

Math 2173 Spring 2021 Recitation Handout 11

Group Member 1: _____

Group Member 2: _____

Group Member 3: _____

Group Member 4: _____

Group Member 5: _____

Group Member 6: _____

Below is a checklist of instructions to follow when completing this assignment. Failure to follow these directions will result in penalty on your final score and/or in some problems not being graded. If multiple directions are not followed, then it is also possible that the assignment will not be accepted for any credit at all. Please contact your TA or make a post on the discussion boards for this course if you have any questions about this assignment or these directions.

Sohail Farhangi: farhangi.3@osu.edu, Pan Yan: yan.669@osu.edu, Yilong Zhang: zhang.6100@osu.edu

Checklist of Instructions	
	Please clearly write the names of all group members working on this assignment in the spaces allotted above.
	This assignment must be completed by a group of 3, 4, 5, or 6 members.
	This assignment is to be uploaded to gradescope as a pdf file no later than 11:59 PM EST on Sunday, April 11.
	The assignment will be uploaded by 1 group member, and that group member will be responsible for manually entering the names of all other collaborators into gradescope.
	This assignment must be completed using this template. You may either print this template to write on it and then scan it (pages ordered correctly) into a pdf file, or you may write directly on the template using programs such as notability.
	If you need more space than what is given to solve a given problem, then you will find blank pages provided at the end of this template. At the end of each problem section of this assignment you will find a space in which to indicate on what page your work is continued in case you used additional pages to complete your solution. You must provide the page number on which your work is continued in the allotted space, or write 'N/A' incase you did not use any additional pages.
	On the additional pages, you will also find space in which to indicate which problem the page is being used for, and if the page is used then that space must also be filled.
	To complete this handout, you may use your textbook, class notes, discussions with your TA and group members, and any resources that are available on Carmen. You should not receive any help from the MSLC or people outside of your group when solving these problems. You may discuss these problems on the Carmen discussion boards, but you should not provide your entire solution when answering a such question, you should only give a hint or a helpful idea.

Ungraded Optional Problem 15.3.52: Conservation of energy.

Suppose an object with mass m moves in a conservative force field given by $\vec{F} = -\nabla\phi$, where ϕ is a potential function in a region R . The motion of the object is governed by Newton's second law of Motion, $\vec{F} = m\vec{a}$, where \vec{a} is the acceleration. Suppose the object moves (either in the plane or in space) from point A to point B in R .

(a). Show that the equation of the motion is

$$m\frac{d\vec{v}}{dt} = -\nabla\phi.$$

(b). Show that

$$\frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{1}{2} \frac{d}{dt}(\vec{v} \cdot \vec{v}).$$

(c). Take the dot product of both sides of the equation in part (a) with $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$ and integrate along a curve between A and B . Use part (b) and the fact that \vec{F} is conservative to show that the total energy (kinetic plus potential) $\frac{1}{2}m|\vec{v}|^2 + \phi$ is the same at A and B . Conclude that because A and B are arbitrary, energy is conserved in R .

Ungraded Optional Problem 16.1.41: Initial value problems. Solve the following initial value problem using the given general solution.

$$y'' - y' - 20y = 0; \quad y(0) = -3, \quad y'(0) = 3;$$

General solution $y = c_1e^{5t} + c_2e^{-4t}$.

Problem 1. (10 Points): Find the work required to move an object along the line segment from $(1, 1, 1)$ to $(8, 4, 2)$ through the force field \vec{F} given by

$$\vec{F} = \frac{\langle x, y, z \rangle}{x^2 + y^2 + z^2}.$$

Problem 15.3.27. (10 points): Evaluate the line integral $\int_C \nabla\phi \cdot d\vec{r}$ for $\phi(x, y) = xy$ and $C : \vec{r}(t) = \langle \cos(t), \sin(t) \rangle$, for $0 \leq t \leq \pi$ in two ways.

- (a). Use a parametric description of C and evaluate the integral directly;
 - (b). Use the Fundamental Theorem for line integrals.
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Problem 16.1.66. (10 points): As will be shown in Section 16.4, the equation $y'' + py' + qy = f(t)$, where p and q are constants and f is a specified function, is used to model both the mechanical oscillators and electrical circuits. Depending on the values of p and q , the solutions to this equation display a wide variety of behavior.

Consider the equation

$$y'' + 9y = 8 \sin(t).$$

(a). **(4 points)** Verify that the following equations have the given general solutions

$$y = c_1 \sin(3t) + c_2 \cos(3t) + \sin t.$$

(b). **(4 points)** Solve the initial value problem with the given initial conditions $y(0) = 0$, $y'(0) = 2$.

(c). **(2 points)** Graph the solutions to the initial value problem, for $t \geq 0$.

(Modified) Problem 15.3.55 (20 points): Let \vec{F} be the vector field

$$\vec{F} = \langle f(x, y), g(x, y) \rangle = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle.$$

It is a rotational vector field with the graph below

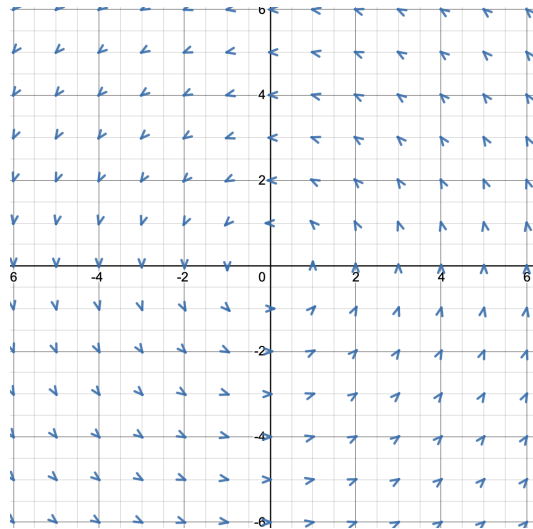


FIGURE 1. vector field \vec{F}

- (1) **(2 points)** Find the domain R of \vec{F} .
- (2) **(4 points)** Is the domain R connected? Is R simply connected?
- (3) **(4 points)** Show that $\frac{\partial g}{\partial x} = \frac{\partial f}{\partial y}$.

- (4) **(4 points)** Let C_a be the parameterized circle $\vec{r}(t) = \langle a \cos(t), a \sin(t) \rangle$, $0 \leq t < 2\pi$ of radius $a > 0$. Show that the integral

$$\int_{C_a} \vec{F} \cdot d\vec{r} = 2\pi.$$

- (5) **(3 points)** Is \vec{F} a conservative vector field on R ? If so, please explain. Otherwise, please explain why it doesn't contradict the result in (3).
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- (6) **(3 points)** Let R_1 be the region $R_1 = \{1 \leq x \leq 2, 1 \leq y \leq 2\}$. Is \vec{F} a conservative vector field on R_1 ? Please explain.
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