Math 2173 Spring 2021 Recitation Handout 8

Group Member 1: _____

Group Member 2: _____

Group Member 3: _____

Group Member 4: _____

Group Member 5: _____

Group Member 6: _____

Below is a checklist of instructions to follow when completing this assignment. Failure to follow these directions will result in penalty on your final score and/or in some problems not being graded. If multiple directions are not followed, then it is also possible that the assignment will not be accepted for any credit at all. Please contact your TA or make a post on the discussion boards for this course if you have any questions about this assignment or these directions.

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Checklist of Instructions	
	Please clearly write the names of all group members working on this assignment in the spaces allotted above.
	This assignment must be completed by a group of 3, 4, 5, or 6 members.
	This assignment is to be uploaded to gradescope as a pdf file no later than 11:59 PM EST on Sunday, March 14.
	The assignment will be uploaded by 1 group member, and that group member will be responsible for manually entering the names of all other collaborators into gradescope.
	This assignment must be completed using this template. You may either print this template to write on it and then scan it (pages ordered correctly) into a pdf file, or you may write directly on the template using programs such as notability.
	If you need more space than what is given to solve a given problem, then you will find blank pages provided at the end of this template. At the end of each problem section of this assignment you will find a space in which to indicate on what page your work is continued in case you used additional pages to complete your solution. You must provide the page number on which your work is continued in the alloted space, or write 'N/A' incase you did not use any additional pages.
	On the additional pages, you will also find space in which to indicate which problem the page is being used for, and if the page is used then that space must also be filled.
	To complete this handout, you may use your textbook, class notes, discussions with your TA and group members, and any resources that are available on Carmen. You should not receive any help from the MSLC or people outside of your group when solving these problems. You may discuss these problems on the Carmen discussion boards, but you should not provide your entire solution when answering a such question, you should only give a hint or a helpful idea.

Ungraded Optional Problem 14.7.43: Let D be the region that is bounded by the paraboloid $z = 16 - x^2 - 4y^2$ and the xy-plane. Use the change of variables $x = 4u\cos(v), y = 2u\sin(v), z = w$ in order to evaluate

$$\iiint_D z dV. \tag{1}$$

Ungraded Optional Problem: Change variables for the following integral with the given transformation.

$\iint_R xy\, dA$, where R is the square with vertices $(0,0), (3,3), (6,0), (3,-3)$.

Use $x = u + v$, $y = u - v$.

(a) Sketch the original region of integration R in the xy-plane and the new region S in the uv-plane using the given change of variables.

(b) Find the limits of integration for the new integral with respect to u and v.

$$\_\_\_ \leq u \leq \_\_\_, \qquad \_\_\_ \leq v \leq \_\_\_.$$

(c) Compute the Jacobian $J(u, v)$.

(d) Set up and evaluate the new integral over the region S.

Problem 14.7.32 (8 points): Let R be the region bounded by the lines $y - x = 0, y - x = 2, y + x = 0, y + x = 2$. Use a change of variables to evaluate

(2) $$\iint_R \sqrt{y^2 - x^2} dA.$$

Problem 14.7.42 (12 points): Find the volume of the solid D that is bounded by the planes $y-2x=0, y-2x=1, z-3y=0, z-3y=1, z-4x=0$, and $z-4x=3$.

Problem 14.7.57 (Parabolic coordinates, 30 points): This problem has parts **a.**-**g.** spread out across the following pages. Your solutions to parts **a, b,** and **f** should include (hand drawn or computer generated) pictures. Part **g.** is optional and ungraded. Please clearly separate your work for the different parts of this problem and label the parts of your work accordingly.

Consider the Transformation T from the uv-plane to the xy-plane given by $T(u, v) = (u^2 - v^2, 2uv)$.

a. Show that the lines $u = a$ in the uv-plane map to parabolas in the xy-plane that open in the negative x-direction with vertices[1] on the positive x-axis.[2] Compare the images of the lines $u = a$ and $u = -a$ under T.

b. Show that the lines $v = b$ in the uv-plane map to parabolas in the xy-plane that open in the positive x-direction with vertices on the negative x-axis.[3] Compare the images of the lines $v = b$ and $v = -b$ under T.

c. Evaluate $J(u, v)$.

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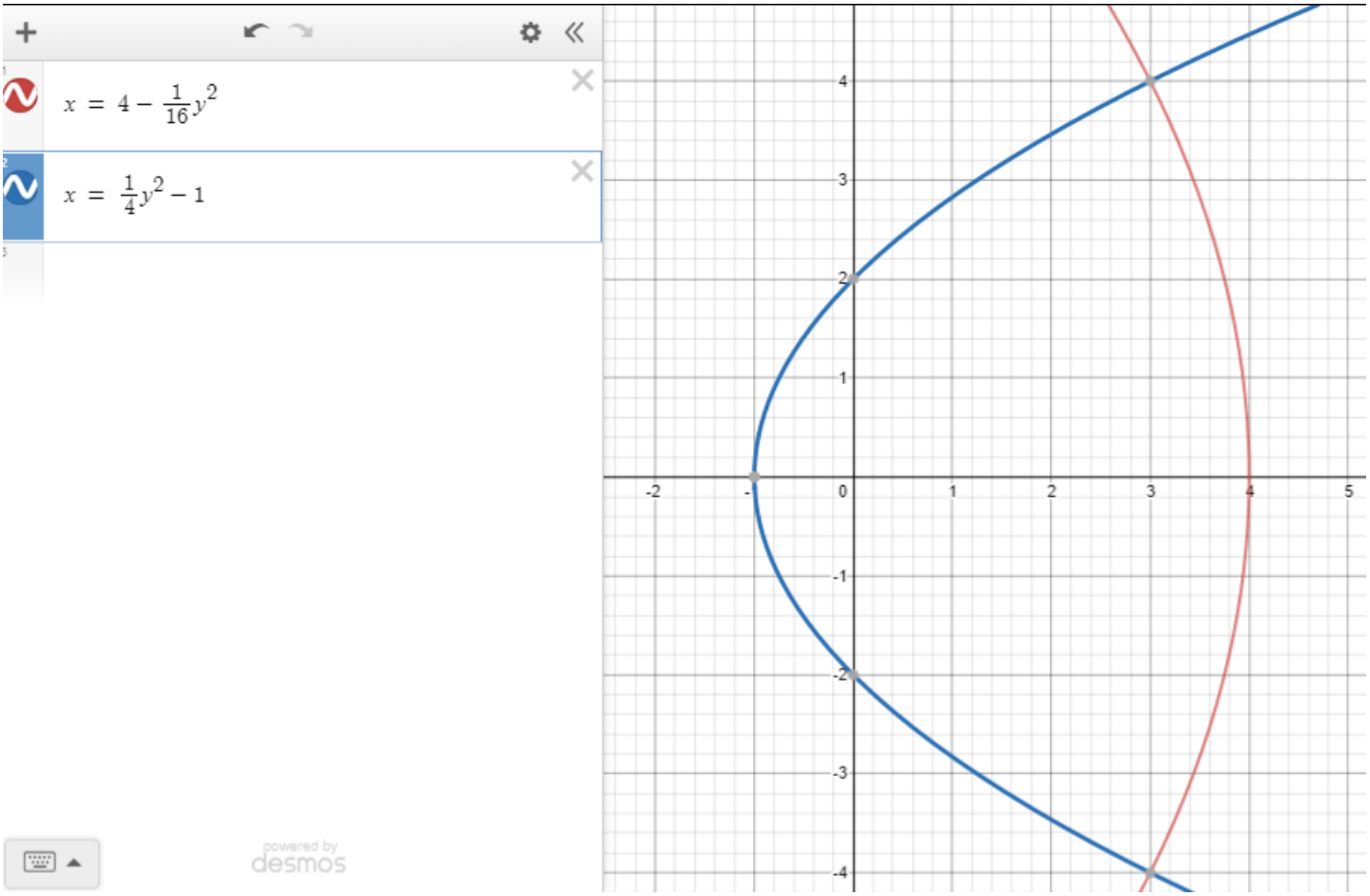
[1] The vertex of the parabola $y = x^2$ is the point $(0, 0)$ and the vertex of the parabola $x = y^2$ is also $(0, 0)$.

[2] You have to show that the curve $\vec{r_1}(v) = (a^2 - v^2, 2av)$ represents the same curve as $x - c = -by^2$ for some positive numbers b and c.

[3] You have to show that the curve $\vec{r_2}(u) = (u^2 - b^2, 2ub)$ represents the same curve as $x - c = by^2$ for some positive number b and some negative number c.

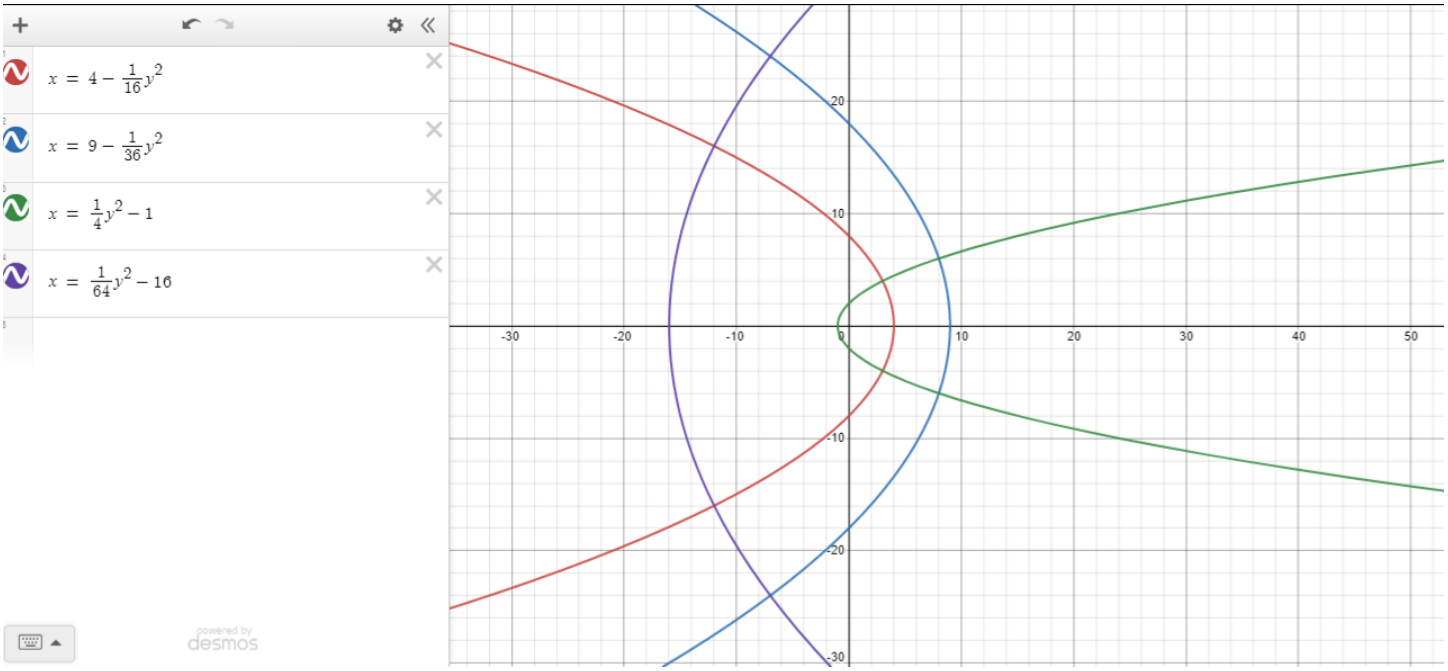
Our work for problem 14.7.57 parts **a.-c.** is continued on page Page 8

d. Use a change of variables into parabolic coordinates to find the area of the region in the xy-plane bounded by the curves $x = 4 - \frac{1}{16}y^2$ and $x = \frac{1}{4}y^2 - 1$. Sketch a picture of the new region of integration as well.



Our work for problem 14.7.57 part **d.** is continued on page Page 10

e. Use a change of variables into parabolic coordinates to find the area of the curved rectangle above the x-axis bounded by $x = 4 - \frac{1}{16}y^2, x = 9 - \frac{1}{36}y^2, x = \frac{1}{4}y^2 - 1$, and $x = \frac{1}{64}y^2 - 16$. Sketch a picture of the new region of integration as well.



Our work for problem 14.7.57 part **e.** is continued on page

f. Describe the effect of the transformation $(u, v) \mapsto (2uv, u^2 - v^2)$ on horizontal and vertical lines in the uv-plane.[4]

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[4] Remember that the transformation $(x, y) \mapsto (y, x)$ reflects points in the xy-plane across the line $y = x$. It will also help to use the results of parts **a.** and **b.** of this problem.

Our work for problem 14.7.57 part **f.** is continued on page Page 13

g. Show that the parabolas that are the images of the lines $u = a$ and $v = b$ under $T(u, v) = (u^2 - v^2, 2uv)$ are orthogonal to eachother.

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