Math 2173 Spring 2021 Recitation Handout 13 Solutions

Group Member 1: Sohail Farhangi

Group Member 2: ______________________________

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Group Member 4: ______________________________

Group Member 5: ______________________________

Group Member 6: ______________________________

Below is a checklist of instructions to follow when completing this assignment. Failure to follow these directions will result in penalty on your final score and/or in some problems not being graded. If multiple directions are not followed, then it is also possible that the assignment will not be accepted for any credit at all. Please contact your TA or make a post on the discussion boards for this course if you have any questions about this assignment or these directions.

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	Checklist of Instructions
	Please clearly write the names of all group members working on this assignment in the spaces allotted above.
	This assignment must be completed by a group of 3, 4, 5, or 6 members.
	This assignment is to be uploaded to gradescope as a pdf file no later than 11:59 PM EST on Sunday, April 25.
	The assignment will be uploaded by 1 group member, and that group member will be responsible for manually entering the names of all other collaborators into gradescope.
	This assignment must be completed using this template. You may either print this template to write on it and then scan it (pages ordered correctly) into a pdf file, or you may write directly on the template using programs such as notability.
	If you need more space than what is given to solve a given problem, then you will find blank pages provided at the end of this template. At the end of each problem section of this assignment you will find a space in which to indicate on what page your work is continued in case you used additional pages to complete your solution. You must provide the page number on which your work is continued in the alloted space, or write 'N/A' incase you did not use any additional pages.
	On the additional pages, you will also find space in which to indicate which problem the page is being used for, and if the page is used then that space must also be filled.
	To complete this handout, you may use your textbook, class notes, discussions with your TA and group members, and any resources that are available on Carmen. You should not receive any help from the MSLC or people outside of your group when solving these problems. You may discuss these problems on the Carmen discussion boards, but you should not provide your entire solution when answering a such question, you should only give a hint or a helpful idea.

Ungraded Optional (Modified) Problem 16.3.35: Determine if the following statements are true and give an explanation or counterexample.
(1) To find a particular solution of the equation $y'' - 4y = t^3$, you should use the trial solution $y_p = At^3$.

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(2) To find a particular solution of the equation $y'' + y' - 6y = \sin(t)$, you should use the trial solution $y_p = A\sin(t)$.

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(3) To find a particular solution of the equation $y'' + 10y' + 25y = e^{5t}$, you should use the trial solution $y_p = Ae^{5t}$.

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(4) To find a particular solution of the equation $y'' + 4y = \cos(2t)$, you should use the trial solution $y_p = A\sin(2t) + B\sin(2t)$.

Ungraded Optional Problem 16.3.40: Find the general solution of the following equation and solve the given initial value problem.

$$y'' - y = 2e^{-t}\sin(t);\ y(0) = 4, y'(0) = 0. \tag{1}$$

Draw the graph of the solution.

Problem 16.3.19 (10 points): Find a particular solution of the following equation.

$$y'' - y' - 6y = \sin(t) + 3\cos(t). \tag{2}$$

Solution: We see that the corresponding homogeneous equation of equation (2) is

$$y_c'' - y_c' - 6y_c = 0. \tag{3}$$

By examining the characteristic equation of equation (3), we see that

$$0 = r^2 - r - 6 = (r-3)(r+2) \to y_c(t) = c_1e^{-2t} + c_2e^{3t}. \tag{4}$$

Since $\sin(t) + 3\cos(t)$ is unrelated to $y_c(t)$, we may proceed to use the method of undetermined coefficients without any adjustments. We use the trial solution of $y_p(t) = A\sin(t) + B\cos(t)$ since $\{\sin(t), \cos(t)\}$ is a linearly independent set of functions that 'generates' any function which is a derivative of $\sin(t) + 3\cos(t)$. We also note that $y_p'(t) = A\cos(t) - B\sin(t)$, and $y_p''(t) = -A\sin(t) - B\cos(t)$. Plugging y_p, y_p', and y_p'' into equation (2) we see that

$$\sin(t) + 3\cos(t) = y_p'' - y_p' - 6y_p \tag{5}$$

$$= \underbrace{-A\sin(t) - B\cos(t)}_{y_p''(t)} - (\underbrace{A\cos(t) - B\sin(t)}_{y_p'(t)}) - 6(\underbrace{A\sin(t) + B\cos(t)}_{y_p(t)}) \tag{6}$$

$$= (-7A + B)\sin(t) + (-A - 7B)\cos(t) \to \begin{aligned} -7A + B &= 1 \\ -A - 7B &= 3 \end{aligned} \tag{7}$$

$$\to A = -\frac{1}{50}\Big((-A - 7B) + 7(-7A + B)\Big) = -\frac{1}{50}(3 + 7\cdot 1) = -\frac{1}{5} \tag{8}$$

$$\to B = 1 + 7A = -\frac{2}{5} \to y_p(t) = \boxed{-\frac{1}{5}\sin(t) - \frac{2}{5}\cos(t)}. \tag{9}$$

Problem 16.3.22 (10 points): Find a particular solution of the following equation.

$$y'' + y = \cos(2t) + t^3. \tag{10}$$

Solution: We see that the corresponding homogeneous equation for (10) is

$$y_c'' + y_c = 0. \tag{11}$$

By examining the characteristic equation of equation (11), we see that

$$0 = r^2 + 1 \to r = \pm i \to y_c(t) = c_1' e^{-it} + c_2' e^{it} = c_1 \sin(t) + c_2 \cos(t). \tag{12}$$

Since $\cos(2t) + t^3$ is unrelated to $y_c(t)$, we may proceed to use the method of undetermined coefficients without any adjustments. We use a trial solution of

$$y_p(t) = A\sin(2t) + B\cos(2t) + Ct^3 + Dt^2 + Et + F, \text{ and observe that} \tag{13}$$

. . . .

$$y_p'(t) = 2A\cos(2t) - 2B\sin(2t) + 3Ct^2 + 2Dt + E, \text{ and} \tag{14}$$

. . . .

$$y_p''(t) = -4A\sin(2t) - 4B\cos(2t) + 6Ct + 2D. \tag{15}$$

Plugging y_p, y_p', and y_p'' into equation (10), we see that

. . . .

$$\cos(2t) + t^3 = y_p'' + y_p = \underbrace{-4A\sin(2t) - 4B\cos(2t) + 6Ct + 2D}_{y_p''(t)} + \underbrace{A\sin(2t) + B\cos(2t) + Ct^3 + Dt^2 + Et + F}_{y_p(t)} \tag{16}$$

. . . .

$$= -3A\sin(2t) - 3B\cos(2t) + Ct^3 + Dt^2 + (E + 6C)t + (2D + F) \tag{17}$$

. . . .

$$(18) \qquad \rightarrow \begin{array}{rcl} -3A & = & 0 \text{ (by comparing the } \sin(2t) \text{ terms)} \\ -3B & = & 1 \text{ (by comparing the } \cos(2t) \text{ terms)} \\ C & = & 1 \text{ (by comparing the } t^3 \text{ terms)} \\ D & = & 0 \text{ (by comparing the } t^2 \text{ terms)} \\ E + 6C & = & 0 \text{ (by comparing the } t \text{ terms)} \\ 2D + F & = & 0 \text{ (by comparing the constant terms)} \end{array}$$

$$(19) \qquad \rightarrow (A, B, C, D, E, F) = (0, -\frac{1}{3}, 1, 0, -6, 0)$$

$$(20) \qquad \rightarrow y_p(t) = \boxed{-\frac{1}{3}\cos(2t) + t^3 - 6t}.$$

Problem 16.3.30 (8 points): (related to the Modified Ungraded problem 16.3.35 (4)) Find a particular solution of the following equation.

$$y'' + 4y = \cos(2t). \tag{21}$$

Solution: We see that the corresponding homogeneous equation of equation (21) is

$$y_c'' + 4y_c = 0. \tag{22}$$

By examining the characteristic equation of equation (22), we see that

$$0 = r^2+4 \to r = \pm 2i \to y_c(t) = c_1'e^{-2it}+c_2'e^{2it} = c_1\sin(2t)+c_2\cos(2t). \tag{23}$$

Normally, the method of undetermined coefficients would have us use the trial solution $y_p(t) = A\sin(2t) + B\cos(2t)$. However, we see in this case that $y_p(t) = y_c(t)$ (after renaming some variables), so we have to adjust our trial solution to obtain $y_p(t) = ty_p(t) = At\sin(2t) + Bt\cos(2t)$. Noting that

$$y_p'(t) = A\sin(2t) + 2At\cos(2t) + B\cos(2t) - 2Bt\sin(2t) \tag{24}$$

$$= (-2Bt + A)\sin(2t) + (2At + B)\cos(2t), \text{ and} \tag{25}$$

$$y_p''(t) = -2B\sin(2t) + 2(-2Bt + A)\cos(2t) + 2A\cos(2t) - 2(2At + B)\sin(2t) \tag{26}$$

$$= (-4At - 4B)\sin(2t) + (-4Bt + 4A)\cos(2t). \tag{27}$$

Plugging $y_p(t), y_p'(t)$, and $y_p''(t)$ into (21), we see that

$$\cos(2t) = y_p'' + 4y_p \tag{28}$$

(29)

$$= \underbrace{(-4At - 4B)\sin(2t) + (-4Bt + 4A)\cos(2t)}_{y_p''(t)} + 4\underbrace{At\sin(2t) + Bt\cos(2t)}_{y_p(t)}$$

. . . .

(30)

$$-4B\sin(2t) + 4A\cos(2t)$$

. . . .

(31)

$$\rightarrow \begin{array}{rcl} -4B & = & 0 \\ 4A & = & 1 \end{array} \rightarrow (A, B) = (\frac{1}{4}, 0) \rightarrow y_p(t) = \boxed{\frac{1}{4}t\sin(2t)}.$$

Problem 16.3.37 (12 points): Find the general solution of the following equation and solve the given initial value problem.

$$y'' + y = 4\sin(2t); \quad y(0) = 1, y'(0) = 0. \tag{32}$$

Draw the graph of the solution and determine the period of the function. (You may seek help from graphing website/software. Think about why the graph behave in that way and how is that related to the solution function.)

Solution: We see that the corresponding homogeneous equation for (32) is

$$y_c'' + y_c = 0. \tag{33}$$

By examining the characteristic equation of equation (33), we see that

$$0 = r^2 + 1 \to r = \pm i \to y_c(t) = c_1' e^{-it} + c_2' e^{it} = c_1 \sin(t) + c_2 \cos(t). \tag{34}$$

Since $4\sin(2t)$ is unrelated to $y_c(t)$, we may proceed to use the method of undetermined coefficients without any adjustments.Using a trial solution of $y_p(t) = A\sin(2t) + B\cos(2t)$, we observe that $y_p'(t) = 2A\cos(2t) - 2B\sin(2t)$ and $y_p''(t) = -4A\sin(2t) - 4B\cos(2t)$. Plugging y_p, y_p', and y_p'' into equation (32), we see that

. . . .

$$4\sin(2t) = y_p'' + y_p = \underbrace{-4A\sin(2t) - 4B\cos(2t)}_{y_p''(t)} + \underbrace{A\sin(2t) + B\cos(2t)}_{y_p(t)}. \tag{35}$$

. . . .

$$= -3A\sin(2t) - 3B\cos(2t) \to \begin{matrix} -3A & = & 4 \\ -3B & = & 0 \end{matrix} \tag{36}$$

. . . .

$$\to (A, B) = (-\frac{4}{3}, 0) \to y_p(t) = -\frac{4}{3}\sin(2t). \tag{37}$$

. . . .

Now that we have found a particular solution $y_p(t)$ to equation (32), we see that $y(t) = y_p(t) + y_c(t)$ is the general solution to equation (32). After explicitly writing down the general solution $y(t)$, we will make use of the given initial values to finish the given initial value problem.

$$\rightarrow y(t) = y_p(t) + y_c(t) = -\frac{4}{3}\sin(2t) + c_1\sin(t) + c_2\cos(t) \tag{38}$$

$$\rightarrow y'(t) = -\frac{8}{3}\cos(2t) + c_1\cos(t) - c_2\sin(t) \tag{39}$$

$$\begin{array}{rcccl} 1 & = & y(0) & = & c_2 \\ 0 & = & y'(0) & = & -\frac{8}{3} + c_1 \end{array} \rightarrow (c_1, c_2) = (\frac{8}{3}, 1) \tag{40}$$

$$\rightarrow y(t) = \boxed{-\frac{4}{3}\sin(2t) + \frac{8}{3}\sin(t) + \cos(t)}. \tag{41}$$

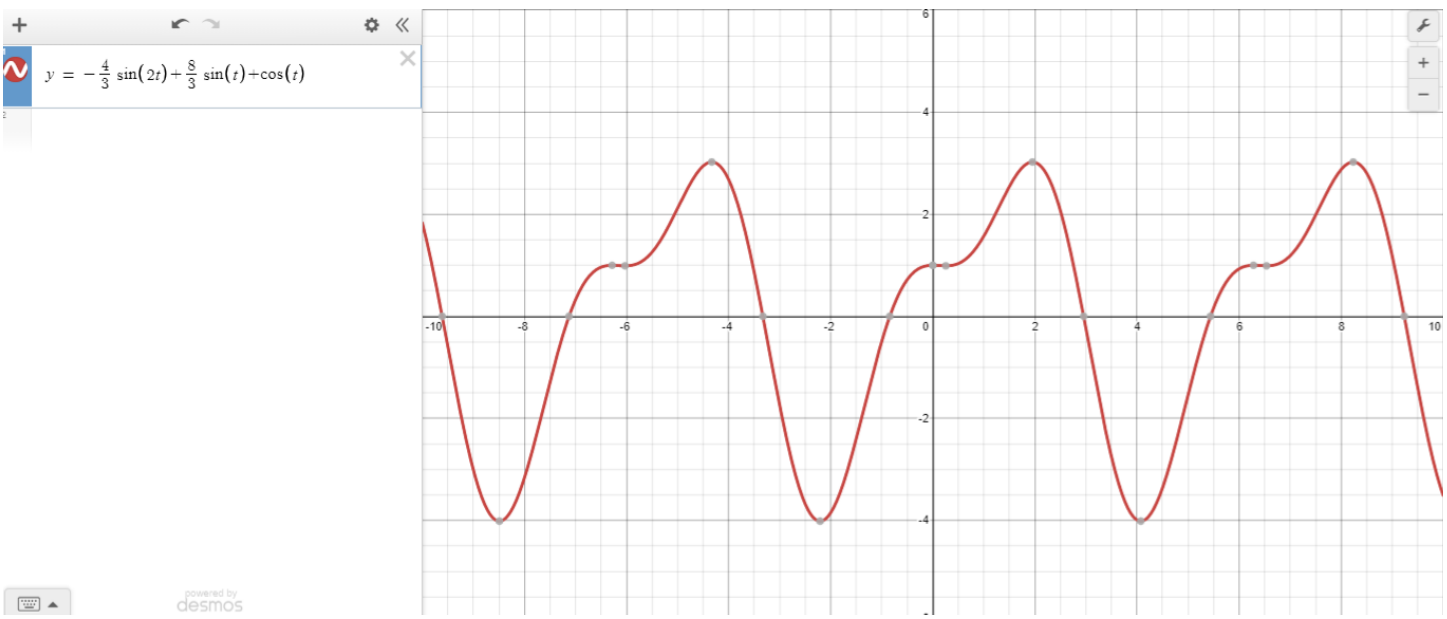

Figure 1. A graph of the solution to the initial value problem given in (32).

It is clear that $y(t+2\pi) = y(t)$, so $y(t)$ is periodic with a period of at most 2π. Based on the graph of $y(t)$, we see that the period of $y(t)$ is not smaller than 2π, so the period of $y(t)$ must be exactly 2π.