

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

**Problem 1** [3 pts] True or False

**Directions:** CIRCLE ALL of the statements that are TRUE. No explanation is necessary. Note that there may be several statements that are true for each question! This question is worth 3 pts, with 1 deducted for each incorrect answer. You cannot score below 0 for this problem.

Suppose that  $\{a_n\}_{n \geq 1}$  is a sequence and  $\sum_{n=1}^{\infty} a_n$  converges to  $L > 0$ . Let  $s_n = \sum_{k=1}^n a_k$ .

A.  $\lim_{n \rightarrow \infty} a_n = L$

B.  $\lim_{n \rightarrow \infty} a_n = 0$

C.  $\lim_{n \rightarrow \infty} s_n = 0$

D.  $\lim_{n \rightarrow \infty} s_n = L$

E.  $\sum_{n=1}^{\infty} s_n$  MUST diverge

F.  $\sum_{n=1}^{\infty} (a_n + 1) = L + 1$

G. The divergence test tells us  $\sum_{n=1}^{\infty} a_n$  converges to  $L$ .

**Problem 2** [3 pts]

Determine whether the series:

$$\sum_{k=1}^{\infty} 2^{2-2k}$$

converges or diverges and if it converges, give its value. JUSTIFY your answer!

$$\sum_{k=1}^{\infty} 2^{2-2k} = \sum_{k=1}^{\infty} 2^2 \cdot \frac{1}{2^{2k}} = \sum_{k=1}^{\infty} 2^2 \left(\frac{1}{2^2}\right)^k = \sum_{k=1}^{\infty} 2^2 \cdot \left(\frac{1}{4}\right)^k$$

The ratio is  $r = \frac{1}{4}$ ;  $|r| < 1$ . So the series converges to

$$\frac{\text{1st term}}{1-r} = \frac{2^{2-2 \cdot 1}}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

( You can also use  $\frac{a_{k+1}}{a_k}$  to find  $r$  :

$$r = \frac{a_{k+1}}{a_k} = \frac{2^{2-2(k+1)}}{2^{2-2k}} = 2^{2-2k-2-2+2k} = 2^{-2} = \frac{1}{4} . )$$