The Law of Excluded Middle Is Synthetic A Priori, If Valid

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§1. INTRODUCTION

In the semantic anti-realist tradition deriving from the writings of Michael Dummett there is a clear need to follow through the consequences of one’s stance on sentence meanings as determinate and compositional and as beholden to rules of logical usage.

The anti-realist presses broadly intuitionistic logical reforms; the realist resists these. The realist does not wish to be methodologically deprived. He is unwilling to give up the strictly classical inferences of classical logic. The anti-realist replies that the new level of methodological discipline is in fact liberating and that the realist is not entitled to those strictly classical inferences anyway. The situation, however, has been clouded by a failure (on both sides) to appreciate exactly what is at stake. I intend to spell out here exactly how the realist can be allowed to have his cake but also be asked to improve his manners when eating it. In doing so, however, I remain an anti-realist. I insist, with Dummett, on the manifestation requirement in the theory of meaning; and I advocate also even farther-reaching logical reform than is customary for a Dummettian.
The anti-realist’s views on meaning invite one to rehabilitate the analytic-synthetic distinction. Moreover, his favored theory of meaning is an inferentialist one. This affords the prospect of examining certain mathematical theories more closely in order to see whether some of their content is actually analytical in nature. Shouldn’t we classify as analytic at least those theorems whose proofs use only meaning-specifying rules\(^3\) for the mathematical operators and predicates of the theory? In advancing an affirmative answer to this question, the anti-realist should be prepared also to abandon a dogma that has long held sway in discussions of analyticity: the dogma that no analytic principles could ever commit one logically to the existence of any entities. I would urge that this dogma is false; for it is an adequacy requirement on a theory of meaning that certain expressions be revealed as committing one, by their very meanings, to the existence of certain (necessary) existents.

The prospect is therefore opened up of (i) classifying various mathematical truths as analytic even when they carry existential commitment (albeit only to necessary existents, such as natural numbers) and (ii) reclassifying various logical principles as synthetic if they cannot be justified, on an anti-realist account, as true by virtue of their meanings alone. We see, then, that there is an important continuing debate for realists and anti-realists over the consequences of their disagreement about the nature of logico-linguistic meaning.

In this paper I shall explore prospect (ii).\(^4\) I shall argue that the Law of Excluded Middle is a synthetic principle.\(^5\) This flies in the face of all the conventional wisdom, among analyticity theorists, to the effect that all the so-called ‘logical truths’ of classical logic are analytic. For the anti-realist, logic is indeed analytic; but there is much less to logic than meets the realist’s eye.

\section*{§2. ANALYTICITY}

A sentence’s truth-value is in general determined by its meaning and by how things are (“the way the world is”):

\[
\begin{align*}
\text{World} & \quad \quad \text{Meaning of } \phi \\
\left\{ \right. & \quad \quad \rightarrow \quad \text{truth-value of } \phi.
\end{align*}
\]

The world cannot confer a truth-value on a meaningless sentence; likewise, no meaningful sentence can have its meaning play no role at all in determining its truth-value. But it is perfectly possible, in these general and abstract terms, for the truth-value of a sentence to depend solely on its meaning, so that the world may, as it were, be ignored.\(^6\) In such a case, the sentence is ana-
lytic. If true, it will be so by virtue of its meaning alone and will tell us nothing about the world. By contrast, if the sentence, though true, is not analytic—that is, if it is synthetic—then the world must have made a contribution to determining its truth. When the world thus has a look in, we have to look out: a true synthetic sentence is telling us something about the world.

This strikes me as the fairest way to generalize Kant’s own definition of analyticity, which all commentators agree is overly limited. Kant did not consider the many logically more complex kinds of sentences besides those of subject-predicate form. Yet at least some of these more complex sentences can also be analytic, in the spirit intended to be captured by Kant’s inadequate definition. It is not enough, for any completely general account of analyticity, to speak of “the predicate B [in a singular judgment of the form “A is B”] [belonging] to the subject A, as something which is (covertly) contained in this concept A,”7 and it is not enough to say that the judgment “[adds] nothing through the predicate to the concept of the subject, but merely [breaks] it up into those constituent concepts that have all along been thought in it . . . ”. The later, Fregean, definition of an analytic sentence as one which can be obtained from a logically true sentence by substituting synonyms was motivated by the understandable need for greater generality.9 Frege’s more general definition tried to focus, quite rightly, on the matter of meaning (through the invocation of synonymy, or sameness of meaning); for that is what is at issue in Kant’s talk of conceptual containment in the logically simple case of a singular judgment of subject-predicate form.10 Yet one fair complaint that can be made about both the spirit and the letter of Frege’s generalized notion of analyticity is that it presupposes logic, in the sense that it takes for granted some fixed logical system (in Frege’s case, a classical higher-order logic) and appeals to its set of logically true sentences in generating a more inclusive set of analytic sentences. The logically true sentences themselves form a core within the wider set of analytic sentences.

Frege’s method is more or less what Carnap followed in The Logical Syntax of Language. The catch, though, was that Carnap still uncritically presupposed classical logic as the repository of unvarnished analyticity-by-virtue-of-logical-meanings.

Both Frege and Carnap, I would argue, had matters the wrong way round. We need, rather, a notion of truth-by-virtue-of-meaning (that is, analyticity) from which it will follow, nontrivially, that the logical truths of one’s chosen logical system are indeed analytic. The proper order of business should therefore be: first, characterize the meanings of the logical operators; secondly, survey the synonymies; thirdly, work out what logic is justified by the meanings of the logical operators, and hence what sentences count as logically true; fourthly and finally, use the method of substituting synonyms to expand from the set of logical truths just determined (and now legitimated
as analytic) to the set of analytic truths in general. The outcome of this method should be exactly the set of sentences for whose determination as true the world (as in the picture given above) drops out as irrelevant.

Once we realize that (as Dummett has often stressed) the proper context for talk of meaning is talk of speakers’ knowledge of meaning, the theory of meaning becomes an epistemology of linguistic understanding. So the notion of analyticity becomes epistemic but without collapsing into the notion of being knowable a priori. According to the epistemic conception, a sentence is analytic just in case one’s grasp of its meaning alone should suffice, in principle, for one’s being able to constitute a warrant for its assertion.

The notion of epistemic analyticity, as it applies to logic, is not new. John McDowell, for example, expresses the matter as follows, when he says that the verificationist conception

requires a conception of the truths of logic, not as true solely in virtue of the senses of the logical constants—without assuming the principle of bivalence, there is no telling which sentences have that status—but as knowable solely in virtue of the senses of the logical constants.

The term “epistemically analytic” comes from Paul Boghossian. McDowell’s formulation of epistemic analyticity, though, masks an important ambiguity, which is neither noted nor appreciated by Boghossian. We need to distinguish between grasp of meaning which would ensure the discovery of a warrant to assert and grasp of meaning which would suffice merely for the constitution (or reconstitution) of a warrant to assert. The latter (weaker) notion is to be preferred. That is, a sentence ϕ is epistemically analytic just in case (a) there is a warrant Π for its assertion and (b) grasp of ϕ’s meaning suffices, in principle, for one to reconstitute Π as a warrant for ϕ’s assertion, were one ever to be made aware of Π.

Note that I am not requiring that one who understands a given analytic sentence ψ should be able to decide, by virtue of that understanding alone, to decide whether ψ is true or false. Rather, I am requiring only that if the understander is presented with a purported warrant or refutation of ψ, then she should be able to assess whether that construction is indeed what it purports to be—namely, a warrant (or a refutation) of ψ—and that, having been presented with the right kind of warrant, the understander would be able to reconstitute it for herself by relying only on her grasp of meaning (hence, not on any of her empirical experience).

The idea can be clarified by parrying two would-be counterexamples. The first concerns visual experience. The statement “If this is red, then this is colored” might be asserted on the basis of a visual experience of the thing t in question as red, hence as colored. That is, one could experience the thing t as colored—C(t)—and from that infer R(t) ⊃ C(t). The warrant based in one’s experience would have the structure

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from experience

\[
\frac{C(t)}{R(t) \supset C(t)} \supset \text{-introduction}
\]

But this is a case of a warrant that is tinged with elements from experience. There is another warrant for the same conclusion that is not so tinged. In this warrant, one assumes \( R(t) \) “for the sake of argument” and infers to \( C(t) \) by means of a rule of inference within the language; then one discharges the assumption \( R(t) \) by an application of \( \supset \)-introduction:

\[
\frac{R(t)}{C(t)} \text{(1)} \supset \text{-introduction}
\]

\[
\frac{C(t)}{R(t) \supset C(t)} \supset \text{-introduction}
\]

Once apprised of this warrant, the speaker would be able to reconstitute it on the basis of her mastery of the language alone. Note that it has no undischarged assumptions—no initial elements of experience serving as premisses on which the conclusion depends.

Our second example concerns counting. One can imagine our subject looking at her toes and counting them, finding ten in all. She then asserts, “There are at least ten things.” Here we shall not try to regiment the warrants, because space is limited. The argumentative point, against a more sim-ple-minded account of epistemic analyticity than ours, is that for the constructive logicist and analyticity theorist, the statement “There are at least ten things” is a theorem of logicist arithmetic.

The proof of the theorem would run as follows. There exists the number of all non-self-identical things, which we call zero; there is then the number of all things identical to zero, namely one; and so on. These are all distinct—because of the requirement that if the number of \( F \)s is identical to the number of \( G \)s, then there must be a one-to-one correspondence between the \( F \)s and the \( G \)s. For, in the case of “the number of things identical to zero, . . . , \( n \)” and “the number of things identical to zero, . . . , \( m \)” (as \( F \) and \( G \) respectively, for distinct natural numbers \( n \) and \( m \)), there cannot be such a one-to-one correspondence.

But all that the second example means is that there is one empirically tinged warrant (the toe-counting one) that has certain assumptions (reporting visual experience) undisputed and that has as its conclusion “There are at least ten things”; and there is another warrant—the abstract, logic-numerical one—that has no undisputed assumptions and that leads (admittedly by more sophisticated reasoning) to the same conclusion. My contention is that, once apprised of this latter warrant, our subject should, ideally, be able to reconstitute it on the basis of her grasp of meanings alone. It is empirically untinged.

Let us now take stock by generalizing the lessons we have learned from
these two would-be counterexamples to the claim of the analyticity theorist. For \( \psi \) to be analytic, the understander’s assessment and reconstitution of a warrant for \( \psi \) should be able to proceed on the basis of her grasp of \( \psi \)’s meaning alone. In order for the understander of an analytic \( \psi \) to appraise purported warrants or refutations of \( \psi \), and then to reconstitute a successful warrant establishing \( \psi \) as analytic, she should not require any sensory experience, nor any perceptual beliefs, nor have recourse to any theoretical beliefs that have themselves been formed, ultimately, on the basis of empirical evidence.

Note that these remarks serve only to underscore the point that an analytic truth will also be a priori. As Kant himself said, “it would be absurd to found an analytic judgment on experience. . . . [T]here is no need to appeal to the testimony of experience in its support.” Kant, in making this remark, was certainly not identifying analyticity with aprioricity.

Nor do we intend to conflate analyticity with aprioricity. Analyticity remains for us, as for Kant, a matter of meaning. A priori knowledge, by contrast, remains for us, as for Kant, knowledge that is absolutely independent of all empirical experience. This leaves open the possibility of synthetic a priori judgments. With our understanding of the a priori status of any judgment, however, we eschew appeal to Kant’s problematic criteria of absolute necessity and strict universality. We work instead only with the central negative definition that emphasizes independence from empirical experience.

Now Kant’s favorite and prima facie most secure kind of synthetic a priori knowledge is mathematical knowledge, based on formal proofs employing abstract constructions. At B15–16 in the Critique of Pure Reason, he passed from the too swiftly established syntheticy of the simple arithmetical statement “7 + 5 = 12” to the sweeping conclusion that arithmetical propositions are always synthetic. There is a strong case, however, for saying that some areas of mathematics—such as number theory—contain epistemically analytic statements. Indeed, once one has properly analyzed how arithmetical knowledge is generated by appropriately chosen (and meaning-constituting) rules, one is in a strong position to gainsay Kant even on his chosen example “7 + 5 = 12.” The rules in question are ones that Kant himself never had in mind and was never able to consider. They figure in what I have elsewhere called, in the spirit of Frege’s foundational approach to arithmetic, a “constructive logicist” derivation of the Peano-Dedekind axioms. These rules are perfectly meaning-constituting and meaning-respecting as far as the arithmetical terms are concerned. Therefore, since they suffice for a proof of “7 + 5 = 12,” that sentence is analytic.

Not only can some sentences of arithmetic be analytic, they can also carry existential commitments. These commitments are not vicious, though, since the (abstract) entities that are involved—namely, the natural numbers—
are necessary existents. Epistemically analytic statements need not be any of
the following: decidable, obvious, free of existential commitments.\textsuperscript{17}

This is not to say that all of mathematics turns out (\textit{pace} Kant!) as ana-
lytic rather than synthetic. It is only to say that at least some mathematical
propositions may well be analytic rather than synthetic, while yet others
remain synthetic. Only a closer, careful look will tell us exactly which ones
are which.

Mutatis mutandis: perhaps not all of the so-called “logical truths” (of
classical logic) are analytic. Let us now examine this possibility more closely.

\textbf{§3. THE PREVIOUS STATUS OF THE LAW
OF EXCLUDED MIDDLE}

On standard accounts of the analytic-synthetic distinction, all the so-called
“logical truths” of classical logic are held to be analytic. That is why Frege’s
logicist reduction of classical arithmetic—had it succeeded—would have
shown arithmetic to be analytic after all and not, as Kant thought, synthetic.
Anyone who accepted the analytic-synthetic distinction as tenable and useful
would have been inclined to give as examples of analytic principles of logic
both the Principle of Non-Contradiction ($\neg(\phi \& \neg\phi)$) and the Law of Excluded
Middle ($\phi \vee \neg\phi$). This co-classification is one that would have survived every
one of the major revisions proposed for the contents of “Kant’s Boxes”:

\begin{tabular}{|c|c|}
\hline
A Priori & A Posteriori \\
\hline
\textbf{Analytic} & X \\
Logic & KANT’S BOXES\textsuperscript{18} \\
\textbf{Synthetic} & \\
Arithmetic \\
Geometry & \\
\hline
\end{tabular}

As is well-known, the Logical Positivists sought first to evacuate the syn-
thetic a priori box. They were confident in the “logicist” reduction of arith-
metic afforded by Russell and Whitehead’s \textit{Principia Mathematica} (so that
arithmetic was reclassified as analytic), and they were impressed by the role
played by non-Euclidean geometries in relativistic physics (so that geometry
was reclassified as a posteriori). Next came Gödel’s first incompleteness
theorem for arithmetic, causing those who still held to the analytic-synthetic
distinction to reconsider the Logical Positivists’ classification of arithmetic
as analytic. Perhaps arithmetic was synthetic after all? But then came Quine’s
“Two Dogmas of Empiricism,” and it was no longer fashionable to think
about arithmetic (or any other area of discourse, for that matter) in these
terms.\textsuperscript{19}
§4. THE LAW OF EXCLUDED MIDDLE
AND THE PRINCIPLE OF BIVELENCE

The reforming anti-realist insists on manifestationism and holds that logic
should be analytic. His analysis of meaning renders only broadly intuition-
istic logical principles as analytically acceptable. Accordingly, he rejects or
eschews as logical principles all strictly classical principles of inference that
do not belong to the justified canon. He is committed to the view that the lat-
ter principles, if necessarily true or necessarily truth-preserving, are synthetic.
That, indeed, is the thesis to be defended here: strictly classical logical prin-
ciples are synthetic a priori, if valid. Note that this is, in a certain sense, a
vicarious claim put forward by the perspicuous anti-realist “on behalf of” the
realist. The anti-realist is not going to accept the strictly classical logical prin-
ciples; so, in particular, he is not going to claim, himself, that they are a pri-
ori. Rather, he is offering this Kantian perspective to the realist in order for
the latter to make better sense of her own commitment to those principles.20
That is why my title is conditional: the Law of Excluded Middle is synthetic
a priori, if valid.21

To establish the thesis I shall, as the title suggests, concentrate on the
classicist’s Law of Excluded Middle,

(LEM) \( \phi \lor \neg \phi \),

according to which every instance of the displayed schema is a necessarily
ture sentence of the object language whose (necessarily) truth is known a pri-
or. I shall proceed on the assumption that this schematic law gives exact
expression, within the logic, to the metalinguistic Principle of Bivalence:

(Biv) For every sentence \( \phi \) of the object language, exactly one
of \( \phi \), \( \neg \phi \) is true.

I am aware that there are supervaluational systems, say, for which Bivalence
does not hold but in which the Law of Excluded Middle is nevertheless held
to be correct. Such systems shall be set aside as irrelevant for the argumenta-
tive concerns of this paper. If need be, I shall address my argument only to
those classicists for whom LEM gives precise expression, within their sys-
tem, to Biv.22

Even so, one can still acknowledge an obvious difference between the
Principle of Bivalence and the Law of Excluded Middle. It is this:

1. Biv is a single statement, in the metalanguage; it is an
   explicit universal generalization about all sentences of the
   object language.

2. LEM is a schema, with infinitely many instances in the
   object language. Each instance is a logical axiom for the
classical logician. So LEM is really an infinite set of (classical) laws in the object language, all having the same form \( \phi \lor \neg \phi \). In speaking of the Law of Excluded Middle, one is saying that any such instance \( P \lor \neg P \) is held to be true a priori, prescinding from what we may know as to the truth-value of \( P \) itself.

We are maintaining that these instances \( P \lor \neg P \) are better thought of as being held to be (necessarily) true on a priori rather than on “logical” grounds. Indeed, the holding true (as a matter of necessity) of every such instance (or the holding true, as a matter of necessity, of any such instance \( P \lor \neg P \), absent any proof or refutation of \( P \)) expresses an essentially metaphysical belief. This belief is that the world is determinate in every expressible regard (or at least, in the case of a particular instance of LEM, determinate in the respects answering to the propositional content \( P \) that is involved). Such a belief is synthetic, since its content cannot be known to be true simply on the basis of the meanings of the logical expressions \( \lor \) and \( \neg \).

§5. LOGIC IS ANALYTIC; BUT WHICH LOGIC IS THE CORRECT LOGIC?

Logic is a canon of inference, consisting of laws and/or rules of inference, whose justification turns on questions of meaning alone. That is, which logic is the correct logic is an analytic matter. The correct logic, it turns out, is \textit{intuitionistic relevant} logic (IR). Its correctness derives from the fact that its rules of inference are exactly faithful to the meanings of the usual logical operators (\( \neg, \land, \lor, \supset, \exists, \forall \)). The meanings in question are those that can be grasped on the basis of observable behavior and whose grasp can be manifested fully in observable behavior. It is a well-known anti-realist contention that those meanings do not underwrite the validity of LEM. This means that the correct logic will be some form of intuitionistic logic. I contend further that this theoretically demanding analysis of meaning reveals \textit{intuitionistic relevant} logic to be the correct logic. That is, the laws of intuitionistic relevant logic (or the inferences permitted therein) are analytic, and, indeed, these laws and inferences are all the analytic laws and inferences there are.

IR results from two lines of reform against the full classical canon. The first one intuitionizes or constructivizes the logic by paying heed to epistemic constraints on truth. Truth is knowable and consists in the existence of an effectively checkable construction, or warrant, establishing the sentence in question as true. This leads one in the direction of an intuitionistic logic as the logic of such constructions.

The second line of reform relevantizes the logic by paying heed to an
epistemic constraint on logical consequence or deducibility. The epistemic constraint in question can be explained very briefly as follows. Suppose that in the process of searching for a proof (call it Σ, say) of a given argument Γ: ψ, we have made some progress and have reached a stage at which we have posed for ourselves the deductive subproblem Δ: φ. We are now looking for a proof Π, say, for this subproblem. Proof Π will then feature as a subproof of Σ. Suppose that we are fortunate enough, subsequently, to find a proof Π′ of a stronger result Δ′: φ or Δ′: ∅ (where Δ′ is a subset of Δ). The Principle of Non-Forfeiture of Epistemic Gain states that we should be able to use the proof Π′ of the stronger result (instead of the proof Π of the weaker result) as a subproof within Σ, without thereby destroying the proofhood of the overall proof Σ itself. Moreover, the overall result obtained by placing Π′ within the wider proof environment Σ that is already built up (on the assumption, of course, that all other deductive subproblems are solved positively) should be a proof of Γ′: ψ or Γ′: ∅, for some subset Γ′ of Γ. That is, the overall result that is established by Σ (when it contains Π′ as a subproof) should be at least as strong (logically) as the would-be result Γ: ψ for which a proof was being sought in the first place. The epistemic constraint on deducibility is that logic’s labor should not be lost. This principle of non-forfeiture uniquely determines IR as the correct fragment of intuitionistic logic.²⁶

§6. ON IDEAL JUSTIFICATIONS

The reforming anti-realist is not so much intent on depriving the realist of his classical tools but, rather, on having the realist acknowledge the true nature of his use of them or of his appeal to them. It is a methodological ideal, going back to Aristotle, that the provenance of any knowledge claim should admit of regimentation. That is, the justificatory pedigree of any knowledge claim should be able to be laid bare in all its details, open to critical inspection and appraisal. Strictly logical steps of inference should be able to be labeled as such; inductive leaps likewise. Sometimes the local appropriateness of a particular step might be legitimated only by appeal to more global considerations about sensitivity to the available evidence, and the use of explanatory foils (as in the case of inference to the best explanation).²⁷ But, so the venerable ideal goes, we should be able to get absolutely clear just what we are up to at each and every inferential step on the way to a new knowledge claim.²⁸

In this context, what I am recommending here is this: The realist should be prepared to identify every so-called “logical” step in any passage of his reasoning that is not, on our strict anti-realist view, analytic and must be prepared to label such steps as synthetic. “Logical” steps that are not really ana-
lytic are (by definition) synthetic; hence, they should be acknowledged as such. If (and only if) this is done, we can always be clear as to how and when the justification of a knowledge claim involves recourse to an essentially metaphysical conception of reality. The importation of a metaphysical conception of reality (such as that of the realist) is something that goes strictly beyond the adducing of observable evidence or the plying of meaning-respecting routes of logical (or mathematical) reasoning. When the realist uses the Law of Excluded Middle on some instance \( P \lor \neg P \), he must be prepared to acknowledge: “Here I presuppose, and give expression to, the metaphysical view that reality is determinate (in the respect \( P \)).”

The benefits of regimentation are not to be underestimated. Many ‘simple’ steps in ordinary classical reasoning, such as, say,

\[
\neg \forall x F(x) \\
\exists x \neg F(x) \\

\neg (A \land B) \\
\neg A \lor \neg B
\]

are compounds of the simpler steps that count as basic in a system of natural deduction. Among the latter are the rules of introduction and elimination of the logical operators, plus—in the classical case—at least one strictly classical rule of negation. In everyday reasoning, which goes unregimented (or insufficiently regimented), the full brunt of the “classicism” borne by the latter negation rules is obscured. It will not strike the logically untutored eye that it is just one isolable kind of strictly classical step that is persistently responsible for the classical nature of the reasoning throughout. Given that negations co-occur with the other logical operators in complex patterns among which we navigate by the cruder likes of the compound moves just displayed above, it is all too easy and inviting to view all the logical operators as imbued or tinged with some imagined “classical ingredient” of meaning. Given that that is indeed so, one can say that the finer analysis of the structure of logical deduction that is afforded by the work of Gentzen has not yet been milked for its final philosophical insights. Among these insights, I contend, is how the contribution of a realist metaphysical outlook can be logically corralled. It can be separated off from the other basic rules, all of which contribute to the exhaustive specification of the meanings of the logical operators.

The realist should not be embarrassed by the above recommendation concerning regimentation. For there should be nothing to hide, and indeed much to be gained, from being forthright and clear about exactly where one’s metaphysical conception intrudes into proffered justifications of knowledge claims. The realist does not shrink from asserting his realism. So why, then, should he be at all reluctant to concede that it would be quite in order to track the epistemic impact of his realism across the justifications of his knowledge claims? Nor, if the realist is willing to do this, should the anti-realist wish, any longer,
to deprive him of anything. For the anti-realist would now be confronted with a range of knowledge claims and their respective justifications in which any realist metaphysical tinctures have openly left their stain. It would be up to the anti-realist to treat these justifications with the requisite caution. Their conclusions could be “bracketed” or could be seen as depending—at least insofar as these stained justifications of them are concerned—upon the metaphysical conception of the realist.

§7. EPISTEMIC ANALYTICITY AND INEXTRICABILITY

Now of course the Quinean will complain at this point that we are underestimating the problem of (alleged) inextricability of belief and meaning. The Quinean will demand watertight behavioral criteria by means of which one will be able to “factor out” the respective contributions of (factual) belief and of (conceptual) meaning to the determination of the truth-value of any sentence. And the Quinean will complain that one cannot, in principle, effect such a factorization: not, at least, if one accepts that all there is to the grasp of meaning is what can be disclosed in observable behavior.

The anti-realist’s response to the Quinean is as follows. The behavioral-manifestation requirement is precisely what leads the anti-realist to his account of the meanings of the logical operators. That account is informed by the further requirement of harmony between the conditions justifying an assertion and the commitments on which an audience can rely, when the assertion is made. This harmony requirement finds natural expression, in the case of any logical operator, in the reduction procedure that is associated with the operator. (It is the repeated application of these reduction procedures that enables one to turn any given proof into one in normal form.) The logic that is determined by the rules displaying such harmony is what turns out to be analytically justifiable. Moreover, we can reasonably require, for attribution of grasp of meaning, that the speaker be able to manifest that grasp fully in observable behavior by the exercise of suitable recognitional capacities. Careful examination of the various ways in which the italicized phrase can be made more precise reveals that one cannot hold both to Bivalence and to manifestationism—at least for any undecidable discourse. Hence, for any undecidable discourse (such as that of arithmetic), the Quinean, by virtue of being a manifestationist, should eschew Bivalence!

As for the objection that the factual and conceptual contributions to the determination of the truth-value of a sentence are inextricable, my reply is that any “extrication” on offer would be a defeasible, interpretative hypothesis. There is a valuable role for such interpretative hypotheses. Like any other high-level scientific hypotheses, they perch gingerly atop a mountain of sup-
porting, but logically inconclusive, behavioral evidence, and they are liable
to be toppled as new evidence accrues. But the distinction they postulate—
namely, a distinction between a semantic contribution from within the lan-
guage and a factual contribution from nonlinguistic, worldly states of affairs
in the determination of a sentence’s truth-value—is one that we would wish
to be able to make in principle. Part of the test of the overall acceptability of
an interpretative theory is just how the sentential chips would fall with respect
to the distinction as finally settled. That there is such a distinction to be made
is presupposed by the very project of interpreting linguistic utterances. That
there might be disagreements as to where, precisely, such a distinction is
finally to be drawn is to be expected from the nature of the enterprise. That
interpreters in disagreement would wish to draw the distinction in different
places should not surprise one, since the precise placement of the distinction
informs the particular interpretation that any one interpreter favors. If one is
in the business of imputing contents (to linguistic utterances and/or mental
states), then one is also in the business of trying to account for when sen-
tences are asserted (or when mental states are occupied) wholly by virtue of
the speaker’s grasp of the contents with which they are laden, and not by
virtue also of some worldly input or opinion. That is to say, the question of
where to draw the analytic-synthetic distinction is firmly on the agenda, even
within the project of radical interpretation. Note also that the Quinean con-
fines himself to the role of the alien observer: one encountering the linguis-
tic community from the outside, having had no experience within it. If,
however, one adopts the standpoint of the reflective participant in a linguis-
tic practice, one will have sharper intuitions about the precise demarcation
of the analytic-synthetic divide.

§8. HOW TO INFLATE THE CORRECT LOGIC
TO OBTAIN CLASSICAL LOGIC

Intuitionistic relevant logic (IR) is properly contained within both intuition-
istic logic and classical logic. Classical relevant logic (CR) is obtained from
IR by adding the Law of Excluded Middle. Full classical logic (C) is then
obtained from classical relevant logic by adding the rule of \(\text{ex falso quodli-
bet}\), according to which a contradiction (or absurdity) implies any sentence
whatevsoever:

\[
\begin{align*}
(\text{EFQ}) & \quad \bot \\
\phi & \\
\end{align*}
\]

Equivalently, one could formulate EFQ as a single metalinguistic principle
to the effect that any contradiction entails any proposition whatsoever.
The system IR is the canon of inference that is based fully on the meanings of the logical operators, and those meanings are exhausted in sustaining just the inferences afforded by IR. The system CR of classical relevant logic can be thought of as a canon of inference that is based partly on the meanings of the logical operators but residually—and very importantly—based on an extremely high-level synthetic principle: the metaphysical principle that reality is determinate. Moreover, the principle is claimed to be known a priori, and it would indeed be a priori if it were valid. In understanding and classifying matters this way, the anti-realist, of course, cannot endorse this principle; indeed, he eschews it. So the principle cannot be a priori for the anti-realist. Rather, the anti-realist has to represent the principle as a priori for the realist. He does so by way of best explanation of the realist's persistent use of the principle in his classical reasoning. For the realist certainly does not treat the principle as, say, an inductive generalization that is based on past experience. Rather, the realist asserts the principle with no justification at all. This is meant here in two dovetailing senses. First, from the anti-realist's point of view, the realist has no justification (and reprehensibly so) for his reliance on the principle; second, the realist, for his part, denies that any justification need or could be provided. Ironically, what Kant said about natural laws could be adapted here:

Even [the principle of determinacy], viewed as [a principle] of the empirical employment of understanding, [carries] with [it] an expression of necessity, and so [contains] at least the suggestion of a determination from grounds which are valid a priori and antecedently to all experience.\textsuperscript{34}

With the principle of determinacy, however, one might urge that there is more than just a suggestion to this effect, so far as the realist is concerned; but that the envisaged determination is degenerate or trivial.

Even for the Kantian, LEM would not need any transcendental deduction of its a priori concepts, for there aren't any a priori concepts specific to LEM! The logical concepts of disjunction and negation are governed by their meaning-specifying introduction and elimination rules, and that is all that needs to be said on that score. Naturally, we are entitled to use those logical operators, and there's the end of it, as far as any transcendental deduction of concepts goes.

Interestingly, however, the same cannot be said for the Principle of Bivalence. The latter principle makes explicit, at the metalevel, what is only implicit in the classicist's use of LEM (via its various instances) in his deductive reasoning. The move to explicitness comes with the introduction of the concept of truth: Bivalence tells us that every declarative sentence is either true or false (equivalently, either the sentence or its negation is true). What, then, about the need for a transcendental deduction of the a priori concept of
truth itself? Fortunately, that would be a problem for the realist who wishes to make matters explicit, not for the anti-realist. But even the realist has the option of refusing to be drawn into the need to make matters explicit. He could, if he wished, stay resolutely within the language governed by LEM. He need not invoke the notion of truth in any attempt to render explicitly, in a single statement, what his allegiance to LEM (through his multiple applications of it) amounted to. The realist who cleaves to LEM without codifying that commitment explicitly in the form of Bivalence is the one who lends himself most clearly to the construal being proposed here: namely, that his use of LEM is expressive of his metaphysical outlook and that within his system LEM should be regarded as having the status of a synthetic a priori principle.

§9. THE STATUS OF EFQ

According to the realist’s principle of determinacy, the truth-value of any declarative statement is determined by reality in advance of our investigations, and that truth-value could attach to the statement quite independently of us and our beliefs and also independently of our available means of coming to know what is the case. To the extent that this account of metaphysical realism seems merely metaphorical, it is Dummett’s suggestion that one take the espousal of the Principle of Bivalence to be the more precise and definitive expression of the metaphysical view in question. Since we are here equating allegiance to Bivalence with the adoption, in one’s logic, of the Law of Excluded Middle, we shall equally regard a priori reliance on the latter law as fully expressive of the realism in question. The system CR is really all that the classicist needs for any of his deductive purposes. He does not need the full system C of classical logic. For we have a metatheorem to the effect that

$$\Delta \vdash_c \phi \iff \text{for some } \Gamma \subseteq \Delta, \text{ either } \Gamma \vdash_{cr} \phi \text{ or } \Gamma \vdash_{cr} \bot.$$  

This metatheorem tells us that, given any classical result, CR can produce that result or possibly an even stronger one (namely, by not using all of the given premisses or, indeed, by showing that some of the premisses are jointly inconsistent). Thus there is good reason to complain that the extra irrelevantist trappings of the full system C serve only to obscure the essential logical connections that even the classicist would want to bring out, between the premisses and conclusion of a classical argument. How are these irrelevantist trappings acquired?

The system C of full classical logic goes even further than CR by adopting *ex falso quodlibet*. This inferential principle is the precise expression of an epistemological view for which I shall now try to articulate a more
metaphorical understanding—parallel to the way that the precise LEM could be rendered, metaphorically, as the metaphysical picture just given above. Figuratively speaking, EFQ says that our thinking about reality is a seamless whole—that any one bad apple will ruin the whole barrel. It warns one that an absurdity encountered or generated in one’s thinking in any one area of cognitive activity immediately spills over to contaminate one’s thinking in any other area. EFQ says that no contradiction is an island. It says that incoherencies cannot be quarantined. EFQ speaks of a true disaster-mentality. It says that every little spill of contaminant will poison the water table of the whole planet and, indeed, of every other planet in the universe. It says that any local muddle in one’s thinking entails the global collapse or trivialization of all of one’s theories concerning all other subject matters. EFQ says that there is really only one inconsistent theory—namely, the whole language.

The opponent of EFQ, however, can point out that it is bad enough to encounter a contradiction, locally, in one’s system of beliefs. Discovering that absurdity lurks in one’s thoughts about some subject matter is in itself sufficient reason to diagnose the problem—locally—and apply an appropriate remedy to one’s thought about that subject matter. It is quite unnecessary to be prompted to undertake such revision by having some epistemic sword of Damocles, in the form of EFQ, dangling above one’s already chastened and willing head.

It is rather ironic that the realist who is given to speaking of carving reality at the joints does not seem to appreciate that with joints there come relatively disjoint domains of things, properties, and facts—domains among which EFQ is not just an inferential license but is inferentially licentious. EFQ strikes one as carrying more of an epistemological import than a metaphysical import. It says something very general about one’s thinking or theorizing, rather than something about reality itself. This is for the simple reason that reality itself (pace Hegel and the latter-day Priest) cannot harbor any contradictions. So EFQ cannot even be interpreted as a universal law about ideal types of states of affairs. If we are looking for a picture, then, to capture the import of EFQ as a logical principle, it would have to be something more like “true thoughts about the world form a seamless web” rather than “reality itself is a seamless whole.”

Determinacy in rebus and seamlessness in cogitationibus are the two pictures rejected by the anti-realist who regards IR as the correct logic on analytic grounds. The metaphysical and epistemological picture that arises from eschewing both LEM and EFQ is that of states of affairs being determine only to the extent that they are, in principle, knowable; and of theories being sufficiently discomfited by the production of a contradiction to warrant immediate and local revision. One can acknowledge the urgent need for such revision without subscribing to the dogma, absurd (though not self-
contradictory!) in itself, that such a contradiction in one’s theory, if left untouched, would entail epistemic commitment to any and every statement within that local discourse and within every other discourse, no matter how disconnected or disparate its subject matter may be.

EFQ is not analytic. Any truth which is not analytic is synthetic. True synthetic principles tell us something about the world. Thus EFQ, if true, tells us something about the world. But we have just satisfied ourselves that EFQ is really an epistemological principle, which tells us something about the structure of supposedly rational thought about the world. The conclusion that is invited is therefore that the proponent of EFQ is committed to including the structure of rational thought as itself part of the world. This would be once again to see EFQ as a metaphysical principle.

But the opponent of EFQ says that, far from being true, EFQ is false; indeed, he would like to say that it is necessarily false. Isn’t it just patently obvious that it is not the case, on a pre-formal and intuitive understanding of entailment, that any contradiction entails every proposition whatsoever? This should be an adequacy constraint on any formal explication of the notion of entailment and on any formulation of a system of deduction that is intended to do justice to the pre-formal and intuitive notion. Accordingly, one can maintain that there are indefinitely many straightforward counterexamples to EFQ. An inconsistency in one’s thought about Julius Caesar—one which, say, implies that he had sixteen teeth on his death and implies also that he had seventeen teeth on his death—cannot entail commitment to the view that the moon is made of Camembert or that the last television commercial beamed out before the Earth explodes will be from Coca-Cola. This makes EFQ, in addition to being a red herring, a very different kettle of fish from LEM. For LEM, while not necessarily true, is immune to counterexample. That is why the anti-realist has to be careful simply to avoid commitment to LEM and not to deny it.

§10. SOME IRONIES AND A NEW UNDERSTANDING

I would like to point out some ironies that have emerged from this discussion.

First, the very behaviorism in the theory of meaning that is urged by Quine is expressed by the Dummettian anti-realist in the form of the manifestation requirement. This requirement leads one to embrace the analytic-synthetic distinction rather than to reject it.

Secondly, one of the favorite, paradigmatic examples, for a Kantian, of synthetic a priori principles—viz., LEM—turns out to be synthetic.

Thirdly, many of the paradigmatic examples, for a Kantian, of synthetic a priori principles—such as “7 + 5 = 12”—turn out to be analytic.

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Mathematical intuitionism, given its Kantian roots, could well be expected to be synthetic; but, for the Dummettian, intuitionistic logic at least and arguably also of mathematics (such as Heyting arithmetic, that is, the intuitionistic Peano-Dedekind theory of the natural numbers) turn out to be analytic. It is the strictly classical residue of classical logic which, for the Dummettian, turns out to be synthetic!

The final irony is this: If the argument given here succeeds, we shall have extricated, in the case of LEM, the (anti-realistically licit) strand of meaning from the strand of (metaphysically realist) belief motivating LEM’s acceptance. And we shall have done so in the case of a principle which for the Quinean—with his quietist acceptance of classical logic—would have been analytic if any principle is capable of being so.

§11. RECONSTRUING THE DISPUTE BETWEEN
THE CLASSICIST AND THE INTUITIONIST

To the extent that Kant wanted to find a proper role for metaphysics by insisting on the existence of synthetic a priori principles, he missed a great opportunity by not singling out LEM as prime among these. (But of course, in his day a proper understanding and systematic formulation of classical logic, especially as a system of natural rules of inference, was not yet available.) LEM is much more interesting than any particular mathematical truth, such as “7 + 5 = 12.” It has a high-level and very general import, and on reflection it strikes one as one of the most “metaphysical” principles one could countenance. It does not require, for the explanation of its a priori status, any of the further controversial doctrines about the respective roles of the sensibility and of the understanding; nor does it involve maintaining any dogma about a scheme-content distinction.

The dispute between the realist and the anti-realist, we have suggested, can be alleviated by having the realist acknowledge, when using LEM, that it expresses an a priori metaphysical conviction on his part. It is not analytic, and of course it cannot be thought to be held a posteriori, given the nature of its use in classical reasoning. So it is synthetic a priori, if universally true. And, strangely enough, though he swears no allegiance to it, the anti-realist cannot, for his part, claim that any instance of LEM is false.37

Finally, regarding LEM as synthetic a priori provokes an interesting and far-reaching reconsideration of the exact nature of the dispute between the realist and the anti-realist (or between the classical mathematician and the intuitionistic mathematician) over the correct choice of logic. Dummett would have it that the classicist wants to regard the intuitionist as one who treats the logical operators as having unusual, “constructive” meanings,
which are distinct from their more usual “classical” meanings. Moreover, according to Dummett, the intuitionist would object that there just aren’t any classical meanings of the kind that the classicist imagines he is able to attach to the logical operators.38

According to Dummett, the classicist charges the intuitionist with changing the subject matter, while the intuitionist charges the classicist with consorting with incoherent, non-existent meanings.

We are now in a position to see that both of the charges that Dummett represents the two sides as leveling are wrong. On the view defended here, there is no “change of subject matter” (concerning the “logical meanings” of the operators); nor should the classicist be denied the meanings actually borne by the logical operators, even as he presses them into strictly classical patterns of inference. For we have seen that there really are only the meanings answering to the anti-realist’s constraints, meanings that legitimate exactly the system IR. It is those meanings in which the classicist is trafficking even as he plies his strictly classical routes of reasoning. And it is for that very reason that the strictly classical moves such as LEM (or Dilemma) give expression to a realist metaphysical conception of the relationship between graspable propositional contents and the world. It is the anti-realist’s instantiating contents P for which the realist’s assertion of instances P ∨ ¬P of LEM is, synthetically for the realist, telling us something about the world. In propounding the determinacy of reality, the realist is not investing the logical operators ¬ and ∨ with any further “classical” ingredients of meaning. Rather, the realist is making a synthetic a priori claim about reality itself, appropriately aided thereto by the anti-realistically licit meanings that are bestowed on the logical operators by the workings of the other logical rules besides the classical negation rules. Thus, it would be philosophically more insightful to parse this last phrase as “classical (negation rules)” rather than as “(classical negation) rules.” The “classical (negation rules)” (and all the other strictly classical compound moves to which they give rise) do not reveal in, or exploit from, or confer upon the logical operators any extra, supposedly “classical” ingredient of meaning. Rather, applications of strictly classical rules make only grand metaphysical claims about the determinacy of reality, and indeed they do so by virtue of having only such meanings as are recognized by the anti-realist, and are adequately codified in IR.

We have now attained a new state of understanding of the realist’s deductive ways. Much of his deductive meandering is based on meaning (as is that of the anti-realist); but a separable and easily identifiable part of the realist’s why’s and wherefore’s is based on (or gives expression to) a strictly metaphysical conception of how things determinately are. It is this metaphysical conception that clinches the dichotomous nature of his logic. One must assume that if disabused of that conception, he would refrain from giving it
expression in his reasoning. He would cut back to using only the canon of IR. And in doing so, he would be using the logical operators with precisely those meanings that they had borne all along. It was those meanings plus the now relinquished metaphysical conception that inflated the canon so as to include strictly classical moves. Those moves were based on a realist metaphysics, not on “classical meanings.”

§12. WHAT IF . . . ?

It might be amusing and instructive to ask, “What if the above neo-Dummettian considerations had been available to Kant? How might subsequent philosophical developments have turned out?” Let us call the characters influenced by these considerations Kant∗, Frege∗, etc. We are to imagine a rather prescient contemporary of Kant∗, called Mikhail Dommatt, who teaches Kant∗ a few of the things available to Dummett.

My tongue-in-cheek conjecture is that Kant∗ would still have insisted on the importance of the synthetic a priori. He would have included LEM alongside real analysis and Euclidean geometry. He would have been excited to learn from Dommatt about advances, since Aristotle, in propositional logic. Dommatt could have imparted to him the Gentzen-style rules of natural deduction for (intuitionistic) propositional logic, without teaching Kant∗ anything about quantifiers.

Frege∗ would in due course have sought to reduce what we now think of as Heyting arithmetic to his (intuitionistic) class theory. And Frege∗’s analysis of quantification, and discovery of first-order (intuitionistic) logic, would have earned him his influential place in analytical∗ philosophy.

Russell∗ would still have discovered his paradox in Frege∗’s system, for its proof is entirely intuitionistic. (Frege∗, by the way, might also have pointed out that his method of reduction, if successful, could be extended to reduce classical arithmetic to classical class theory, though both would be tinged by the synthetic a priori status of LEM.) Russell∗ would have furnished his theory of types, in both intuitionistic39 and classical forms, in order to avoid the paradox in Frege∗’s system.

The early Wittgenstein* would have written his Tractatus Logico-Philosophicus* under Russell∗’s direction, but would have had to invent possible world semantics instead of the relatively trivial truth-tables, thereby earning himself a much more impressive reputation as a logician. The later Wittgenstein* would not have written anything, having been depressed to find that all his best ideas had been anticipated in the work of a relatively obscure contemporary of Kant∗’s named Mikhail Dommatt. Ludwig* would have stayed in Norway and bought it.50
The Logical Positivists*, in rejecting all forms of the synthetic a priori, would have had to reject that most famous form, the Law of Excluded Middle. Carnap* could still have written "Weg mit der Metaphysik!" in his notes for his Bauhaus lecture back in the late 1920*s. Popper* would have claimed that there is no such thing as induction and that the only important feature of scientific theories is their falsifiability. He would have pointed out, quite correctly, that LEM is superfluous and that it does not increase our ability to subject scientific theories to the strongest possible tests. But as the relations of confirmation and support between evidence and hypotheses were paid more attention (pace Popper*), Logical Positivism* would in due course have become anti-realist Logical Empiricism.

Gödel* would have shown (within an intuitionistic metatheory) the undecidability and essential incompleteness of both Heyting arithmetic and classical (Peano-Dedekind) arithmetic. Tarski* would have put forward his justly celebrated account of the concept of truth in formalized languages, also using only an intuitionistic metatheory. He* would have shown that his theory of truth meets his adequacy condition without ever having to appeal to LEM. He* would also have shown that the notion of arithmetical truth is not arithmetically definable (and he would have added a footnote to the effect that this result cannot be circumvented by “going classical” with one’s arithmetical theory).

A precocious and enterprising young American called Quine* would have come to Prague in the early 1930*s to visit the already famous Carnap*, and he would in due course have presided over the merging of the American pragmatist tradition with that of the Logical Positivists*. He would have written an influential logic text called Methods of Logic*, in which (so some* would later say) he regretfully over-emphasized intuitionistic theoreomhood at the expense of intuitionistic deducibility. But many* would have been dazzled by his thesis of the Radical Indeterminacy of Translation. His masterpiece, Word and Object*, would have “established” such indeterminacy, even while conceding that a principle of charity requires us to read our own intuitionistic logic into the language of the jungle-dwellers. Quine* would also have been credited with the acid observation that the small minority of logicians who want to use LEM run the risk of “changing the subject matter.”

Church* would have collapsed Ayer*’s criterion of cognitive significance in his Journal of Symbolic Logic* review of Language, Truth, and Logic*, using only intuitionistic logic to do so. There would have been further attempts to provide a criterion, notably an attempt by Carnap* in 1956*. But it would have been unjustly neglected in the mood of pessimism summarized by Hempel* in an influential article entitled “Problems and Changes in the Empiricist* Criterion of Cognitive Significance.”*

As the Gödel*-Tarski* results sank in, philosophers* would have tempered their rejection of the synthetic a priori status for all truths of arithmetic.
Perhaps, they would think, the independent Gödel sentences, at least, are synthetic, for in their proofs (when formalized at the metalevel) we* need recourse to a truth predicate that occurs in new instances of the axiom schema of induction. Yet there is no occurrence of such a predicate in the independent sentence itself; so how, then, could its truth be grasped on the basis of its meaning alone? But they would not have reconsidered their eschewal of Bivalence and the Law of Excluded Middle, since by now they would have been convinced that LEM is epistemologically otiose.

Then, as the emphasis in American universities reverted to the history of eighteenth*- and nineteenth*-century philosophy*, the works of the little-known Mikhail Domnatt would have come to light. Philosophers* would have discovered therein the antidote required for the meaning skepticism of Quine*. The analytic-synthetic distinction would have been reinstated, and new attempts would have been made to explicate the important notion of cognitive significance.

As it happens, such developments* would have culminated in more or less the view that I think is correct now. And we would have reached it even after having sent Michael Dummett in a time machine back to Königsberg in 1775. Why, then: it must be correct!

NOTES

1. I am indebted, for helpful comments on an earlier draft, to Alan Code, Vittorio Höele, Geoffrey Keene, Robert Kraut, Joe Salerno, and Stewart Shapiro. I am grateful also to Christopher Hill for his helpful editorial suggestions and requests for clarifications of certain points.

2. On Dummett’s characterization, semantic realism (for a discourse) is the view that every declarative sentence of the discourse is determinately true or false and that this can be so, for a given sentence, independently of our means of coming to know what its truth-value is. This is the brand of realism I shall be considering in this paper. The issue at hand is how we should best taxonomize its logical and epistemological ramifications.

3. Note that not all rules are necessarily meaning-specifying rules. The anti-realist’s theory of harmony and separability places quite strong constraints on what can serve as a meaning-specifying rule. It is by no means the case that any old set of axioms “implicitly defining” the notions involved in them count as meaning-specifying. (I include axioms as a special case of rules: they are rules with no premisses.)

4. Prospect (i) has been explored in my The Taming of the True (Oxford: Oxford University Press, forthcoming), ch. 9.

5. This claim is hinted at in only one of Dummett’s writings, so far as I have been able to determine; but it remains undeveloped, and its consequences unexplored. In “Frege’s Distinction between Sense and Reference,” in Truth and Other Enigmas (Cambridge, Mass.: Harvard University Press, 1978), Dummett writes:

[Frege’s] first fundamental idea is that the condition for a complex sentence to be true depends solely upon its composition out of atomic sentences. . . . This idea is not seriously challenged, at least by anyone
accepting a Fregean syntax. . . . [T]he second fundamental principle of classical [Fregean] semantics is that the condition for the truth of each sentence is, determinately, either fulfilled or unfulfilled. We can regard this as a metaphysical assumption—an assumption of the existence of an objective reality independent of our knowledge. We can, equally, regard it as an assumption in the theory of meaning, namely that we succeed in conferring on our sentences a sense which renders them determinately true or false (120–21).

Note that Dummett would himself appear to favor the view that bivalence is an assumption in the theory of meaning, rather than being a more straightforwardly metaphysical claim.

6. Robert Kraut has objected that no argument has been offered for this possibility. Here, then, is an argument of the kind that is required: The sentence “If Kraut is right, then Kraut is right” does not depend on the world for its truth-value. Here is another argument: “If Kraut is wrong, then Kraut is wrong” does not depend on the world for its truth-value. Kraut should now see that there is, in fact, an infinity of arguments establishing the possibility claimed.


8. Ibid.


10. Notoriously, however, Frege’s definition fails to confer analytic status upon ‘All red things are colored’ and ‘All bodies are extended’.

11. It was Vittorio Hösle who made me mindful of this danger.


14. I owe this example to Stewart Shapiro.


17. Anyone who maintains that first-order logical truths are analytic is already committed to the view that analytic statements need not be decidable or obvious. But it is decidable and obvious that analytic statements need not be free of existential commitments either: consider the analytic claim that the number of non-self-identical things is zero. In *The Taming of the True*, ch. 9, an account of analyticity is developed that warrants all of these claims.

18. Throughout this discussion we make no mention of the (usually synthetic and a posteriori) status of empirical laws or of those among them (such as the law of causation) that Kant reckoned to be synthetic a priori. We want to show the variety of views on offer regarding only the status of mathematics and logic.

19. For a detailed discussion of the vicissitudes of classification and of the tenability of the analytic-synthetic distinction itself, see my *The Taming of the True*, ch. 9. In this brief sketch, I am perforce ignoring the subtlety of distinguishing between formal and physical geometry, and I am ignoring also the fate of other Kantian synthetic a priori principles besides those of arithmetic and geometry, such as the Principle of Causation.

20. I am grateful to Alan Code for eliciting this clarification.

21. Stewart Shapiro has asked (personal communication), “On what grounds can the realist claim to know the Law of Excluded Middle a priori?” The fact that he believes the world to be determinate does not give him license to claim that the law can be known
independently of experience. “But we need knowledge or knowability to get apriority.” That, I would want to say, is the realist’s problem, not mine. The realist may in the end have to settle for saying something like, “Look, the spade simply turns with the Law of Excluded Middle, and I know it.” The imaginative work being done here on behalf of the realist (or, at least, in the interests of a better understanding of what realism amounts to) is aimed at securing classical principles such as the Law of Excluded Middle a “most favored possible status.” If an objector wished to suggest that the law should be regarded, rather, as a posteriori rather than a priori, well then—well and good! The cards would then at least be out on the table, and the strictly classical parts of logic would no longer be among the face cards.

22. I owe to Stewart Shapiro the observation that LEM is equivalent to Biv if the Tarskian T-sentences are assumed. So the supervaluationist has to demur from the latter.

23. Stewart Shapiro has suggested (personal communication) that “LEM is largely epistemic. It does not (only) say that the world is determinate but that it is cooperative. Every proposition is either knowable or refutable.” My response here would be: Yes, but what do you think it is about the world that is supposed to make the latter epistemic claim true? Wouldn’t it be precisely the alleged determinacy of the world? (One has to bear in mind here that “knowable” means “knowable in principle,” which involves a certain amount of idealization.)

24. An impression of impudence in making this claim might be dispelled by any Anti-Realism and Logic and The Taming of the True.

25. This is not the occasion to rebut the classicist’s objection that of course he can manifest his grasp of LEM as an allegedly logical truth. For the difficulties to be faced by the classicist on this score, the reader is referred to my Anti-Realism and Logic.

26. For more details, see The Taming of the True, ch. 10.

27. See the illuminating discussion in Peter Lipton, Inference to the Best Explanation (London: Routledge, 1991).

28. This requirement of “perfectibility of proof” applies as much to a coherentist epistemology as it does to a foundational one. For even the coherentist is appealing to the logical interrelatedness of sentences in her system when showing that all her beliefs cohere. And if she needs classical logic to do this, then she is basing coherence itself, at least in part, on the assumed determinacy of the world; so even she should think again about the metaphysical implications of her methodological norms.

29. Remember that it is a consequence of our re-fashioning of the analytic-synthetic distinction that a great deal of mathematical inference is revealed as analytic. Indeed, we even allow that an analytic truth might entail the existence of certain entities, albeit only necessarily existing entities. For more details, see The Taming of the True, ch. 9.

30. Along with the Law of Excluded Middle, there are other, equivalent, classical negation rules whose application would call for a similar acknowledgment. They are the Law of Double Negation, the rule of Classical Reduction, and the rule of Dilemma.

31. Although Brandom has picked up on the harmony requirement in his book Making It Explicit (Cambridge. Mass.: Harvard University Press, 1994), he does not follow through, as Dummett does, on the implications this may have for logical reform. Brandom remains a quietist about classical logic, for want of a more detailed examination of what the requirement of harmony, properly understood and prosecuted, entails.

32. For a fuller account of harmony, reduction, and constructive validity of arguments, see my Anti-Realism and Logic.

33. For a detailed argument for this claim, see The Taming of the True, ch. 7.

34. Kant, op. cit., B198.

35. It is not logically possible for a world to contain a contradiction. But ideal types of states of affairs—ones containing, say, perfectly spherical solids, frictionless and perfectly plane surfaces, perfectly straight and rigid rods, etc.—are logically, even if not physically, possible.
36. Thus, one may be totally unmoved by objections to the effect that such and such well-known modal or model-theoretic definition of logical consequence has EFQ as a necessary and “idealizing” consequence concerning logical consequence itself. EFQ will always be found to be at work at the meta-level in these “justifications” of EFQ at the object level.

37. For a more careful canvassing of the possibilities that are open to the anti-realist with respect to Bivalence, however, see The Taming of the True, ch. 6.


The affront to which the [intuitionist’s] challenge [to fundamental accustomed modes of reasoning] gives rise is quickly allayed by a resolve to take no notice. The challenger must mean something different by the logical constants; so he is not really challenging the laws that we have always accepted and may therefore continue to accept. . . . [The intuitionist] denies that [the classical mathematician] has hold of a coherent meaning, . . . He acknowledges that he attaches meanings to mathematical terms different from those the classical mathematician ascribes to them; but he maintains that the classical meanings are incoherent and arise out of a misconception on the part of the classical mathematician about how mathematical language functions. Thus the answer to the question how it is possible to call a basic logical law in doubt is that, underlying the disagreement about logic, there is a yet more fundamental disagreement about the correct model of meaning, that is, about what we should regard as constituting an understanding of a statement (17).

39. He would thereby have made Martin-Löf* somewhat superfluous in this brief history*.

40. Possible world semantics would have been invented by the early Wittgenstein*. There would have been no later Wittgenstein* to write about rule following. Ignorance prevailed (in the period between 1960* and 1980*) of the works of Mikhail Dommat. Thus the character Kripke* did not need to be invented.