Unit Commitment with an Enhanced Natural Gas-Flow Model

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Abstract—The interdependency of electric power and natural gas systems requires co-ordinated operational planning. We propose a unit commitment model that integrates a second-order-cone relaxation of dynamic natural gas flows. The model is enhanced by using convex envelopes of bilinear terms, which tighten the relaxation. By fixing the binary variables at their optimal values, we obtain electricity and natural gas locational marginal prices as the dual variables of electricity- and natural gas-flow-balance equations, respectively. The interdependence between these sets of prices is discussed. Numerical results from two test systems validate the solution-quality and computational-efficiency benefits of the proposed modeling methodology.

Index Terms—Power system operations, natural gas, unit commitment, second-order cone programming

NOMENCLATURE

Indices, Sets, and Functions

\( C(m) \) set of natural gas compressors connected to node \( m \)

\( E(i) \) set of power system buses directly connected to bus \( i \)

\( E_B(i) \) set of transmission lines

\( E_c(i) \) set of generating units connected to bus \( i \)

\( G(m) \) set of natural gas nodes connected to node \( m \)

\( G_B \) set of natural gas pipelines

\( G_P(m) \) set of natural gas-fired generating units connected to node \( m \)

\( G_r(m) \) set of natural gas suppliers connected to node \( m \)

\( i, j \) indices of power system buses in set, \( \Omega_E \)

\( k \) index of natural gas compressors in set, \( G_C \)

\( m, n \) indices of natural gas-system nodes in set, \( \Psi_G \)

\( \text{REF} \) reference bus of the power system

\( t \) index of time periods in set, \( T \)

\( v \) index of generating units in set, \( \Omega \)

\( w \) index of natural gas suppliers in set, \( \Psi_S \)

\( \Omega_G \) set of natural gas-fired generating units

\( \Omega_R \) set of coal-fired generating units

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Parameters and Constants

\( C_{EL} \) value of lost electric load [$/p.u.$]

\( C_{GL} \) value of lost natural gas load [$/Mm^3$]

\( C_{G,v} \) variable production cost of coal-fired unit \( v \) [$/p.u.$]

\( C_{O,v} \) non-fuel variable operation and maintenance cost of natural gas-fired unit \( v \) [$/p.u.$]

\( C_{S,w} \) variable production cost of natural gas supplier \( w \) [$/Mm^3$]

\( C_{SD,v} \) shutdown cost of generating unit \( v \) [$/shutdown$]

\( C_{SU,v} \) start-up cost of generating unit \( v \) [$/start-up$]

\( F_{L,m,t} \) non-generation-related natural gas demand at node \( m \) in time period \( t \) [Mm$^3$/h]

\( F_{C,k}^{\max} \) natural gas-transportation limit of compressor \( k \) [Mm$^3$/h]

\( F_{S,w}^{\max} \) maximum natural gas supply of supplier \( w \) [Mm$^3$/h]

\( F_{S,w}^{\min} \) minimum natural gas supply of supplier \( w \) [Mm$^3$/h]

\( F_{S,w}^{\ramp} \) ramping limit of natural gas supplier \( w \) [Mm$^3$/h/(time period)]

\( K_{m,n} \) line-pack parameter of pipeline connecting nodes \( m \) and \( n \) [(Mm$^3$/h)/bar]

\( L_{\min} \) minimum total line-pack in natural gas system [Mm$^3$]

\( P_{L,i,t} \) electric demand at bus \( i \) in time period \( t \) [p.u.]

\( F_{G,v}^{\max} \) maximum output of generating unit \( v \) [p.u.]

\( F_{G,v}^{\min} \) minimum output of generating unit \( v \) when it is online [p.u.]

\( F_{G,v}^{\ramp} \) ramping limit of generating unit \( v \) [p.u.]/(time period)]

\( W_{m,n} \) Weymouth constant of pipeline connecting nodes \( m \) and \( n \) [(Mm$^3$/h)/bar]

\( \eta_v \) heat rate of natural gas-fired unit \( v \) [Mm$^3$/h/p.u.]

\( \vartheta_k \) conversion efficiency of natural gas compressor \( k \)

\( \pi_{\max}^{m} \) maximum natural gas pressure at node \( m \) [bar]

\( \pi_{\min}^{m} \) minimum natural gas pressure at node \( m \) [bar]

\( \rho_k^{\max} \) maximum compression ratio of natural gas compressor \( k \)

\( \rho_k^{\min} \) minimum compression ratio of natural gas compressor \( k \)

\( \sigma_{i,j} \) susceptance of transmission line connecting buses \( i \) and \( j \) [p.u.]

Variables

\( F_{C,k,t} \) natural gas flow in time period \( t \) through compressor \( k \) [Mm$^3$/h]
non-generation-related natural gas demand at node \( m \) that is served in time period \( t \) [Mm³/h]

\( F_{G,v,t} \) fuel consumed by natural gas-fired generating unit \( v \) in time period \( t \) [Mm³/h]

\( F_{m,n,t} \) natural gas flow in time period \( t \) through pipeline connecting nodes \( m \) and \( n \) [Mm³/h]

\( \bar{F}_{m,n,t} \) average natural gas flow in time period \( t \) through pipeline connecting nodes \( m \) and \( n \) [Mm³/h]

\( F_{S,w,t} \) natural gas supplied in time period \( t \) by supplier \( w \) [Mm³/h]

\( L_{m,n,t} \) line-pack in time period \( t \) in pipeline connecting nodes \( m \) and \( n \) [Mm³/h]

\( P_{D,i,t} \) electric demand at bus \( i \) that is served in time period \( t \) [p.u.]

\( P_{G,v,t} \) active power produced in time period \( t \) by generating unit \( v \) [p.u.]

\( u_{G,v,t} \) binary variable that equals 1 is generating unit \( v \) is online in time period \( t \) and equals 0 otherwise

\( y_{G,v,t} \) binary variable that equals 1 is generating unit \( v \) is started up in time period \( t \) and equals 0 otherwise

\( z_{G,v,t} \) binary variable that equals 1 is generating unit \( v \) is shutdown in time period \( t \) and equals 0 otherwise

\( \theta_{i,t} \) in \( \pi_{k,t} \) inlet pressure of natural gas compressor \( k \) in time period \( t \) [bar]

\( \pi_{m,t} \) natural gas pressure at node \( m \) in time period \( t \) [bar]

\( \pi_{out} \) outlet pressure of natural gas compressor \( k \) in time period \( t \) [bar]

\( \tau_{k,t} \) natural gas consumed by natural gas compressor \( k \) in time period \( t \) [Mm³/h]

I. INTRODUCTION

Electric power and natural gas systems are becoming increasingly interdependent\[1,\ 2\]. This is driven by the low cost of natural gas-fired generating units. Moreover, many natural gas-fired units can provide the operating flexibility that high penetrations of renewable energy require \[3,\ 4\]. Despite the growing interdependencies between these systems, they are typically planned and operated independently of one another. This lack of co-ordination can give rise to suboptimal operating decisions and can even raise security, reliability, or resilience issues. For instance, the United States experienced a large-scale electricity- and natural gas-service disruption in February, 2011, which highlights the challenges that this interdependency creates \[5\].

Given this context, co-ordinating the operation of the two systems is becoming increasingly important. The technical literature provides a number of approaches to such co-ordination. Liu et al. \[7\] propose a security-constrained unit commitment model that incorporates natural gas-pipeline constraints. The model is solved using Benders’ decomposition, wherein the natural gas flows are represented using linear subproblems. Liu et al. \[8\] incorporate a transient natural gas-flow model, using a bilevel modeling approach. Their work takes the operation of the power system to be the upper-level problem, and includes natural gas-flow feasibility in the lower level. Zhao et al. \[9\] develop a two-stage stochastic unit commitment problem that includes natural gas-supply and -price uncertainties. Zhang et al. \[10\] propose a stochastic unit commitment model that considers transmission and generator outages and demand response. Correa-Posada et al. \[11\] consider transmission and pipeline contingencies within an integrated unit commitment model. He et al. \[12\] propose a two-stage robust unit commitment model that accounts for the natural gas system in making power system-operation decisions. Antenucci and Sansavini \[13\] investigate the impacts of transient natural gas-system constraints on a stochastic unit commitment model with \((N-1)\) contingency constraints.

A major challenge that these works contend with is that natural gas flows are highly nonlinear and non-convex. Some works \[9\]–\[11\] ignore these complexities (i.e., model linear natural gas flows or approximate them as being piecewise linear) while others \[7\], \[8\], \[13\] use nonlinear optimization models, which raise tractability issues. Another approach is to convexify the flow equations. Doing so allows some of the nonlinearities to be captured, while mitigating the challenges that non-convexity raises. Sanchez et al. \[14\] propose using a second-order-cone (SOC) relaxation to represent natural gas flows for expansion planning of natural gas and electric power systems. Other works \[12\] employ SOC relaxations for modeling optimal power and natural gas flows in co-ordinated operational-planning problems. Chen et al. \[15\] develop both steady-state and transient natural gas-flow models that employ SOC relaxations. The objective functions of their models are tailored to ensure tight solutions. Wang et al. \[16\] propose a market-clearing model for natural gas that uses an SOC relaxation of the network.

SOC relaxations of natural gas-flow models represent a tradeoff between fidelity and computational tractability. The methods in the existing literature that employ SOC relaxations do leave some important gaps. Many methods\[12\]–\[14\], \[15\] employ steady-state natural gas-flow models, whereas dynamic flows are more appropriate for operational planning \[17\]. Secondly, many of the models in the existing literature that employ SOC relaxations yield solutions with relatively large feasibility gaps. This implies that the models may be unsuitable for operating a system without heuristic refinement of a solution to find meaningful operating decisions.

Our work seeks to fill this gap in the literature on SOC relaxations of natural gas flows. Specifically, we propose a unit commitment model that has embedded within it dynamic natural gas-flow equations. This unit commitment problem is a mixed-integer nonlinear optimization problem. We then employ an enhanced SOC relaxation that provides tighter feasibility bounds compared to natural gas-flow models in the existing literature that employ SOC relaxations. The enhanced SOC relaxation is based on convex envelopes of bilinear terms in the nonlinear natural gas-flow equations. With this relaxation, our unit commitment problem becomes a mixed-integer second-order cone problem (MISOCP). Using a small four-node example and a large case study that is based on the IEEE 118-bus test system, we demonstrate the performance of our enhanced MISOCP compared to the standard SOC relaxation that is in the literature, demonstrating its tighter feasibility gaps. We also examine the impacts of congestion in the natural gas and power system on locational marginal
prices (LMPs) in the two systems.

The remainder of this paper is organized as follows. Section II presents the mixed-integer nonlinear unit commitment model with integrated natural gas-flow equations. Section III details the enhanced SOC relaxation, which yields the MIS-OC. Sections IV and V summarize the results of the example and case study, respectively. Section VI concludes.

II. MODEL FORMULATION

We present here the formulation of our ‘base’ model (i.e., the mixed-integer nonlinear optimization problem without any relaxation of the natural gas-flow constraints). This model includes a linearized dc representation of power flows and nonlinear and non-convex dynamic natural gas-flow constraints [18]. The power flows could be represented using a nonlinear ac model Doing so would raise further tractability issues, in addition to those that arise from representing natural gas flows. Linearized dc models are normally used for day-ahead power system operation, which is the envisioned use of our proposed model. Moreover, our focus is on modeling natural gas flows, to which the representation of power flows is not germane.

The model is formulated as:

\[
\begin{align*}
\min & \sum_{t \in T} \left( \sum_{v \in \Omega} (C_{SU,v}y_{G,v,t} + C_{SD,v}z_{G,v,t}) \right) \\
& + \sum_{v \in \mathcal{R}} C_{G,v}P_{G,v,t} + \sum_{v \in \Omega_G} C_{O,v}P_{G,v,t} \\
& + \sum_{w \in \mathcal{S}_G} C_{S,w}F_{S,w,t} + \sum_{i \in \Omega_G} C_{EL} \cdot (P_{L,i,t} - P_{L,i,t}^D) \\
& + \sum_{m \in \mathcal{W}_G} C_{GL} \cdot (F_{L,m,t} - F_{L,m,t}^D)
\end{align*}
\]

s.t. \[
\begin{align*}
\forall i & \in \Omega_G, t \in T; \\
\sigma_{i,j} \cdot (\theta_{i,t} - \theta_{j,t}); & \\
\forall i & \in \mathcal{E}_G, t \in T; \\
0 & \leq P_{L,i,t}^D \leq P_{L,i,t}; \forall i \in \mathcal{E}_G, t \in T; \\
\forall v & \in \mathcal{O}_G, t \in T; \\
y_{G,v,t} \cdot z_{G,v,t} & = u_{G,v,t} - u_{G,v,t-1}; \forall v \in \mathcal{O}_G, t \in T; \\
\forall \mathcal{V} & \in \mathcal{W}_G, t \in T; \\
0 & \leq P_{L,i,t}^D \leq P_{L,i,t}^\text{max}; \forall i \in \mathcal{E}_G, t \in T; \\
\forall (i,j) & \in \mathcal{E}_G, t \in T; \\
\theta_{\text{REF},t} & = 0; \forall t \in T; \\
0 & \leq F_{L,m,t}^D \leq F_{L,m,t}; \forall m \in \mathcal{W}_G, t \in T; \\
\forall k & \in \mathcal{C}(m) \\
\sum_{w \in \mathcal{O}_G} F_{S,w,t} - \sum_{k \in \mathcal{C}(m)} F_{C,k,t} & = F_{G,v,t} \\
\forall m & \in \mathcal{W}_G, t \in T; \\
\bar{F}_{m,n,t} & = \frac{1}{2} (F_{m,n,t} - F_{n,m,t}); \\
\forall (m,n) & \in \mathcal{W}_G, t \in T; \\
\bar{F}_{m,n,t}^2 - \bar{F}_{m,n,t}^2 & = \pi_{m,n,t}^2 - \pi_{n,m,t}^2; \forall (m,n) \in \mathcal{W}_G, t \in T
\end{align*}
\]

Objective function (1) gives the total cost of operating the power and natural gas systems. The first two terms represent the start-up and shutdown costs, respectively, of generating units. The third term represents the variable cost of coal-fired units while the fourth term represents the non-fuel variable cost of natural gas-fired units. The fifth term represents natural gas-production costs. This term implicitly includes the cost of supplying fuel to natural gas-fired units. The final remaining terms represent the costs of curtailing electric and natural gas demands, respectively.

The model has three sets of constraints. The first set, constraints (4)–(9), pertain to the operation of the electric power system. Constraints (4) impose load balance at each bus. Constraints (3) and (4) impose capacity and ramping limits, respectively, on the generating units. Constraints (5) define the start-up and shutdown variables for the generating units in terms of changes in the corresponding ‘online’ state variables while constraints (6) enforce integrality of these variables. Constraints (7) define power flows along each transmission line in terms of differences in the phase angles at its ends and constraints (8) set the phase angle at the reference bus equal to zero. Constraints (9) limit load served at each bus by demand.

The second set of constraints, (10)–(22), pertain to the natural gas system. Constraints (10) impose nodal flow balance. Constraints (11) define the average flow in each pipeline in terms of flows in each direction. Constraints (12) relate these average natural gas flows to the change in squared pressure between the two ends of each pipeline. We assume that \( F_{m,n,t} \geq 0 \), meaning that we know the direction of the flows \textit{a priori}. This is a reasonable assumption in day-ahead operations [19]. Constraints (13) determine the line-pack on pipelines based on the upstream and downstream pressures at their ends. Constraints (14) give the relationship between changes in flows and line-pack in pipelines. Constraints (15) compute the fuel consumption of natural gas-driven compressors in the network. Constraints (16) impose minimum and maximum compressor ratios while constraints (17) impose flow limits. Constraints (18) and (19) impose capacity and ramping limits,
respectively, on natural gas suppliers. Constraints (20) limit the nodal pressures. Constraint (21) imposes a minimum line-pack level in the final time period of the optimization horizon, thereby ensuring that the natural gas in the network is not depleted. Constraints (22) limit load served at each node by nodal demand.

The final set of constraints, (23), couple the two systems through the fuel consumption of natural gas-fired units.

III. ENHANCED SOC-BASED RELAXATION OF NATURAL GAS-FLOW MODEL

Integrated model (1)–(23) is a mixed-integer nonlinear optimization problem that has a non-convex continuous relaxation. Specifically, tractability issues arise from constraint set (12), which can be equivalently written as:

\[ F_{m,n,t}^2 / W_{m,n}^2 \leq \pi_{m,n,t}^2; \forall (m,n) \in G_B, t \in T \tag{24} \]

\[ F_{m,n,t}^2 / W_{m,n}^2 \geq \pi_{m,n,t}^2; \forall (m,n) \in G_B, t \in T. \tag{25} \]

A standard technique to convexify such a model is to relax (25), thereby replacing (12) with (24). Doing so yields a MISOCP, which can be solved using off-the-shelf software tools. However, the solutions that are obtained from such a MISOCP may not be sufficiently tight, in the sense that they yield non-trivial violations of constraint set (12).

Building off of this approach, we propose employing an enhanced SOC relaxation that includes a convex relaxation of (25). To convexify (25), we replace the bilinear terms that appear in the inequalities with their convex envelopes (20), (21). To do so, we define two sets of variables, \( a_{m,n,t} \) and \( b_{m,n,t} \), which are defined as the sums and differences of the pressure at the two ends of each pipeline via the equalities:

\[ a_{m,n,t} = \pi_{m,n,t} + \pi_{m,n,t}; \forall (m,n) \in G_B, t \in T \tag{26} \]

\[ b_{m,n,t} = \pi_{m,n,t} - \pi_{m,n,t}; \forall (m,n) \in G_B, t \in T. \tag{27} \]

We also define two new sets of auxiliary variables, \( \kappa_{m,n,t} \) and \( \lambda_{m,n,t} \). The convex relaxation that we propose is given by:

\[ \kappa_{m,n,t} \geq \lambda_{m,n,t}; \forall (m,n) \in G_B, t \in T \tag{28} \]

\[ \kappa_{m,n,t} \geq \bar{F}_{m,n,t}^2; \forall (m,n) \in G_B, t \in T \tag{29} \]

\[ \kappa_{m,n,t} \leq (F_{\max}^{m,n,t} + F_{\min}^{m,n,t}) \bar{F}_{m,n,t} - F_{\max}^{m,n,t} F_{\min}^{m,n,t}; \forall (m,n) \in G_B, t \in T \tag{30} \]

\[ \lambda_{m,n,t} \geq a_{\min}^{m,n,t} b_{m,n,t} + b_{\min}^{m,n,t} a_{m,n,t} - a_{\min}^{m,n,t} b_{\min}^{m,n,t}; \forall (m,n) \in G_B, t \in T \tag{31} \]

\[ \lambda_{m,n,t} \geq a_{\max}^{m,n,t} b_{m,n,t} + b_{\max}^{m,n,t} a_{m,n,t} - a_{\max}^{m,n,t} b_{\max}^{m,n,t}; \forall (m,n) \in G_B, t \in T \tag{32} \]

\[ \lambda_{m,n,t} \leq a_{\min}^{m,n,t} b_{m,n,t} + b_{\min}^{m,n,t} a_{m,n,t} - a_{\min}^{m,n,t} b_{\min}^{m,n,t}; \forall (m,n) \in G_B, t \in T \tag{33} \]

\[ \lambda_{m,n,t} \leq a_{\max}^{m,n,t} b_{m,n,t} + b_{\max}^{m,n,t} a_{m,n,t} - a_{\max}^{m,n,t} b_{\max}^{m,n,t}; \forall (m,n) \in G_B, t \in T. \tag{34} \]

Thus, (28), (29) ‘replaces’ (25), inasmuch as it imposes the necessary relationship between \( \kappa_{m,n,t} \) and \( \lambda_{m,n,t} \). Fig. 1 shows the convexified approximation of \( F_{m,n,t}^2 \) that is given by (29) and (30).

Constrants (31)–(34) impose analogous bounds on \( \lambda_{m,n,t} \). To see this, first note that from the definition of \( a_{m,n,t} \) and \( b_{m,n,t} \), we have that \( a_{m,n,t} b_{m,n,t} = \pi_{m,n,t}^2 - \pi_{m,n,t}^2 \). Constrants (31)–(34) impose bounds on \( \lambda_{m,n,t} \) that are related to \( a_{m,n,t} \) and \( b_{m,n,t} \). Visualizing these bounds is challenging, because \( a_{m,n,t} b_{m,n,t} \) is a surface and (31)–(34) are hyperplanes in a three-dimensional space. Fig. 2 shows \( a_{m,n,t} b_{m,n,t} \) and (31)–(34) for the special case in which \( a_{m,n,t} = b_{m,n,t} \). The figure shows that (31)–(34) provides the tightest convex envelope that contains \( a_{m,n,t} b_{m,n,t} \).

A. Tightened Enhanced SOC Relaxation of Natural Gas Flows

Figs. 1 and 2 show that the tightness of the proposed convex envelopes that are given by (29)–(34) depend on the

Fig. 1. Convexified approximation of \( F_{m,n,t}^2 \) that is given by (29) and (30).

Fig. 2. Convexified approximation of \( \pi_{m,n,t}^2 - \pi_{m,n,t}^2 \) that is given by (31)–(34).
chosen values of $F_{m,n,t}^{\text{min}}$, $F_{m,n,t}^{\text{max}}$, $a_{m,n,t}$, $a_{m,n,t}^{\text{max}}$, $b_{m,n,t}^{\text{min}}$, and $b_{m,n,t}^{\text{max}}$. This is because (29)–(34) allow $\kappa_{m,n,t}$ and $\gamma_{m,n,t}$ to lie anywhere within the convex envelopes that are given by the constraint sets. The farther the true values of $F_{m,n,t}^{\text{min}}$ and $F_{m,n,t}^{\text{max}}$, $a_{m,n,t}$, $a_{m,n,t}^{\text{max}}$, $b_{m,n,t}^{\text{min}}$, and $b_{m,n,t}^{\text{max}}$, respectively, the less accurate the resulting relaxation is.

As such, one can tighten the enhanced relaxation by first solving the MISOCP with a starting set of values for $F_{m,n,t}^{\text{min}}$, $F_{m,n,t}^{\text{max}}$, $a_{m,n,t}$, $a_{m,n,t}^{\text{max}}$, $b_{m,n,t}^{\text{min}}$, and $b_{m,n,t}^{\text{max}}$. These values can then be updated to:

\[
F_{m,n,t}^{\text{min}} \leftarrow (1 - \epsilon) F_{m,n,t}^{*}, \forall (m,n) \in G_B, t \in T; \quad (35)
\]

\[
F_{m,n,t}^{\text{max}} \leftarrow (1 + \epsilon) F_{m,n,t}^{*}, \forall (m,n) \in G_B, t \in T; \quad (36)
\]

\[
a_{m,n,t} \leftarrow (1 - \epsilon) a_{m,n,t}^{*}, \forall (m,n) \in G_B, t \in T; \quad (37)
\]

\[
a_{m,n,t}^{\text{max}} \leftarrow (1 + \epsilon) a_{m,n,t}^{*}, \forall (m,n) \in G_B, t \in T; \quad (38)
\]

\[
b_{m,n,t}^{\text{min}} \leftarrow (1 - \epsilon) b_{m,n,t}^{*}, \forall (m,n) \in G_B, t \in T; \quad (39)
\]

\[
b_{m,n,t}^{\text{max}} \leftarrow (1 + \epsilon) b_{m,n,t}^{*}, \forall (m,n) \in G_B, t \in T; \quad (40)
\]

where $\epsilon \in (0,1)$ is a control parameter and $F_{m,n,t}^{*}$, $a_{m,n,t}^{*}$, and $b_{m,n,t}^{*}$ are the optimal values that are obtained with the initial values of $F_{m,n,t}^{\text{min}}$, $F_{m,n,t}^{\text{max}}$, $a_{m,n,t}$, $a_{m,n,t}^{\text{max}}$, $b_{m,n,t}^{\text{min}}$, and $b_{m,n,t}^{\text{max}}$. The MISOCP can be re-solved with the updated values in (29)–(40). Indeed, the updating in (35)–(40) can be repeated iteratively to further tighten the relaxation.

### B. Comparison of Natural Gas-Flow Models

We compare the performance of three models in our example and case study. The first, which we hereafter refer to as MINLP, is given by (1)–(23). The second, which we hereafter refer to as MISOCP, is given by (1)–(11), (13)–(23), and (24). The third, which we hereafter refer to as eMISOCP (enhanced MISOCP), is given by (1)–(14), (13)–(23), (24), and (26)–(34). We also examine a Tightened eMISOCP, in which the convex envelopes are updated once using (35)–(40).

We measure the performance of these models in terms of computation time and solution quality. Solution quality is measured in two ways. First, the percentage difference in the optimal objective-function value between the MINLP and each of MISOCP and eMISOCP measures how accurately the two relaxations represent the true cost of operating the power and natural gas systems. Second:

\[
V_{m,n,t} = \frac{\pi_{m,n,t}^2 - \pi_{n,t}^2 - F_{m,n,t}^2/W_{m,n}^2}{\pi_{m,n,t}^2}, \quad (41)
\]

measures the p.u. amount by which each of the MISOCP and eMISOCP solutions violate constraint (12) in time step $t$ for the pipeline connecting nodes $m$ and $n$.

### C. Electric and Natural Gas LMPs

The MISOCP and eMISOCP have convex continuous relaxations. As such, if we fix the binary variables in these models to their optimal values, the resulting continuous relaxation becomes an SOC problem. Because the strong-duality theorem applies to SOC problems, they are guaranteed to have well defined dual variables. The dual variables that are associated with constraint set (3) are standard electric LMPs, which are differentiated by time period and bus. Analogously, the dual variables that are associated with constraint set (10) give natural gas LMPs, which are differentiated by time and node.

### IV. Example

This section summarizes the results of a four-bus/four-node example, the topology of which is shown in Fig. 3. Buses, nodes, loads, natural gas supplies, and generators are labeled using the same notation as in the model formulation. Natural gas-fired unit 2 couples the two systems. All of the pertinent data are provided in an online supplement. We examine system operation in a base case as well as two additional cases in which non-generation-related natural gas demands are increased by 10% and 20% relative to the baseline.

![Fig. 3. System topology of the example in Section IV.](https://doi.org/10.6084/m9.figshare.6025340.v1)

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>OBJECTIVE FUNCTION VALUES [$M$ MILLION] AND GAPS [%] FOR EXAMPLE IN SECTION IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Base Line</td>
</tr>
<tr>
<td>MINLP</td>
<td>3.296</td>
</tr>
<tr>
<td>MISOCP</td>
<td>3.286</td>
</tr>
<tr>
<td>eMISOCP</td>
<td>3.291</td>
</tr>
<tr>
<td>Tightened eMISOCP</td>
<td>3.295</td>
</tr>
</tbody>
</table>

Figs. 5 and 6 illustrate the interdependencies between natural gas and electric LMPs and how the cost of operating...
the two systems are interrelated. Fig. 5 shows hourly load-weighted natural gas LMPs that are obtained from the Tightened eMISOCP. All three of the cases result in high LMPs in hours 9–12, which is more pronounced in the cases with higher natural gas demands. This is due to congestion in the natural gas system, which results in unavoidable curtailment of natural gas demand. This results in natural gas-fired unit 2 having a higher operating cost during these hours, which yields the increased electric LMPs that are shown in Fig. 6.

The three models are programmed using version 24.7 of the GAMS mathematical modeling software package. The MINLP is solved using DICOPT and the MISOCP and eMISOCP are solved using CPLEX. All of the models are solved on a computer with a 1.9 GHz Intel Core processor and 4 GB of memory. The MINLP, MISOCP, eMISOCP, and Tightened eMISOCP take approximately 6.1 s, 1.3 s, 1.7 s, and 3.3 s, respectively, to solve.

V. CASE STUDY

This section summarizes the results of a case study, which consists of the IEEE 118-bus system, which is coupled with the 48-node natural gas system that is shown in Fig. 7. Nodes, natural gas supplies and loads, and power system nodes that have natural gas-fired units (which couple the systems) are labeled using the same notation that is used in the model formulation. Natural gas compressors are represented by the trapezoids. Natural gas-system data are obtained from the work of Wu et al. [22]. The nine natural gas-fired units constitute 36% of the total generating capacity in the power system.

Table II and Fig. 8 summarize the improved quality of solutions given by the eMISOCP relative to the MISOCP. Table II illustrates that the eMISOCP yields a more accurate estimate of the cost of the operating the two systems. Fig. 8 shows, as a representative example, the constraint violations for all of the natural gas pipelines in hour 15. The table and figure show that the improved performance of the eMISOCP and Tightened eMISOCP carry over to this larger case study.

To further explore interactions between the two systems, we consider three cases with different amounts of transmission capacity available in the electric power system. Specifically, we consider a base case, which corresponds to the IEEE 118-bus system and two additional cases in which all branches are assumed to have transmission capacities that are 20% and 40% below the baseline.
Fig. 7. Natural gas-system topology of the case study in Section V.

Table II

<table>
<thead>
<tr>
<th>Model</th>
<th>Value</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>MINLP</td>
<td>44.29</td>
<td>—</td>
</tr>
<tr>
<td>MISOCP</td>
<td>43.81</td>
<td>1.1</td>
</tr>
<tr>
<td>eMISOCP</td>
<td>43.92</td>
<td>0.8</td>
</tr>
<tr>
<td>Tightened eMISOCP</td>
<td>44.22</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Fig. 9 shows day-ahead load-weighted electricity LMPs in the three cases when the Tightened eMISOCP is employed. The third case, with 40% less transmission capacity relative to the baseline, has the highest overall prices, due to extreme transmission congestion. Fig. 10 shows the amount of natural gas-fired generation in the three cases. Because natural gas is a relatively expensive generation fuel (compared to coal), natural gas-fired generators are only used in the case study when lower-cost alternatives cannot be. Fig. 10 shows that transmission congestion exacerbates the need to use natural gas-fired generation, leading to the higher electric LMPs that are shown in Fig. 9. The increased reliance on natural gas-fired units in the two cases with lower transmission capacity leads to higher natural gas LMPs, as shown in Fig. 11. Thus, the natural gas and electric LMPs tend to increase together.

Fig. 8. Violations of constraints (12) for all natural gas pipelines in hour 15 in the case study in Section V.

Fig. 9. Load-weighted average electric LMPs obtained from applying Tightened eMISOCP to the case study in Section V.

Fig. 10. Natural gas-fired electricity produced as a percentage of total electric load from applying Tightened eMISOCP to the case study in Section V.

The case study is implemented using the same computational environment that the example is. The MINLP, MISOCP, eMISOCP, and Tightened eMISOCP each require approximately 375 minutes, 41 minutes, 46 minutes, and 74 minutes of computation time, respectively. Contrasting these computation times with the example in Section IV shows that the MISOCP and eMISOCP scale better than MINLP does. Moreover, use of the eMISOCP introduces a tradeoff. While it provides higher-quality solutions than the MISOCP, this entails an added computational cost.
VI. CONCLUSION

This paper presents a unit commitment model that integrates an enhanced convex relaxation of dynamic natural gas-flow equations. The relaxation is obtained by using convex envelopes of bilinear terms in the flow equations. This allows us to model ‘both sides’ of the equality. This can be contrasted with other convexification techniques, which only include one of the two inequalities that equivalently define the natural gas-flow equations. Electric and natural gas LMPs can be obtained by fixing the binary variables in the eMISOCP to their optimal values. Test results demonstrate that the proposed eMISOCP yields higher quality solutions compared to the MISOCPR, especially when the convex envelopes are tightened.

We investigate the interdependencies between prices in the two systems. We find that congestion in one system can affect prices in the other. Moreover, this effect can be bidirectional. In the example in Section IV, high natural gas demands force curtailment of natural gas loads, which significantly increases natural gas LMPs. This, in turn, makes natural gas-fired units more expensive, decreasing their use while at the same time increasing electric LMPs. These dynamics are reversed in the case study in Section V. There, natural gas-fired units are relatively expensive and are only used to produce energy if absolutely necessary (i.e., other lower-cost units are capacitated or transmission congestion requires the use of natural gas-fired units). Limited transmission capacity exactly forces such increased use of the natural gas-fired units. This increases both electric and natural gas LMPs (the latter effect owing to increased demand for natural gas due to electricity-production needs).

REFERENCES


