NEIL TENNANT

FORMAL GAMES AND FORMS FOR GAMES

1. AMBIGUITY AND LOGICAL FORM

The sentence

(1) John saw her duck

is one of those clear cases of ambiguity that can be decided by intuition alone. It is associated with two logically independent interpretations or senses. These are, roughly,

(1.i) John saw her bend quickly downwards
(1.ii) John saw her water-bird.

Interestingly, it is not just the lexical ambiguity of “duck” which enters here. There is also structural ambiguity. Precisely what structures are involved will be a matter for theoretical discussion, but it is clear that the difference is that between John’s seeing her doing something, and his seeing something of hers.

The assignment of senses to sentences is not one-to-one. Consideration of active and passive sentences, and (1) above show us that the assignment is many-many. Every student learning how to translate English sentences into logical notation discovers this early on.

We also learn how limited are the resources of expression in standard first order logic by comparison with English. There are clear, unambiguous English sentences that cannot be translated into the language of first order logic. We recognise this in our talk of ‘fragments’ of English which enjoy the metamodality of logical parsability. Moreover, it is not just a case of lacking structurally similar logical translations, with inverted iotas instead of Russellian quantified complexes doing duty for definite articles. The limitations of first order logic are more serious than that. The problem is that, even allowing for such excesses as a million occurrences of an existential quantifier to cope with one occurrence of the word “million”, there are sentences that cannot be translated into first order logical notation at all.

This is so even with a minimal constraint upon such translation: that the logical translation and the English sentence have the same truth-conditions (where for simplicity I am assuming the English sentence to be unambiguous). There are unambiguous English sentences whose truth-conditions cannot be captured by any sentence of (existing) formal languages.

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Such sentences inspire the invention of systems to deal with them. There are many in the literature: systems for plurality quantification, restricted quantification, higher order quantification, the generic "the", quantification into opaque contexts, and so on. Not only quantification calls for increased scope in this way. Mass nouns, adverbs, attributive adjectives, modalities and many other grammatical or logical categories are fast being provided formal semantics or logics under one "programme" or another.

This raises the *prima facie* possibility that we may find an English sentence one of whose senses can be captured by translation into an existing formal system, but another of whose cannot. But this would not be a ground for refusing to acknowledge the ambiguity. Another possibility is that we may find an ambiguous sentence none of whose senses can be so captured, but whose senses have a common logical consequence which can be. But this would not be a ground for refusing to acknowledge the ambiguity, and recognizing only the common logical consequence as the sense of the sentence. The proper response in both these cases must be to try to refine and extend existing formal systems so as eventually to provide distinguishing representations for the intuitively competing senses of the ambiguous sentence.

Thus far I assume agreement among logicians and linguists. But within the perspective provided above we can anticipate a divergence between them. This lies in what they respectively regard as the "existing formal systems", and what they take these to be designed to achieve. Take, for example, sentence (1). A modern logician would probably be happy in offering a logical translation only for sense (1.ii). Given the present state of development of his formal systems, he is unlikely to cope with sense (1.i). The logic of perception and of events, and of perception of events is still in its infancy. The logician should shy from sense (1.i), aware that the adequacy of any particular representation depends on the overall adequacy of the system of representations for sentences in the English fragment to which (1) relevantly belongs. The adequacy of the whole system is judged, basically, in two complementary ways. The first is that each logical representation of the (or a) sense of a sentence must capture the appropriate truth-conditions. It may do, perhaps, as an input for the optimal generation of a Tarskian biconditional in a theory of truth in the well-known way. The second is that the logical representations, with their truth-conditions duly determined, should be correctly tethered in the field of logical consequence, that relation depending on their internal structure alone. The correct connection between ambiguity and entail-
ment is to be found in our readiness to multiply considerations of the form "If one takes S on such-and-such a reading, it has so-and-so entailments. On the other hand, if one takes S on this other reading, it will have these other entailments..."

That, briefly, is how a logician regards his formal systems. For a linguist, however, matters may be different. The structures he attributes to sentences are those relevant to his own theoretical concerns, the paramount one in the current paradigm being the rule-governed generation of "surface sentences" from "deep structures" for the English fragment in question. The selection of phrase structure rules and transformations and the basic categories of expression for the grammar will provide further explanation of such matters as grammatical function, degrees of grammaticality and so on. In this model we are supposed also to find an explanation of ambiguity. If two distinct deep structures map to the same surface structure then the latter will be ambiguous.

The deep structures provided in the grammar therefore held out a certain semantic promise. It was not long before generative semanticists suggested that these origins of transformational pedigrees should coincide with logicians' formal representations of the relevant senses of sentences.

If generative semantics is correct, we all have changes to make. The linguist has to produce deep structures that can be assigned truth-conditions and ordered in the relation of logical consequence. The logician, reciprocally, has to provide formal representations with enough grammatical detail to allow them to surface. If, however, linguists and logicians collaborate in this way they will be running a three-legged race, labouring under the combined adequacy constraints upon their theorizing.

With this word of methodological caution I shall look at some of Kempson and Cormack's proposals in due course. But first I would like to locate the concept of ambiguity in a general theoretical setting. To begin with, some notions and notations.

English sentences S will be the kind of thing which suffer from ambiguity if anything can. Ambiguity will be between (or among) different readings R₁,..., Rₙ of S. Readings will also variously be called senses, meanings or interpretations. These are the kind of thing too much of which brings on an attack of ambiguity. A context C for S is determined by the intentions and beliefs of all concerned, the surrounding text, and so on. S might be pre-contextually ambiguous among R₁,..., Rₙ. A context C might eliminate some of these readings. A theory of ambiguity will be concerned with how this takes place. S in
context might still be ambiguous among some of the \( R_i \). I shall then call \( S \) thoroughly ambiguous. Each competing reading of \( S \) in context \( C \), with \( C \), determines a proposition \( P \) – the proposition expressed by \( S \) on that reading in context \( C \).

I shall regard logical forms of sentences as corresponding to, or capturing, readings. They are theoretical devices which determine what propositions are expressed in given contexts. Thus we shall use the notation \( R_1, \ldots, R_n \) for logical forms.

To summarize these ideas consider the following schematism.

\[
\begin{array}{c}
R_1 \\
S + C \\
R_i \\
\vdots \\
R_n \\
\end{array}
\]

Pre-contextually \( S \) may be ambiguous among \( R_1, \ldots, R_n \). Once given context \( C \) it may then be ambiguous among the chosen fewer \( R_i, \ldots, R_j \). These readings, in context, then determine respective propositions \( P_i, \ldots, P_j \). We must either account for pre-contextual selection of readings or radically redescribe our understanding of sentences in context and the point of assigning sentences logical forms.

Logical relations are properly understood as holding between the logical forms, or readings, of sentences. An argument is assessed for validity on the assumption that the context is held constant for the premisses and conclusion. Thus we may have a valid argument whose premisses and conclusion are highly context dependent – involving, say, indexicals – the validity of which is evident even though, since we are not considering its premisses and conclusion in any particular context, we are therefore not considering any particular propositions that they express. As a simple example consider the valid argument

\[
\text{I am a richer person than he} \\
\text{The richer one is, the more heavily one is taxed} \\
\text{I am taxed more heavily than he is.}
\]

We can see that it is valid without considering any particular propositions expressed by the premisses and conclusion in any particular context.

To illustrate our schematism above, consider once more the ambiguous sentence “John saw her duck”. Call the readings given above \( R_1 \) and \( R_2 \). Suppose the context \( C \) contains the information that the parties
discussed were playing cricket in the Australian desert. Then we select $R_1$, the reading concerning downward movement, or – as I have only now discerned! – another reading $R_3$ to the effect that John saw her dismissed without scoring. Indeed, in this context $S$ may remain ambiguous between $R_1$ and $R_3$. For the thrown ball which she ducked to avoid might have been the one which hit her stumps to bring her innings to a scoreless close. Suppose now that the context $C$ were different, containing the information that the only woman under discussion was a sad figure in orthopaedic irons who doted on members of a menagerie of water-fowl tended by the local vet John. I think this would incline us to $R_2$. Once context has helped us to select certain readings, further information available in context, such as the reference of “John” and the feminine demonstrative pronoun then determines the propositions respectively expressed by the sentence on those readings.

2. Kempson and Cormack's Proposals

Kempson and Cormack would have us believe that if the only possible readings of $S$ are $R_1$ and $R_2$, and $R_1$ entails $R_2$, then $S$ is not ambiguous between $R_1$ and $R_2$, but has a single logical form. Their chosen logical form is (that for) $R_2$, the logically weaker reading. In general, their proposal is that where there is a minimum for the partial ordering by entailment of competing readings, “the” reading is that minimum, the logically weakest one. Presumably then the others, if demanded by context, can be recovered from “the” reading by suitable procedures.

One such procedure might be appropriate conjunction. Consider, for example, a free logic in which failure to denote renders atomic prediction false, and singular terms do not have scope. The English sentence

\[
a \text{ does not love } b
\]

has “the” reading corresponding to external negation:

\[
-Lab
\]

and the other readings corresponding to internal negation are recovered by conjoining existence claims with the external negation:

\[
(-Lab) & \exists!a
\]
\[
(-Lab) & \exists!b
\]
\[
(-Lab) & \exists!a & \exists!b
\]

What, however, happens when the partial ordering by entailment of the competing readings has no minimum, no weakest member entailed by
each of them? Then the question of the procedures simply would not arise. But even when we can make some sort of canonical choice of "the" minimum, the conjunctive procedure is severely limited. Consider the sentence

Everyone loves someone.

According to the theoretical sketch above, it is at least pre-contextually ambiguous. If we follow Kempson and Cormack in assigning this sentence the single logical form \((\forall x)(\exists y)L_{xy}\), how are we to represent the reading imposed by Cresswell’s context:

In that country all the people love their president. Isn’t it frightening to be in a place where everyone loves someone?

The strategy of conjoining something extra to the minimal reading does not work here, for there is no way of representing "and it’s the same person in each case" coherently as a further conjunct in logical notation. Kempson and Cormack indeed propose other procedures besides conjunction, about which I shall have more to say below.

Let us for the moment return to the other problem mentioned above. They seek a single logical form for ambiguous quantified sentences even when their proposed competing readings are partially ordered by entailment in such a way that there are local minima but no overall minimum. How is this single logical form to be determined? They give no general guidance here. But they do believe they can give a single logical form for a double quantification such as

(2) Two examiners marked six papers.

This logical form is to be compatible with the two scope-differentiated readings and with the complete and incomplete group interpretations, and is to be separately entailed by each of them. Now on a classical account of entailment one such logical form immediately leaps to mind: \(A \lor \neg A\!\). So perhaps their conditions are necessary but not sufficient. In seeking a single logical form for a sentence \(S\) with purportedly competing readings \(R_1, \ldots, R_n\) we need more than that \(R\) is consistent with and entailed by each \(R_i\). We must demand further that whenever \(R\) is true then at least one of the \(R_i\) is true. The obvious choice for \(R\) would appear to be the disjunction of the \(R_i\), but as they point out, this gives the wrong results when one considers negation. Instead, for (2) Kempson and Cormack propose the logical form

There are at least two examiners and at least six scripts, and at least one of the former marked at least one of the latter.
(Compare their symbolic XIII.) Now suppose the world is such that the words "at least" can be replaced throughout by "exactly" in the logical form given, so that it is true that

There are exactly two examiners and exactly six scripts, and there is exactly one examiner and exactly one script such that he marked and it was marked.

In these circumstances (2) is false on each of the competing interpretations—the two scope-differentiated ones, and the complete and incomplete group readings. So their logical form for (2) simply fails to capture the right truth-conditions. For mixed numerical quantification in general, Kempson and Cormack propose as the logical form of

\[(1) \quad mA's \ R nB's\]
a second-order formula which reads

\[(2) \quad \text{There are at least } mA's \text{ and at least } nB's \text{ and at least one of the former } R's \text{ at least one of the latter.}\]

Then, by the proposed "procedures" of "generalizing" (that is, turning an existential quantifier into a universal) and "uniformising" (switching from $\forall \exists$ to $\exists \forall$) they can "generate" from this formula various others corresponding to the different readings of (1). The following extracts give the flavour of their leading idea:

an analysis of mixed numerical quantification should be in terms of a weakly specified semantic representation which is common to each of the possible distinct interpretations of any such sentence
there should be a stateable procedure for relating these
all sentences of a form corresponding to ("Two examiners marked six papers") have a single logical form, which is then subject to the procedures of generalising and uniformising to yield the various interpretations of the sentence in use
the more detailed interpretations are derived by a set of procedures applying variously to that semantic representation
given the incomplete group interpretation of sentences with two numerical quantifiers, we can predict the existence of the further interpretations by general rules.

But as they concede, "The doubt about this analysis is the status of the procedures themselves". And here I wish to disabuse the reader of any belief prompted by footnote 54 that Kempson and Cormack's "procedures" bear any sort of analogy to game-theoretical moves that I shall describe below. Their procedures simply bash one well-formed formula
of higher order logic into another. Given the resources to generate any one such formula, of course we can "predict the existence" of others. But the mangling of meanings by the procedures makes nonsense of the claim to have found a logical form, or semantic representation "common to each of the possible distinct interpretations". Kempson and Cormack are playing linguistic checkers with a logician's chess set. Despite the drawback of having to account for how, in general, we pass from the surface simplicity of (1) to a complicated conjunction like (2), Kempson and Cormack's proposals suggest extension as a cognitive model for other domains. We might, for example, appeal to the procedures of melting down and re-casting in order to explain inter-dependencies between the bronzes of Donatello and those of Henry Moore.

In more muted sympathy with their project, however, I have a suggestion to make. Kempson and Cormack entertain the possibility that we may have to consider context for more than is usually required, along with the structural and lexical information in a logical form, for the interpretation of an utterance assigned that form. For them, "the proposed semantic representation of the sentence cannot be its total semantic content." My suggestion, similarly, is this. Let (1) have the strikingly parallel but presently rather uninformative "logical form"

\[(3) \quad mx(Ax)_{\neg xy} \quad ny(By)_{Rxy}\]

(which, one hopes, would be an easier starting point for grammatical transformations). Then let context determine (in ways yet to be analyzed) which language game for mixed quantification the utterer of (1) is prepared to play on (3) in defence of his assertion. It is the choice of game that determines the unique interpretation of (3), and hence of (1), to which it is assigned.

I shall describe some of the games (as determined by their constitutive rules) that can be played on (3). In each of them, the existence of a winning strategy for player T is the condition for (3)'s truth on the interpretation determined by the game. Play proceeds by unravelling the sentence (3) until \(Rxy\) is reached and individuals have been assigned to the free variables \(x\) and \(y\). Atomic satisfaction means a win for T. \(x\) and \(y\) below range over \(m-\) and \(n-\)membered sets respectively.

**Game 1.** T chooses a set \(\tilde{x}\) of A's (\(F\) could then challenge the A-ness of any member of \(\tilde{x}\), but for the sake of simplicity we shall assume here and henceforth that he waives his right to enter such sub-games, and gets on with contesting the main predication). T chooses a set \(\tilde{y}\) of B's. \(F\) chooses some member \(d\) of \(\tilde{x}\) and some member \(e\) of \(\tilde{y}\).
A winning strategy for $T$ in this game correlates with the truth of (3) on the complete group interpretation.

*Game 2.* $T$ chooses a set $\bar{x}$ of $A$'s. $F$ chooses some member $d$ of $\bar{x}$. $T$ chooses a set $\bar{y}$ of $B$'s. $F$ chooses some member $e$ of $\bar{y}$.

A winning strategy for $T$ in this game correlates with the truth of (3) on the scope differentiated reading where $mx$ has wide scope. A similar game gives the reading where $ny$ has wide scope.

*Game 3.* $T$ chooses a set $\bar{x}$ of $A$'s, and a set $\bar{y}$ of $B$'s. $F$ chooses either some $d$ in $\bar{x}$ or some $e$ in $\bar{y}$. $T$ respectively chooses some $e$ in $\bar{y}$ or some $d$ in $\bar{x}$.

A winning strategy for $T$ in this game correlates with the truth of (3) on the incomplete group interpretation. Clearly we can devise many games by varying the rules for who makes what choices when. Consider, for example,

*Game 4.* $T$ chooses a set $\bar{x}$ of $A$'s and a set $\bar{y}$ of $B$'s; $F$ chooses some $d$ in $\bar{x}$; $T$ chooses some $e$ in $\bar{y}$, played when $m$ is considerably greater than $n$. This gives the reading “$mA$'s $R'd B$'s from a group of $n$”.

It might be possible to show that certain surface items, stresses and contextual factors indicate the appropriate game the utterer is “prepared to play” on the logical form underlying his utterance. For example, in

(4) \[ mA's \text{ each } R'd nB's \]

we might take “each” to mark the point where, once $T$ has chosen his set $\bar{x}$ of $A$'s, $F$ can intervene with his choice of some $d$ in $\bar{x}$ before $T$ goes on to choose his set $\bar{y}$ of $B$'s. In other words, “each” determines the game for the scope-differentiated reading.

This approach to mixed quantification has the advantage that we keep the deep structure, or logical form, as simple as possible. A possible disadvantage, however, is that there may be games just as simple as those we have described that determine “readings” of (3) requiring complicated prose for their precise expression. This, however, is just as much a drawback for the Kempson-Cormack approach, which allows use of all the apparatus of second order logic.

Our approach, though, arguably deals more easily with more complicated examples where pronouns correspond to bound variables of quantification. Consider a report of a Bedouin feast:

(5) \[ Ten \text{ Bedouin partook of the meat of thirty of their sheep.} \]

Suppose it is widely understood that proper partaking is a matter of having at least one mouthful of mutton from each carcass. Suppose
further that individual ownership of livestock is the tribal custom. Then we read (5) with a complete group interpretation of partaking, but an incomplete group interpretation of ownership. On the proposed logical form

\( (6) \quad 10x(Bx)(Oxy, Pxy) \) 

\( 30y(Sy)(Oxy, Pxy) \)

we presumably play Game 3 with respect to \( Oxy \) but Game 1 with respect to \( Pxy \).

Now suppose our ten Bedouin own 300 sheep altogether (note the surface indicator here for a certain group interpretation!) and have to preen them for the inter-tribal livestock show. Each takes 30 sheep without regard to ownership, and in communal spirit they preen in parallel. Then for the sentence

Ten Bedouin each preened thirty of their sheep

we would appeal to (6) with the scope differential reading for preening, but the incomplete group interpretation of ownership as before. That is, we would allow Game 2 with respect to \( Pxy \) but Game 3 with respect to \( Oxy \), using the sets \( \bar{x}, \bar{y} \) chosen by \( T \) in the course of play in Game 2.

I think the flexibility and simplicity of the game theoretic account has much in its favour, once we abandon the methodological assumption that a logical form, or deep structure, should contain all the information necessary to determine a unique interpretation. The possibility I am canvassing is that optional transformations might introduce certain items (like "each" and "altogether" above) whose effect is to help to determine what game is to be played on the deep structure, thereby fixing the intended reading. One must then deal with the problem of characterizing the relation of logical consequence, perhaps as the intersection of consequence relations \textit{modulo} the various games; but that it is a topic for another paper.

\textit{Department of Philosophy, University of Edinburgh}

\textbf{NOTES}

An ancestor of this paper was read as a reply to an ancestor of Kempson and Cormack’s ‘Ambiguity and Quantification’ at the Scots Philosophical Club Conference in Philosophy of Language in Glasgow, 1978. I am grateful to them for letting me see their copy for publication. But footnote 54 shows that in their generosity they have misunderstood both my criticisms of their proposals and my own suggestions concerning game-theoretic interpretation of quantified sentences.