Math 1172 - Autumn 2015 Name:

Quiz 9 - Take Home Recitation Instructor:

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [2.5 pts] (Projecting a Vector Field Onto a Curve)
In multivariable calculus, many problems require one to find the component of a vector (field) onto a given curve at each point along the curve.

Suppose $\vec{r}(t) = \langle t^2, 4t, 4\sin t \rangle$.

a) [1 pt] Calculate the *unit* tangent vector $\hat{T}(t)$ when $t = 0$.

$\vec{r}\,'(t) = \langle 2t, 4, 4\cos t \rangle$, $\vec{r}\,'(0) = \langle 0, 4, 4\cos 0 \rangle = \langle 0, 4, 4 \rangle$

so $\hat{T}(0) = \dfrac{\vec{r}\,'(0)}{|\vec{r}\,'(0)|} = \dfrac{\langle 0,4,4\rangle}{\sqrt{0^2+4^2+4^2}} = \dfrac{\langle 0,4,4\rangle}{4\sqrt{2}} = \langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

b) [.5 pts] Show that for any vector $\vec{F}$ and *unit* vector $\hat{v}$ that $scal_{\hat{v}}\,\vec{F} = \vec{F} \cdot \hat{v}$

Proof. By definition, $scal_{\hat{v}}\,\vec{F} = \dfrac{\vec{F} \cdot \hat{v}}{|\hat{v}|}$.

But $|\hat{v}| = 1$ since $\hat{v}$ is a unit vector. Hence $scal_{\hat{v}}\,\vec{F} = \vec{F} \cdot \hat{v}$.

c) [1 pt] Suppose $\vec{F} = \langle -1, 2, 4 \rangle$. Find $scal_{\hat{T}(0)}\,\vec{F}$. This is the magnitude of the component of the vector $\vec{F}$ along the curve $\vec{r}(t)$ at $t = 0$.

By b), $scal_{\hat{T}(0)}\,\vec{F} = \vec{F} \cdot \hat{T}(0) = \langle -1, 2, 4 \rangle \cdot \langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= -1 \cdot 0 + 2 \cdot \frac{1}{\sqrt{2}} + 4 \cdot \frac{1}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$.

Problem 2 [1.5 pts] Suppose $\vec{r}(t)$ is a differentiable vector-valued function and $|\vec{r}(t)| = 1$.

a) [.5 pts] (True or False) Is $\vec{r}\,'(t)$ a unit vector for each value of t? Think about this both conceptually and computationally!

False. Counter example: $\vec{r}(t) = \langle 1, 0, 0 \rangle$. Clearly $\vec{r}(t)$ is differentiable and $|\vec{r}(t)| = 1$, but $\vec{r}\,'(t) = \langle 0, 0, 0 \rangle$ with $|\vec{r}\,'(t)| = 0$.

There are other examples, e.g. $\vec{r}(t) = \langle \cos(2t), \sin(2t) \rangle$.

b) [1 pt] Show that $\vec{r}(t)$ and $\vec{r}\,'(t)$ are orthogonal for each value of t.

Hint: $\vec{r}(t) \cdot \vec{r}(t) = 1$ for all t.

Proof. $\vec{r}(t) \cdot \vec{r}(t) = 1$. By the product rule for dot products,

$$\frac{d}{dt}\left(\vec{r}(t) \cdot \vec{r}(t)\right) = \vec{r}\,'(t) \cdot \vec{r}(t) + \vec{r}(t) \cdot \vec{r}\,'(t) = 2\,\vec{r}\,'(t) \cdot \vec{r}(t) = 0$$

so $\vec{r}\,'(t) \cdot \vec{r}(t) = 0$, i.e. $\vec{r}(t)$ and $\vec{r}\,'(t)$ are orthogonal for all t.