Math 1172 - Autumn 2015 Name: ______________________

Quiz 10 - In Class Recitation Instructor: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [6 pts] Suppose $\vec{r}(t) = \langle 3t, 2\sin 2t, 2\cos 2t \rangle$, $0 \le t \le 2\pi$.

a) [1 pt] Find $\vec{r}\,'(t)$ and $|\vec{r}\,'(t)|$ for this curve.

$$\vec{r}\,'(t) = \langle 3, 4\cos(2t), -4\sin(2t) \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{3^2 + (4\cos(2t))^2 + (-4\sin(2t))^2}$$
$$= \sqrt{3^2 + 4^2(\cos^2(2t) + \sin^2(2t))}$$
$$= \sqrt{3^2+4^2} = 5$$

b) [2 pts] Find the length of the curve.

$$(\text{arc})\text{length} = \int_0^{2\pi} |\vec{r}\,'(t)|\,dt = \int_0^{2\pi} 5\,dt = 5(2\pi - 0) = 10\pi$$

c) [3 pts] Is the curve parameterized by arclength? If it is not, find another description of the cruve that uses arclength as a parameter.

The arclength function

$$s(t) = \int_0^t |\vec{r}\,'(t)|\,dt = \int_0^t 5\,dt = 5t\Big|_0^t = 5t - 0 = 5t$$

$s = 5t \neq t \Rightarrow$ The curve is not parameterized by arclength.

$s = 5t \Rightarrow t = \frac{s}{5}$. plugging this in $\vec{r}(t)$, we get

$$\vec{R}(s) = \langle 3\cdot\tfrac{s}{5}, 2\sin(2\cdot\tfrac{s}{5}), 2\cos(2\cdot\tfrac{s}{5}) \rangle$$

$$= \langle \tfrac{3}{5}s, 2\sin(\tfrac{2}{5}s), 2\cos(\tfrac{2}{5}s) \rangle, \quad 0 \le s \le 10\pi.$$

$\vec{R}(s)$ uses the arclength as a parameter.