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Talent Search and Development

Arnold E. Ross

Emeritus Professor of the Ohio State University

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Abstract

Approaches to funding and encouraging talented students in several countries are discussed.

1. Introduction

Every teacher knows that searching for young talent and helping it to come to fruition provides a deeply satisfying and stimulating activity for himself as well as for the student. When a teacher meets a very talented youth, he naturally receives him warmly, encourages and helps him. But my contention is that, considering the realities of our science fiction world (see [1]), an occasional happy encounter with talented youngsters, although always welcome, does not fill the world's acute need for a large reserve of trained talent. I believe it is our duty to take a strong initiative in searching out the very gifted. We must reach out to them when they are still very young and impressionable, and, since talent is no respecter of social status, our search must be carried out with the broadest possible social base in mind, as I have outlined in [2].

With the rapid growth of college enrolments and the concomitant financial pressures for uniformity in the treatment of students, one is tempted to present strong arguments against these unhappy levelling tendencies. I shall, however, resist this temptation for the present, and devote my discussion exclusively to the very young who are also very talented.

The first highly successful program of nurturing of young talent was developed by our Hungarian colleagues at the turn of the century, as mentioned in [1]. The Russian mathematicians followed in their footsteps, and have since created an academic mechanism for talent search and development on an impressively large scale. The competitive examination component of their method has become common practice on an international scale (Olympiads, etc.). A number of related programs of varied duration and design came into being in the U.S.A. after the launching of Sputnik, and in recent years such programs have been introduced in Australia and in India as well.

It is my intention to devote the present account to a summer program in the U.S.A.

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with which I have been intimately associated for the past twenty years (see [3]), and also to two programs of similar nature, one in India and one in Australia, in which it has been my good fortune to participate within the last three years.

2. Summer Programs for talented youngsters

I believe that an effective way of outlining the basic considerations in our program design is to share the trials, tribulations and soul-searching which beset us, as we determined the direction of our effort.

When in the summer of 1957, seven eager and bright high-school youngsters approached us for guidance at the University of Notre Dame, Indiana, U.S.A., independently of each other, our summer program for secondary school teachers of mathematics was a going concern. With the memory of Sputnik still fresh in everyone's mind, our teacher-students were under great pressure to contribute to a reform of science education in their own schoolrooms. We on the faculty were expected to provide them with the mathematical insights and methods appropriate to the raising of a new generation of scientists.

The insertion of the seven youngsters into the secondary school teachers' group promised to test severely our choice of curriculum content as well as our teaching skills. Our young visitors rapidly put our misgivings to rest. They responded enthusiastically to every opportunity for involvement, grasped ideas which were traditionally considered inaccessible to the very young, and mastered relevant techniques so well as to perform outstandingly on the periodic tests taken together with over a hundred participating teacher-students.

It was not long before our high school participants began to relate their new intellectual experiences to their life ambitions; these varied from engineering, to the sciences and mathematics. As time went on our work with the youngsters began to acquire a character all its own; many questions were raised, some of which our colleagues in Hungary and the U.S.S.R. had faced before us, some represented new questions and problems set to us by the rapidly changing world.

Thanks to the enthusiastic support of our teacher-students and to the assistance of many university colleagues who learned about our experiment, the number of able high school participants grew from year to year until it stabilized at about seventy. This limit was imposed primarily by the available material resources. The summer of 1962 was quite typical. Our group of participants consisted of 73 boys and girls who came from 18 states of the U.S.A., represented 71 secondary schools (high schools) and was distributed in a very wide age range from 14 to 17 years.

Whenever the selection of participants relies on some measure of their potential, judged through the quality of their initiative, the intensity of their interest and the degree of their perseverance, one is bound to bring together a group with a wide spectrum of interests. At the outset, therefore, one must confront the question of what purpose should be served by a mathematics program for a collection of young individuals who have in common only eagerness, curiosity, an unbounded (and hitherto undirected) supply of vitality and, possibly, an ultimate destiny in science.

One may decide upon the selection of some mathematical skills generally considered useful in the sciences. One may choose some applications falling within the range of the student's experience and jointly treat the mathematical tools and the ideas in this selected field of application. One may, on the other hand, take advantage of the fact that with a proper choice of material one can remain within the field of mathematics and yet exhibit a whole gamut of problems which will confront every scientist.

Even though we did not intend to neglect any of the above alternatives entirely, we subordinated everything to the aim of providing our young charges with the opportunity of acquiring experience in scientific thinking, while remaining within the field of mathematics. We have never regretted this choice of strategy.

3. The role of number theory in the programs

Since 1957, we have used number theory as the basic vehicle for the development of the student's capacity for observation, invention, the use of language, and all those traits of character which constitute intellectual discipline.

We have chosen number theory because of its wealth of accessible, yet fundamental and deep, mathematical ideas and for its wealth of challenging but tractable problems. Our choice has turned out to be practical. In our treatment of number theory, we make use of the fact that this subject can lead to the study of modern algebra and hard analysis. We have also found that an appropriate treatment of number theory lays the groundwork for the study of combinatorics and of discrete mathematics generally, a range of mathematical ideas which is becoming increasingly important in science and technology.

Although the problem-solving experience which is acquired by the student who prepares himself for challenging competitive examinations is clearly valuable, we have felt that the education of future scientists should also encourage the kind of involvement which develops the student's capacity to observe keenly, to ask astute questions and to recognize significant problems. This last factor is important in scientific education for two reasons. First, the progress of every science depends upon the capacity of its practitioners to ask penetrating questions and to identify important problems. Second, we believe that personal discovery is a vital part of the learning process for every individual eager to gain deep insight into his subject.

Thus, as we work at number theory, our aim is to develop attitudes as well as skills. From the very beginning the participant is given an opportunity to develop his powers of observation, to experiment and to discover significant relations between the objects of his experimentation. He learns to use counter-examples to destroy untenable conjectures, and as his experience grows he begins to provide the security of a proof for the surviving conjectures. As is natural in all fields of human activity, word labels follow the recognition of phenomena; the incentive for the precise and concise use of language comes from the desire to share one's experience with others.

The range of the subject matter studied depends upon the duration of each program. In our India (pilot) Program of three weeks duration (All India Summer Institute in Mathematics, Bangalore, 1973) directed by Professor V. Krishnamurthy of Birla Institute of Technology and Science in Pilani, we were able to bring within the reach of our students an impressive selection of topics in number theory (see the description below), thanks to the assistance of our colleagues, Professor Krishnamurthy himself and Dr T. Sonndarajan of Madurai University.

In our two-week Australian Summer (January) Program in Canberra, directed since 1969 by Professor Larry Blakers of the University of Western Australia, we have used number theory as a systematic introduction to mathematical thinking for the past three summers, each summer learning how to induce an ever deeper involvement of our young charges. The summer of 1977 (January 8–January 21) has been the most satisfying one to date.

The duration of our U.S.A. Summer Program is eight weeks. In it we provide sufficiently varied mathematical activity to make it worthwhile inviting some of its participants to return for the second or even the third summer. The program moved from Ohio State University to the University of Chicago Campus in 1975; Professor Felix Browder served as its director until the summer of 1977 [3].

Eight weeks gives us sufficient time to allow for reasonable scope and depth in our treatment of number theory. The details below give a fair account of the range of ideas with which we deal. Naturally each summer there are variations of emphasis in the treatment of different sections due to a variation of student response to different topics, or to the needs of colleagues discussing some related ideas.

The following list of topics will serve as the subject matter outline: Euclid's algorithm, continued fractions and their uses in constructing algorithms for the solution of linear Diophantine equations in Euclidean rings; divisibility theory; residue class rings $\mathbb{Z}_m$ and in particular the structure of the group of units in $\mathbb{Z}_m$; quadratic congruences, including the law of reciprocity; basic properties of arithmetic functions, including the Moebius inversion formula and some questions regarding $\pi(x)$. A reasonably detailed study is made of other arithmetics such as the arithmetic of polynomials, the domain of Gaussian integers and some other quadratic domains, Euclidean or not; finite fields appear as residue class rings in $\mathbb{Z}_p[x]$; the Pell equation and questions of best approximation arising in the task of approximating irrationals by rationals; construction of the group of units in the arithmetic of some real quadratic fields; geometric methods; Minkowski's theorem on lattice points and symmetric convex bodies; representation of integers by the sum of two and by the sum of four squares; Diophantine equations.

Every opportunity is taken to point out and make use of relationships between arithmetic on the one hand and algebra, geometry and analysis on the other. A strong effort is made to provide the participant with a very large number of problems, many of which call for considerable ingenuity. Extension of some basic results in the arithmetic in $\mathbb{Z}$ to polynomials and to some quadratic fields is undertaken as an exercise of insights and methods put together under the title 'techniques of generalization'. Whenever possible, constructive methods are used, and the student is encouraged to master available algorithms.

4. Special topics in the Indian, Australian and Chicago programs

In our India program of 1973, in addition to number theory some selected topics in combinatorics were discussed by Professor Krishnamurthy. Professor

K. Venkatachaliengar gave a number of talks on the work of Ramanujan and Professor M. Venkataraman of the University of Madurai discussed some fundamental questions concerned with the relationship of mathematics with the sciences.

In the Australian Program different mathematical topics were selected in different years by Dr M. Newman of the Australian National University (ANU) and Dr T. M. Gagen of the University of Sydney and treated in depth during the whole two-week session. In 1977 Dr. Newman discussed the geometric foundations of the concepts of area and volume and Dr Gagen introduced some combinatorial problems. Special one-hour lectures were given by Dr R. Bryce of ANU (The Geometry of Ellipses), by Professor Toby Lewis of CSIRO and University of Hull (Lanchester and his Equations for Conflict), by Mr Ian White of ANU (Mathematics in the Real World—Applications to Fibre Optics) and by Dr J. Gani of the CSIRO Division of Mathematics and Statistics (Simple Mathematical Models in Biology).

In our University of Chicago Summer Program, a course in group theory taught by Professor Jonathan Alperin has served as a companion course to number theory. In 1976 this group theory course was designed so that its first part dealing with fundamentals was accessible to all the first summer participants. Its middle part dealt with the Sylow theorems, with some of their applications, and was significantly more difficult. The last part treating Galois theory represented a quantum jump in difficulty and was accessible only to the most experienced participants. Students were given many problems and interesting examples.

We have felt for some time that our most experienced participants, particularly those who come to us for a second or third summer in succession and whose interests definitely point in the direction of mathematics and science, should have an opportunity to gain some appreciation of the subtler concerns of the experimentalist. They should acquire some feeling, within the limitations of their experience, for the preoccupations of the theoreticians of science. We have been fortunate that our scientific colleagues at the University of Chicago have been willing to help us in this difficult task.

Professor Robert Geroch of the Departments of Mathematics and Physics conducted an elementary seminar in the theory of relativity for a small group of advanced participants. The groundwork was laid down in the summer of 1975 by his course on four-dimensional geometry.

Professor Gerhart Closs, A. A. Michelson Distinguished Service Professor of Chemistry, gave a series of five fascinating lectures on 'Some Simple Applications of Group Theory to Chemical Problems'. These began in the second half of the program on July 26 and were open to all of the 1976 program participants. In preparation for these lectures, however, some of the more experienced participants studied an elementary version of group representation theory and were able to calculate character tables for all the relevant finite groups by the time Professor Closs's lectures began. They thereby shared in the task of building a meaningful bridge from mathematics to chemistry, in the style of contemporary research trends in theoretical chemistry.

Dr David Y. Y. Yun of the IBM Research Laboratory gave a course of twenty lectures on 'Symbolic and Algebraic Computation'. This is a subject of growing importance in the effective use of high speed digital computers in science and technology.

The following special lectures were given in the summer of 1976: 'The First Twenty Field Prize Winners' by Professor Irving Kaplansky. (The Field Prize—a gold medal and cash—is the closest thing in mathematics to a Nobel Prize.) 'Prime Numbers and Brownian Motion' by Professor Patrick Billingsley. 'Can Computers do Algebra and Calculus?' by Dr. Yun. 'The Notion of Derivative and its Generalizations' by Professor Antoni Zygmund.

5. Selection methods for participants in our U.S.A. program

Selection of a group of participants who can benefit from what we do for them has always presented a difficult problem. In response to requests for information about our summer program we send out packets of material containing a description of the program, an extensive questionnaire, and, most important, a set of problems. These problems on elementary mathematical topics are meant to test the respondent's ingenuity and interest, rather than breadth of experience. Almost all of our applicants have had formal academic records of top quality. The applicant's essay, requested in the questionnaire, and the way in which the applicant handles the challenging problems help us to evaluate his real potential better than does his formal record.

Since we look for talent and creativity rather than experience in the applicants, we have had to face the problem of the diversity thus generated in the background of the students selected. This problem is not resolved by restricting participants to a given age level or to a designated secondary school grade. Indeed, the last rather artificial restriction greatly diminishes the effectiveness of our search for talent. A more effective response to the unavoidable variations in experience, temperament, or thrust of major interest in almost every group of talented youngsters, calls for great effort and ingenuity in carrying through the academic program, both in the classroom and outside it. This effort must be made, and its effect makes it well worthwhile.

Although the majority of our participants come to us at the conclusion of their penultimate year in high school, a number of very gifted and very young participants also join the program. It has been our firm principle in serving the basic purpose of talent search and development, that it is best when making contact with these talented young people, to try to maintain our association with them until they enter college. This continued association has a tremendously significant effect upon their intellectual growth. At the same time, a dedicated youngster of proven talent brings to the success of our program at least as much as he or she carries away. Thus the best interest of all the participants, and not merely of the most accomplished, is served if a small group of extremely able participants returns for a second or third summer. These returning students, though undertaking an intellectual program at a higher level than the new participants, contribute immeasurably to the *esprit de corps* of the whole group. By their example, by the quality of the intellectual stimulus which they provide for the ablest newcomers, and by serving as an example of the recognition which we accord to outstanding achievement, they add much to our enterprise.

6. Counsellors and student involvement

An unusually high degree of involvement by the participants is absolutely essential for the success of our program. We have been able to achieve such student involvement through the contagious enthusiasm and dedication of the young people who serve as counsellors and the elder statesmen of the program. Our counsellors are drawn primarily from former graduates of the program who are studying mathematics as undergraduates or graduate students in some of the best colleges and universities of the U.S.A. We have noted the deep impression which the shared responsibility for the success of our program has made upon our young counsellors. Their dedication, their technical competence, and their deep sense of responsibility for their charges contributes greatly to our success. Their task is technically demanding since we are able to work at non-trivial mathematics with our gifted young participants. It also demands a constant association, in the same living quarters, with bright and ambitious youngsters less experienced than themselves. The apprenticeship which our counsellors undergo encourages the development of the best qualities of academic citizenship.

We have always found a discussion of the problems which we face with interested and sympathetic colleagues to be most stimulating and encouraging. I hope that some of our colleagues who share our concerns for talent search and development will find it possible to share their ideas and concerns with us.

References

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