Math 1172 | Name: ____________________

Midterm 3 | OSU username (name.nn): ____________________

Autumn 2014 | Lecturer: ____________________

Recitation Instructor: ____________________

Form A | Recitation Time: ____________________

Instructions

•) SHOW ALL WORK!!! Incorrect answers with work shown may receive partial credit, but unsubstantiated answers may receive NO credit.

•) Give EXACT answers unless asked to do otherwise.

•) You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{5}{\sqrt{12}}$.

•) Calculators are permitted EXCEPT those calculators that have computer algebra systems (CAS) or ability to communicate with others. Furthermore, all memory must be cleared and all apps must be removed. PDA's, laptops, and cell phones are prohibited. Do not have these devices out!

•) The exam duration is 55 minutes.

•) The exam consists of 5 problems starting on page 2 and ending on page 6. Make sure your exam is not missing any pages before you start. Page 7 contains formulas. Page 8 may be used for extra work space.

Problem Number	Maximum Point Value	Score
1	18	
2	16	
3	22	
4	24	
5	20	
Total	100	

Problem 1

[18 pts] True or False. You do not need to show work for problems on this page.

a) [2 pts] The level curves of the function $z = 3x^2 + y^2$ for $z > 0$ are ellipses.

b) [2 pts] If the vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel, then $|\mathbf{u} + \mathbf{v}| = |\mathbf{u}| + |\mathbf{v}|$.

c) [2 pts] For any plane parallel to the yz-plane, the trace of the surface $x^2 - y - 4z^2 = 1$ in that plane is a parabola.

d) [2 pts] Given functions $f(t)$ and $g(t)$ with $f'(t)$ and $g'(t)$ continuous, the curves defined by $\mathbf{r}(t) =< f(t), g(t) >$ and $\mathbf{R}(t) =< g(t), f(t) >$ have the same length over the interval $[a, b]$.

e) [2 pts] Assume that a constant force $\mathbf{F} = \langle 3, -1, 2 \rangle$ (in newtons) is used to move an object from $(2, 0, 2)$ to $(3, 3, 3)$ (distance measured in meters.) Then, the work done by the force is 12 Joules.

f) [2 pts] If $z = f(x, y)$ is differentiable and $x = u + t$ and $y = u - t$, then $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}$.

g) [2 pts] The curve $\mathbf{r}(t) = \langle \cos t, t, \sin t \rangle$ is parameterized by arc length.

h) [2 pts] If the speed $|\mathbf{v}(t)|$ is a constant then the curve $\mathbf{r}(t)$ is a line.

i) [2 pts] $\displaystyle\lim_{(x,y)\to(-2,2)} \frac{x^3y - xy^3}{2(x^2 - y^2)} = -2$

Problem 2

[16 pts] Lines and Planes.

a) [8 pts] Find an equation of the line of intersection of the planes $2x - z = 3$ and $x + y + 3z = 2$.

b) [8 pts] Find an equation of the plane that passes through the points $(2, 0, 3)$, $(-1, 1, 2)$, and $(0, 2, 0)$.

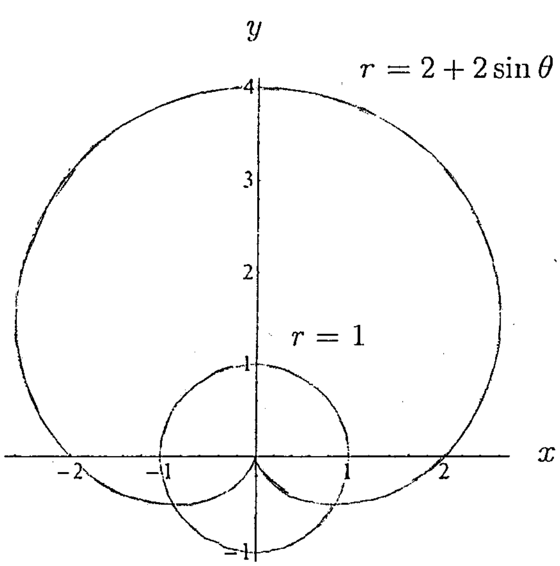
Problem 3

[22 pts] Polar Curves.

a) [6 pts] Given the polar curve $r = 2 + 2\sin\theta$, find $\dfrac{dy}{dx}$ in terms of r and θ.

b) [6 pts] Find all points, (r, θ), on the polar curve $r = 2 + 2\sin\theta$ where the tangent line is horizontal.

c) [10 pts] Set up an integral that represents the area of the region that lies inside the curve $r = 2 + 2\sin\theta$ and outside the circle $r = 1$. DO NOT EVALUATE THE INTEGRAL.



Problem 4

[24 pts] Consider a particle that at time $t = 0$ has position $\mathbf{r}(0) = \langle 1, 4, -1 \rangle$ and velocity $\mathbf{v}(0) = \langle 0, 0, 2 \rangle$. Moreover, the particle has acceleration given by $\mathbf{a}(t) = \langle 6t, -2, 0 \rangle$ for $-\infty < t < \infty$.

a) [6 pts] Find the velocity, $\mathbf{v}(t)$.

b) [4 pts] Find the unit tangent vector at time $t = 1$.

c) [6 pts] Find the position function $\mathbf{r}(t)$.

d) [4 pts] Find the points (if any) where the curve $\mathbf{r}(t)$ intersects the plane $y = 3$.

e) [4 pts] Set up an integral that represents the distance traveled by the particle along the curve over the interval $0 \leq t \leq 7$. DO NOT EVALUATE THE INTEGRAL.

Problem 5

[20 pts] Multivariable Functions.

a) [8 pts] If $f(x, y) = e^{x^2 y} \sin x$, find f_x and f_y.

b) [6 pts] State the domain and range of the function $g(x, y) = \dfrac{1}{x^2 + y^2}$.

c) [6 pts] Use the Two-Path Test to prove that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^3 - y^3}$ does not exist.

A Few Trigonometric Identities

1) $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

2) $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

3) $\cos^2\theta + \sin^2\theta = 1$

4) $\sec^2\theta - \tan^2\theta = 1$

5) $\csc^2\theta - \cot^2\theta = 1$

A Few Reduction Formulas

Assume n is a positive integer.

1) $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

2) $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

3) $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx, \quad n \neq 1$

4) $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx, \quad n \neq 1$