Math 2173 Spring 2021 Recitation Handout 6 Solutions

Group Member 1: Sohail Farhangi

Group Member 2: ______________________________

Group Member 3: ______________________________

Group Member 4: ______________________________

Group Member 5: ______________________________

Group Member 6: ______________________________

Below is a checklist of instructions to follow when completing this assignment. Failure to follow these directions will result in penalty on your final score and/or in some problems not being graded. If multiple directions are not followed, then it is also possible that the assignment will not be accepted for any credit at all. Please contact your TA or make a post on the discussion boards for this course if you have any questions about this assignment or these directions.

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	Checklist of Instructions
	Please clearly write the names of all group members working on this assignment in the spaces allotted above.
	This assignment must be completed by a group of 3, 4, 5, or 6 members.
	This assignment is to be uploaded to gradescope as a pdf file no later than 11:59 PM EST on Sunday, February 28.
	The assignment will be uploaded by 1 group member, and that group member will be responsible for manually entering the names of all other collaborators into gradescope.
	This assignment must be completed using this template. You may either print this template to write on it and then scan it (pages ordered correctly) into a pdf file, or you may write directly on the template using programs such as notability.
	If you need more space than what is given to solve a given problem, then you will find blank pages provided at the end of this template. At the end of each problem section of this assignment you will find a space in which to indicate on what page your work is continued in case you used additional pages to complete your solution. You must provide the page number on which your work is continued in the alloted space, or write 'N/A' incase you did not use any additional pages.
	On the additional pages, you will also find space in which to indicate which problem the page is being used for, and if the page is used then that space must also be filled.
	To complete this handout, you may use your textbook, class notes, discussions with your TA and group members, and any resources that are available on Carmen. You should not receive any help from the MSLC or people outside of your group when solving these problems. You may discuss these problems on the Carmen discussion boards, but you should not provide your entire solution when answering a such question, you should only give a hint or a helpful idea.

Modified Problem 14.4.3 (10 points): Write an iterated integral for $\iiint_D f(x,y,z)dV$, where D is a sphere of radius 9 centered at $(0,0,1)$. Use the order $dV = dzdydx$.

Hint: Start by finding the equation of the the surface of the sphere of radius 9 *centered at* $(0,0,1)$.

Solution: We recall that the equation of the sphere of radius R centered at (a,b,c) is given by

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2, \tag{1}$$

so the equation of the sphere of radius 9 centered at $(0,0,1)$ is given by

$$x^2+y^2+(z-1)^2=81. \tag{2}$$

Since we are considering

$$\iiint_D f(x,y,z)dV=\int_?^?\int_?^?\int_?^? f(x,y,z)dzdydx, \tag{3}$$

we begin by observing that the smallest possible value of x in our region D is -9, and the largest possible value of x in our region D is 9, so $-9\le x\le 9$. We then observe that for each $x\in[-9,9]$, we have

$$y^2+(z-1)^2=81-x^2, \tag{4}$$

so the smallest possible value of y in our region D (corresponding to our chosen value of x) is $-\sqrt{81-x^2}$ and the largest possible value of y in our region D (corresponding to our chosen value of x) is $\sqrt{81-x^2}$, so $-\sqrt{81-x^2}\le y\le\sqrt{81-x^2}$. Lastly, we observe that for each $x\in[-9,9]$ and each $y\in[-\sqrt{81-x^2},\sqrt{81-x^2}]$, we have

$$(z-1)^2=81-x^2-y^2\to z=1\pm\sqrt{81-x^2-y^2}, \tag{5}$$

so the smallest possible value of z in our region D (corresponding to our chosen values of x and y) is $1-\sqrt{81-x^2-y^2}$ and the largest possible value of y in our region D (corresponding to our chosen values of x and y) is $1+\sqrt{81-x^2-y^2}$, so $1-\sqrt{81-x^2-y^2} \leq z \leq 1+\sqrt{81-x^2-y^2}$. It follows that we can describe our region D as

$$(6)\quad D = \Big\{(x,y,z) : -9 \leq x \leq 9, -\sqrt{81-x^2} \leq y \leq \sqrt{81-x^2},\ 1-\sqrt{81-x^2-y^2} \leq z \leq 1+\sqrt{81-x^2-y^2}\Big\}, \text{ so}$$

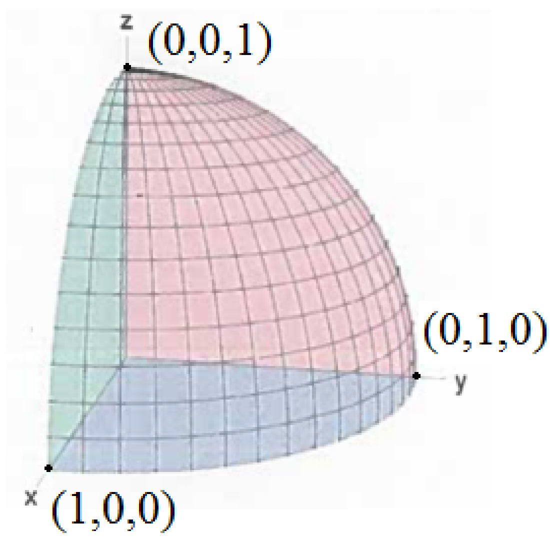
$$(7)\quad \iiint_D f(x,y,z)dV = \boxed{\int_{-9}^{9}\int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}}\int_{1-\sqrt{81-x^2-y^2}}^{1+\sqrt{81-x^2-y^2}} f(x,y,z)dzdydx}.$$

Problem 14.4.4 (10 points): Sketch by hand or graph with a computer program the region of integration for the integral

$$\int_0^1 \int_0^{\sqrt{1-z^2}} \int_0^{\sqrt{1-y^2-z^2}} f(x,y,z)\,dx\,dy\,dz. \tag{8}$$

Note: You may also describe the region of integration in writing instead. If you choose to do this, please write complete sentences and provide a thorough description.

Solution: Since $x^2 + y^2 + z^2 = 1$ is the equation of the unit sphere (centered at (0,0,0)), we may repeat the steps of the previous problem to see that the region of integration is related to the unit sphere. The key difference here is that the smallest possible values of x, y, and z are always 0, so our region of integration ends up being the portion of the unit sphere within the first octant.



Problem 14.4.31 (10 points): Evaluate

$$\int_{1}^{\ln(8)}\int_{1}^{\sqrt{z}}\int_{\ln(y)}^{\ln(2y)} e^{x+y^2-z}\,dxdydz. \tag{9}$$

Solution: We see that

$$\int_{1}^{\ln(8)}\int_{1}^{\sqrt{z}}\int_{\ln(y)}^{\ln(2y)} e^{x+y^2-z}\,dxdydz = \int_{1}^{\ln(8)}\int_{1}^{\sqrt{z}} e^{x+y^2-z}\Big|_{x=\ln(y)}^{\ln(2y)} dydz \tag{10}$$

$$= \int_{1}^{\ln(8)}\int_{1}^{\sqrt{z}} (e^{\ln(2y)+y^2-z} - e^{\ln(y)+y^2-z})dydz \tag{11}$$

$$= \int_{1}^{\ln(8)}\int_{1}^{\sqrt{z}} (2ye^{y^2-z} - ye^{y^2-z})dydz \tag{12}$$

$$= \int_{1}^{\ln(8)}\int_{1}^{\sqrt{z}} ye^{y^2-z}dydz \overset{u=y^2}{=} \int_{1}^{\ln(8)} \frac{1}{2}e^{y^2-z}\Big|_{y=1}^{\sqrt{z}} \tag{13}$$

$$\frac{1}{2}\int_{1}^{\ln(8)} (e^0 - e^{1-z})dz = \frac{1}{2}(z + e^{1-z}\Big|_{1}^{\ln(8)}) \tag{14}$$

$$= \frac{1}{2}(\ln(8) + e^{1-\ln(8)} - (e^{1-1}+1)) \tag{15}$$

$$= \frac{1}{2}(\ln(8) + e^1\cdot e^{-\ln(8)} - e^0 - 1) = \frac{1}{2}(\ln(8) + \frac{e}{e^{\ln(8)}} - 2) = \boxed{\frac{1}{2}\ln(8) + \frac{e}{16} - 1}. \tag{16}$$