Math 1172 - Autumn 2015 Name: ____________

Quiz 10 - Take Home Recitation Instructor: ____________

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [3 pts] A golfer stands 5 m above the fairway and drives a golf ball with an initial velocity of $v_0 = \langle 0, 20, 49\rangle\, m/s$. The golfer wishes to impart slice to the golf ball, which is modeled by an acceleration of $1.2m/s^2$ in the $\hat{x}$ direction. Thus, the acceleration function is given by:

$$\vec{a}(t) = \langle 1.2, 0, -9.8\rangle\,.$$

Assuming $\vec{r}(0) = \langle 0, 0, 5\rangle$, determine:

a) [2 pts] The velocity and position functions.

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle \int 1.2\,dt, \int 0\,dt, \int -9.8\,dt\rangle = \langle 1.2t + c_1, c_2, -9.8t + c_3\rangle$

$\vec{v}(0) = \langle c_1, c_2, c_3\rangle = \vec{v}_0 = \langle 0, 20, 49\rangle$. So $c_1 = 0$, $c_2 = 20$, $c_3 = 49$

and $\vec{v}(t) = \langle 1.2t, 20, -9.8t + 49\rangle$,

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \int 1.2t\,dt, \int 20\,dt, \int -9.8t + 49\,dt\rangle$

$= \langle 0.6t^2 + d_1, 20t + d_2, -4.9t^2 + 49t + d_3\rangle$

$\vec{r}(0) = \langle d_1, d_2, d_3\rangle = \langle 0, 0, 5\rangle$. So $d_1 = 0$, $d_2 = 0$, $d_3 = 5$

and

$\vec{r}(t) = \langle 0.6t^2, 20t, -4.9t^2 + 49t + 5\rangle$

b) [1 pt] The maximum height of the golf ball. height $z(t) = -4.9t^2 + 49t + 5$

Set $z'(t) = -9.8t + 49 = 0 \Rightarrow t = 5$ (critical point)

$0 < t < 5 \longleftrightarrow z'(t) > 0 \longleftrightarrow z(t)$ increases

$t > 5 \longleftrightarrow z'(t) < 0 \longleftrightarrow z(t)$ decreases

$\Rightarrow z(5)$ is the max height

and $z(5) = -4.9 \times 25 + 49 \cdot 5 + 5 = \frac{255}{2} = 127.5$ m.

c) [1 pt] The range of the shot; that is, the distance between where the ball lands and $\langle 0, 0, 0\rangle$. Set $z(t) = 0$ to solve for travel time t.

$-4.9t^2 + 49t + 5 = 0 \iff 49t^2 - 490t - 50 = 0$

$\iff t^2 - 10t - \frac{50}{49} = 0 \iff t^2 - 10t + 25 - \frac{50}{49} - 25 = 0$

$\iff (t-5)^2 = \frac{50}{49} + 25 \iff t = 5 + \pm\sqrt{\frac{50}{49} + 25} = 5 \pm \frac{5\sqrt{51}}{7}$

So $t = 5 + \frac{5\sqrt{51}}{7}$ (time when the ball lands). So range of shot

is $\left|\vec{r}\left(5 + \frac{5\sqrt{51}}{7}\right)\right| = \left|\left\langle \frac{1500}{49} + \frac{30\sqrt{51}}{7}, 100 + \frac{100\sqrt{51}}{7}, 0\right\rangle\right| \approx 211.092$ m