Math 1172

Midterm 3

Spring 2015

Form A

Name: ______________________

OSU username (name.nn): ______________________

Lecturer: ______________________

Recitation Instructor: ______________________

Recitation Time: ______________________

Instructions

•) SHOW ALL WORK!!! Incorrect answers with work shown may receive partial credit, but unsubstantiated answers may receive NO credit.

•) Give EXACT answers unless asked to do otherwise.

•) You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{5}{\sqrt{12}}$ unless asked to do otherwise.

•) NO CALCULATORS. NO CELL PHONES. NO ELECTRONIC DEVICES.

•) The exam duration is 55 minutes.

•) The exam consists of 5 problems starting on page 2 and ending on page 6. Make sure your exam is not missing any pages before you start. Page 7 contains formulas. Pages 7 and 8 may be used for extra work space.

Problem Number	Maximum Point Value	Score
1	18	
2	18	
3	27	
4	17	
5	20	
Total	100	

Problem 1

[18 pts] True or False. You do not need to show work for problems on this page.

a) [2 pts] If $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ and $\mathbf{k} = \langle 0, 0, 1 \rangle$ then $\text{proj}_{\mathbf{k}}\mathbf{v} = \langle 0, 0, v_3 \rangle$.

b) [2 pts] If $\mathbf{n}$ is the normal vector of a plane and $\mathbf{v}$ is the direction vector of a line that lies within that plane, then $\mathbf{n} \cdot \mathbf{v} = 0$.

c) [2 pts] In polar coordinates (r, θ) and $(-r, \theta - 2\pi)$ refer to the same point.

d) [2 pts] The line given by $\mathbf{r}(t) = (3t+1)\mathbf{i} + (4-t)\mathbf{j} + (2t-4)\mathbf{k}$ never intersects the plane given by $2x - y - 3z = 0$.

e) [2 pts] If $\mathbf{v} \times \mathbf{w} \neq \mathbf{0}$ then $\mathbf{v}$ and $\mathbf{w}$ are orthogonal.

f) [2 pts] The length of the curve $\mathbf{r}(t) = \langle \sin(2t), 4, \cos(2t) \rangle$ for $0 \le t \le \pi$ is 2π.

g) [2 pts] If the vector-valued function $\mathbf{r}(t)$ describes a curve that is a straight line then $\mathbf{r}'(t)$ is a constant.

h) [2 pts] If $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are differentiable vector-valued functions such that at $t = 2$ $\mathbf{v}(2) = \langle 2, -3, 0 \rangle$, $\mathbf{v}'(2) = \langle 0, 2, 1 \rangle$, $\mathbf{w}(2) = \langle 5, -3, 0 \rangle$, and $\mathbf{w}'(2) = \langle -2, 1, 0 \rangle$, then we have $\left.\dfrac{d}{dt}(\mathbf{v} \cdot \mathbf{w})\right|_{t=2} = -13$

i) [2 pts] $2 = 1 - \ln 2 + \dfrac{(\ln 2)^2}{2!} - \dfrac{(\ln 2)^3}{3!} + \cdots$

Problem 2

[18 pts] Multiple Choice. For problems on this page you do not need to show your work. There is no partial credit for these problems. In each problem, there may be more than one correct answer that needs to be circled.

a) [6 pts] Given the vector-valued function $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$, circle all of the following statements that must be true.

(i) $|\mathbf{r}'(t)|$ is increasing for $t > 0$.

(ii) $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal for all t.

(iii) $|\mathbf{r}(t)|$ is increasing for $t > 0$

(iv) $\mathbf{r}(t) \times \mathbf{r}(t) = 0$ for all t

(v) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 0$ for all t.

(vi) $\mathbf{r}'(t)$ is a unit vector for all t.

b) [6 pts] Circle all of the following parametric equations that describe the curve that starts at $(0,0)$ and moves along the right side of the parabola $y = 9x^2$.

(i) $x(t) = t,\ y(t) = 9t^2;\ -\infty < t \leq 0$

(ii) $x(t) = t,\ y(t) = 9t^2;\ 0 \leq t < \infty$

(iii) $x(t) = t^2,\ y(t) = 9t^2\ ;\ 0 \leq t < \infty$

(iv) $x(t) = 3t,\ y(t) = 3t^2\ ;\ 0 \leq t < \infty$

(v) $x(t) = t^2,\ y(t) = 9t^4\ ;\ 0 \leq t < \infty$

(vi) $x(t) = 2t,\ y(t) = 36t^2\ ;\ 0 \leq t < \infty$

c) [6 pts] Given the curve described by the parametric equations $x(t) = 2t + 1$, $y(t) = 3t^2 + 5$ for $-\infty < t < \infty$, circle all of the following statements that are true.

(i) There are two points on the curve where the tangent line is horizontal.

(ii) As t increases, $\dfrac{dy}{dx}$ increases.

(iii) The slope of the line tangent to the curve at the point $(-1, 8)$ is -3.

(iv) The graph is a parabola.

(v) There is a horizontal tangent line at the point $(1, 5)$ on the curve

(vi) The slope of the line tangent to the curve at the point $(-1, 2)$ is -3.

Problem 3

[27 pts] Motion in Space.

a) [12 pts] Given that a particle has an acceleration given by $\mathbf{a}(t) = \langle e^t, 0, 2t \rangle$ with an initial velocity $\mathbf{v}(0) = \langle 3, -4, 5 \rangle$ and initial position $\mathbf{r}(0) = \langle 6, 0, 0 \rangle$, find the position function $\mathbf{r}(t)$.

b) [15 pts] Given that a particle has the position function $\mathbf{r}(t) = \langle 3t, 20-t, -t^2+8t+80 \rangle$ (where the z-coordinate is height), find the following.

i) The position of the particle when it is at maximum height.

ii) The speed of the particle when it is at its maximum height.

iii) The acceleration vector when the particle is at its maximum height.

Problem 4

[17 pts] Planes and surfaces.

a) [8 pts] Find an equation of the plane that contains the line $\mathbf{r}(t) = \langle 4t, t+1, 3-t \rangle$ and the point $(0, 3, 5)$.

b) [9 pts] Given the surface described by $x = z^2 - 4y^2$, sketch the trace of the surface in the following planes (and give the equation for each trace.)

i) xy-plane.

ii) xz-plane.

iii) yz-plane.

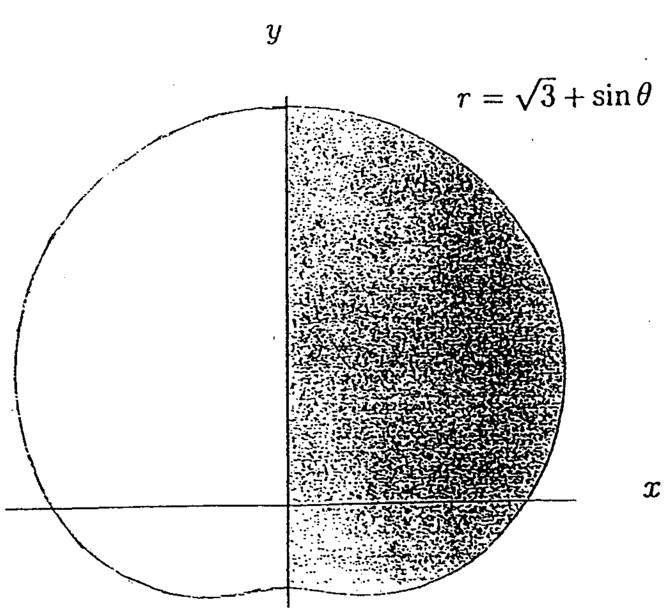
Problem 5

[20 pts] Polar Curves.

a) [6 pts] Given the polar curve $r = \sqrt{3} + \sin\theta$, find $\frac{dy}{dx}$ in terms of θ.

b) [8 pts] Find all points, (r, θ), on the polar curve $r = \sqrt{3} + \sin\theta$ where the tangent line is horizontal.

c) [6 pts] Set up an integral that represents the area of the shaded region. DO NOT EVALUATE THE INTEGRAL.



A Few Trigonometric Identities

1) $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

2) $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

3) $\cos^2\theta + \sin^2\theta = 1$

4) $\sec^2\theta - \tan^2\theta = 1$

5) $\csc^2\theta - \cot^2\theta = 1$

EXTRA WORKSPACE

Do not remove this page.