrary objects. This Fine does. But his *acharnement* about the applications of his theory makes one suspicious of his confessed willingness to go along with a programme of reduction that would put his new found crystals into logical solution. The way to a positive answer to the external question about existence was, after all, the utility of applications of the 'linguistic framework' involved. Yet Fine describes himself as answering negatively what he calls the 'ontologically significant' question about existence. He likens himself to the nominalist who denies the ultimate necessity of number talk for our scientific purposes. But if Fine wishes to deny the ultimate necessity of talk about arbitrary objects for our scientific purposes (namely, achieving a
better understanding of truth conditions and valid reasoning in ordinary and mathematical discourse), we can legitimately ask what the point is of developing the theory in the first place. If we already have our reduction in the orthodox view, why go inventing theories about new categories of ill-behaved objects that are to be reduced to it? And if the reduction is to go a different way, why not aim for its terminus straightaway?

What is Fine’s position qua semantical theorist with regard to the internal question about existence? He calls the question ‘ontologically neutral’ and answers it affirmatively. He likens himself to the nominalist philosopher of mathematics who is convinced that number talk is dispensable, but useful, and who indulges himself in first order arithmetical assertions. The modern theorist will live with his objects, but not really commit himself to them.

I shall not raise here problems that might beset Fine should he wish to use set theory to do the arbitrary object model theory required to treat the set theorist’s own sayings of the form ‘Let x be an arbitrary set . . .’. Instead I wish to draw attention to a position not yet considered as far as ontological issues are concerned. Is the native speaker (as opposed to the semantic theorist studying his utterances about some subject matter) in any way committed to the existence of arbitrary objects? I would say not. Assume the native speaker speaks an arbitrary first order language with no branching quantifiers. If we are concerned to interpret his utterances by elucidating their truth conditions then it seems we have no way of committing him to arbitrary objects that we might invoke to work out what follows from his general claims about the subject matter in question. It is only if the native reasons by means of certain locutions that can be understood in no other way than as referring to arbitrary objects that it becomes plausible to regard him as ‘internally’ committed to their existence.

Suppose now that I am a first order theorist whose surface linearizations of natural deductions in tree form judiciously eschew all apparent reference to arbitrary objects. For example, instead of saying

Let a be an arbitrary object . . . Then Fa. So, since a was arbitrary, for all x Fx

I say rather

Take a . . . Then Fa. But I assumed

for all x Fx

Each of these manners of speaking adequately uses the inferential move of ∀-introduction

∀xFa

where a does not occur in 

∀xFx

which Fa depends

The latter gloss is deep and robust. The former bloating. If I, in the position of the native, give robust renderings of the moves in my language, I am wrong to claim

I have as much reason to affirm that there are objects in this [internal] sense as to affirm that there are numbers.

Fine says (p. 57) that

the question ‘what are they?’ may be taken non-philosophical way, as a request for what objects one is talking about.

And he goes on immediately to to say or to refer to the kind of role that was intended to play.

We may ask here whether he is talking about arbitrary objects or by parts of language for those objects are (perhaps misguided) consider set theory. In his more philosophical characterizes arbitrary objects as abstractions like. Let us not pause to ask how, in his more philosophical would characterize arbitrary sets. Let us take context of set theory, whose intentions might plausibly be to describe uncluttered by arbitrary members, sets that
of truth conditions and valid reasoning in logical discourse), we can legitimately ask if we are developing the theory in the first place. If we are, in the orthodox view, why go out new categories of ill-behaved objects and to it? And if the reduction is to go a year aim for its terminus straightaway?

We are semantical theorist with regard to what about existence? He calls the question 'what about existence? He likens the realist philosopher of mathematics who is a non-scientific realist talk is dispensable, but useful, and who first order arithmetical assertions. The scientist with his objects, but not really commit the problems that might beset Fine should he want do the arbitrary object model theory.

Instead I wish to draw attention to a considered as far as ontological issues are anative speaker (as opposed to the semantic utterances about some subject matter) in the existence of arbitrary objects? I would like to consider a non-existent first order first-order quantifiers. If we are concerned to the consequences by elucidating their truth conditions the no way of committing him to arbitrary quantifiers. I would like to work out what follows from the subject matter in question. It is only if by means of certain locations that can be rarer way than as referring to arbitrary objects possible to regard him as 'internally' committed to.

I am a first order theorist whose surface natural deductions in tree form judiciously reference to arbitrary objects. For example, arbitrary object . . . Then Fa. So, since a was not a, I x Fx.

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I say rather

Take a . . . Then Fa. But I assumed nothing about a. So,
for all x Fx

Each of these manners of speaking adequately represents the inferential move of ∀-introduction

∀x Fx

where a does not occur in any assumption on which F depends

The latter gloss is deep and robust. The former is ontologically bloating. If I, in the position of the native, persevere with deeply robust renderings of the moves in my language game it is simply wrong to claim

I have as much reason to affirm that there are arbitrary objects in this [internal] sense as the nominalist has to affirm that there are numbers.

Fine says (p. 57) that

the question ‘what are they?’ may be taken, in an ordinary, non-philosophical way, as a request for an explanation of what objects one is talking about.

And he goes on immediately to to say one can do no better than refer to the kind of role that arbitrary objects are intended to play.

We may ask here whether he is talking about a role played by arbitrary objects or by parts of language for whose interpretation those objects are (perhaps misguided) being invoked. Consider set theory. In his more philosophical mood Fine characterizes arbitrary objects as abstractions like sets or propositions. Let us not pause to ask how, in his more philosophical mood, he would characterize arbitrary sets. Let us instead ask, in the context of set theory, whose intentions about roles to be played are relevant here. Presumably not those of the set theorist. For his intentions might laudably be to describe the universe of sets uncluttered by arbitrary members, sets that do not depend for
their existence on, or derive their character from any role he intends them to play. So the intentions must be those of the semantic theorist.

Now is it satisfactory to answer the ordinary non-philosophical questioner who asks 'what are arbitrary objects?2 by telling him what role one intends for them as a semantic theorist? Would one be satisfied, upon asking 'what are spirits?' if one's Azande informant told one what role he intended spirits to play in his theory of bad weather and juvenile delinquency? Surely our retort would be that the description of intention is not enough, and that there are further independent methodological constraints to be placed on the theory before a satisfactory answer might be forthcoming. Thus I am interested not so much in the role Fine intends arbitrary objects to play as in the genuinely explanatory role (if any) that they have to be allowed to play if we are to account successfully for the logico-linguistic intentions of native speakers. These intentions are to model and describe reality, and this sometimes in highly schematic fashion, as when they reason about it at first order.

I say 'schematic' for good reason. The orthodox construal of Fine's phrase (p. 65) 'names for arbitrary objects' is 'placeholders for names of actual individuals'. We can extend the discussion above of the rule of $\forall$-introduction to make this clear. The subproof is schematic in a in the following sense. Any term $t$ may be substituted for appropriate occurrences of $a$ in the subproof so as to yield a proof of $Ft$ from the original assumptions. (Similar considerations apply to the subproof in the rule of $\exists$-elimination.3) Thus instead of taking the parameter $a$ as a name for an arbitrary object we may consider it as a placeholder for names of actual individuals. Talk 'on the surface' of arbitrary objects a when presenting the proof may thus be construed as remarks about the logically hygienic pattern of occurrences of the parameter $a$ within the proof, given or planned. We thus have a wholly syntactic option for systematically understanding the behaviour of 'names for arbitrary objects'. At the other extreme, arbitrary objects themselves, if admitted, might be better assimilated to the domain of cognitive psychology. The original discussions of the 'general triangle' were highly psychologicist.4 One might regard arbitrary objects as incomplete mental representations—not so much objects of thought as objects within thought, by

means of which we reason about the world.

We may even be able to merge the two in a best cognitive account will be one that treats relations as clusters of predicates, or 'packing' a relational scheme. Fine himself expects

that to each set of individuals from an arbitrary object with that set as its argument be in each case definable, thereby in effect equating arbitrary objects and sets, in all applications of which he has given us to suppose that only definable subsets matter. We may interpret his arbitrary objects syntactic conditions on their ranges.5 Is not this a form of orthodoxy a post-Fregean in devising a set-theory of arbitrary objects, without offering to take the objects as models of cognitive processes psychological—or indeed any kind of—reality?

NOTES

3 For more detail see my *Natural Logic* (Edinburgh, 1980).
4 For references to an extensive literature see Beth, 'Zur Dreieck' *Kolosse* 48 (1956/57) 361–380.
5 One is reminded here of Hilbert's $\epsilon$-terms and Halpern-Leisenring, *Mathematical Logic and Hilbert's $\epsilon$-Symbol* (Maia, 1957) 19–33 and 113–129.
derive their character from any role he might play in the world. So the intentions must be those of the person who created it, whether they are the agent or the creator.

We may ask, "What are arbitrary objects?" by telling him that they play a role in the world. We may ask "What are spirits?" by telling him that role he intended to play in his society. Surely our use of the idea of a spirit is partly due to the way it is used in the world, and partly due to the way it is used in the world by others. Thus we are not simply abstracting from the world a set of objects with the property of being arbitrary. Rather, we are abstracting from the world a set of objects with the property of being arbitrary objects.

In fact, we are abstracting from the world a set of objects with the property of being arbitrary objects. This set of objects is the set of all arbitrary objects. The arbitrary objects are those objects that are arbitrary in the sense of being arbitrary in the world. The arbitrary objects are those objects that are arbitrary in the world and are also arbitrary in the world in the sense of being arbitrary in the world by others.

We may even be able to merge the two accounts. Perhaps the best 'cognitive account' will be one that treats mental representations as clusters of predicates, or 'pigeonholes' within a relational scheme. Fine himself expects

that to each set of individuals from $I$ there will be an arbitrary object with that set as its range.

thereby in effect equating arbitrary objects with subsets of $I$. But in all applications of which he has given us any inkling it would appear that only definable subsets matter. So can we not reinterpret his arbitrary objects syntactically as the defining conditions on their ranges? Is not Fine himself being too orthodox a post-Fregean in devising a semantic or referential theory of arbitrary objects, without offering schemes of arbitrary objects as models of cognitive processes with some smack of psychological—or indeed any kind of—reality?

NOTES