

Name: _____

April 19, 2021

Procedures for the exam: Please log on to the Main Lecture Zoom room by **10:15 am ET**. You will be split into a breakout room with one of the moderators.

<https://osu.zoom.us/j/96249977566?pwd=Wk5jN1k5c2JmZkZEUTRVOHNhOUJDdz09>

Zoom Meeting ID: 962 4997 7566, Password: 650807

At **10:20 am ET**, the exam will be made available on the Carmen Homepage. At this time, you may download the exam and immediately begin working. You may download, complete, and submit the assignment using an ipad or other tablet device. You may also do your work on paper and then scan and submit your work. If you choose the first option, please use the exam template for your work. If you choose the second option, be sure to clearly label your work. **Unlike the recitation handouts and midterm 1** if you require additional pages to show your work for a given problem, include those pages in order immediately after the problem they pertain to. When you submit your exam to gradescope you are then required to mark the pages of your exam that correspond to the various problems on the exam. Video instructions for how to do this can be found here for the afternoon lecture and here for the morning lecture. If you have questions throughout the exam, you can direct message your moderator using the chat feature in Zoom.

When you have completed the exam, you should send a message to your moderator letting them know that you are no longer writing and are beginning to submit. You may then submit to Gradescope. You must submit the exam to Gradescope by **11:15 am ET**. If you have not finished by 11:10 am ET, an announcement will be made letting you know that you must now begin submitting. In other words, you should spend approximately 1 hour and 40 minutes working on the exam with the remaining five minutes left for submitting the exam.

Exam rules: In order to get credit for the exam, you must be in the Main Lecture Zoom room for the duration of the exam with your webcam on. No books, no notes, no calculators, and no internet resources may be used to complete the exam.

You must show your work. Work on the scrap work page will not be graded unless you indicate otherwise. Your work must be legible, and your final answers must be reasonably simplified.

On some problems, you are asked to use a specific method to solve the problem. On all other problems, you may use any method we've covered. You may not use methods we have not covered.

If at any point you experience technical difficulties, you must immediately e-mail both Dr. Skipper and your recitation instructor. Your email **must** include a complete copy of your exam, even if that is just in the form of photographs taken on your phone.

Good luck!

Problem 1 (20 points): Please determine whether the following statements are true or false. Provide some justification for each of your answers.

- (a) The function $e^{\frac{1}{x}} + \sin(y^x) - 1$ attains its absolute maximum value on the region $R = \{(x, y) \mid (x - 1)^2 + y^2 \leq 1\}$.

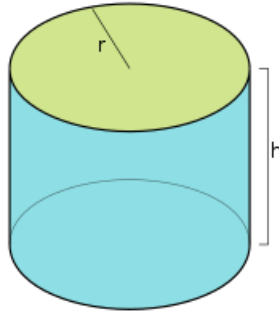
- (b) The vector field $\vec{F} = \langle ax + by, cx + ay \rangle$ is conservative for any real numbers a , b , and c .

(c) For any 3 continuous functions $f, g, h : \mathbb{R} \rightarrow \mathbb{R}$ we have

$$(1) \int_0^1 \int_0^z \int_{-1}^0 f(x)g(y)h(z)dx dy dz \\ = \left(\int_0^1 f(x)dx \right) \left(\int_0^z g(y)dy \right) \left(\int_{-1}^0 h(z)dz \right).$$

(d) The equation $yy' = e^t$ is a first order linear nonhomogeneous differential equation.

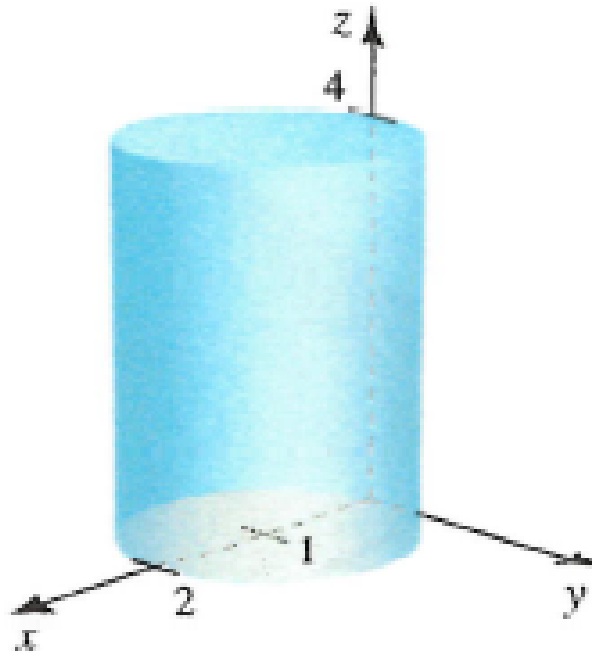
Problem 2 (20 points): Use Lagrange multipliers to find the dimensions of the right circular cylinder of minimum surface area (including the circular ends) with a volume of 32π in³.



Problem 3 (20 points): Use triple integration in Cartesian coordinates to find the volume of the tetrahedron S that has its vertices at $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, and $(0, 0, c)$, where $a, b, c > 0$.

Hint: One of the faces of the tetrahedron lies on the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Problem 4 (20 points): Use triple integration in cylindrical coordinates to find the volume of the solid cylinder E whose height is 4 and whose base in the xy -plane is the disk $\{(r, \theta) : 0 \leq r \leq 2 \cos(\theta)\}$.



Problem 5 (20 points): Find the average value of the function $f(x, y, z) = x$ over the curve \mathcal{C} that is parameterized by

$$(2) \quad \vec{r}(t) = \left\langle 20 \sin\left(\frac{t}{4}\right), 20 \cos\left(\frac{t}{4}\right), \frac{t}{2} \right\rangle, 0 \leq t \leq 4\pi.$$

Problem 6 (20 points): Evaluate

$$(3) \quad \int_C \langle \sqrt[4]{x+6} + \ln(\ln(\ln(e^{e^e} + 4 + x))) - 1, y^3 + 2 + e^{y^2} \rangle \cdot d\vec{r},$$

where C is the curve that is shown in the picture below.

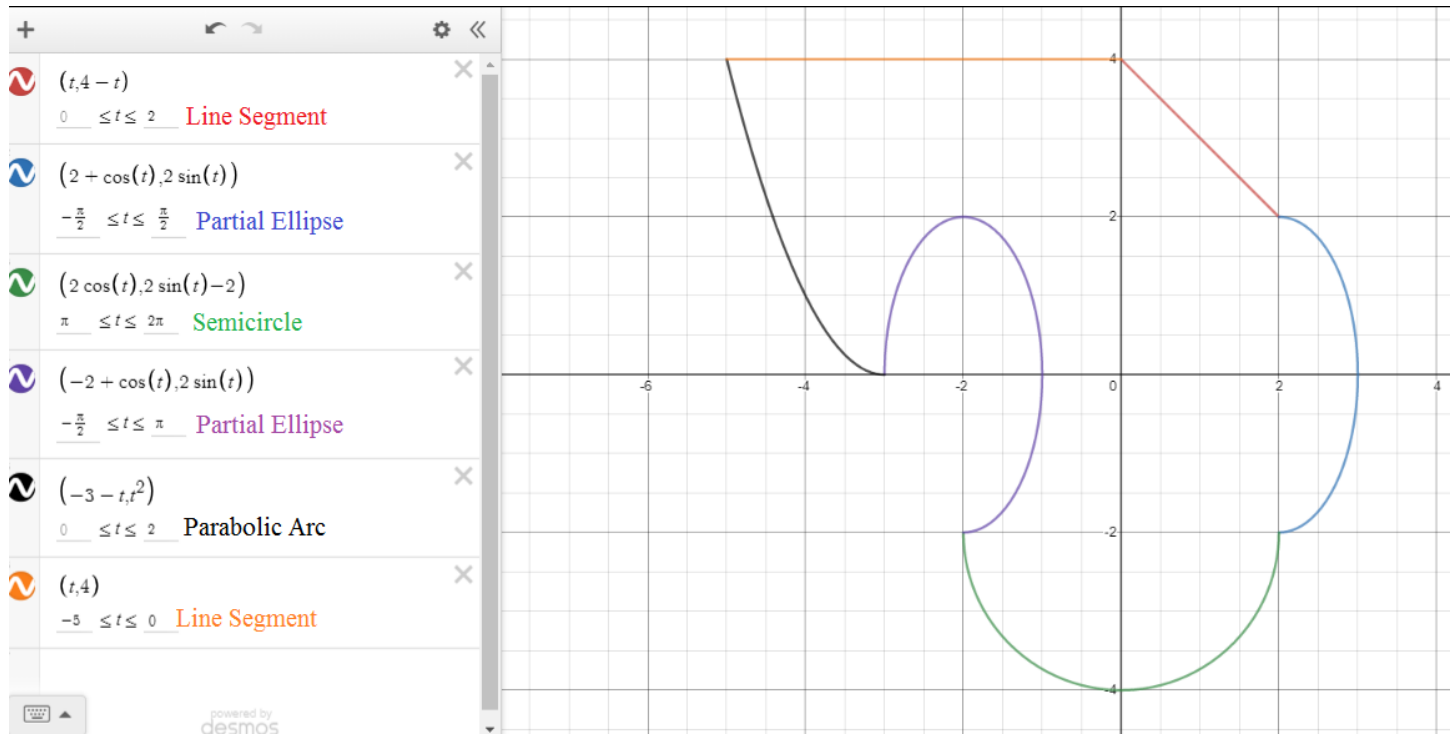


FIGURE 1.

Be sure to show all of your work and give a complete justification of your answer.

Problem 7 (20 points): Let a be a real number.

(a) Find the general solution to equation (4) in terms of a .

(4)
$$y'' - (a + 2)y' + 2ay = 0.$$

(b) Solve the initial value problem given in (5).

(5)
$$y'' - 25y' + 46y; \quad y(0) = 0, \quad y'(0) = 21.$$

Problem 8 (20 points): Find a particular solution to equation (6).

(6)
$$y'' + 4y = t \sin(2t).$$
