We pursue here a method of speculative reconstruction in order to detail what can be learned about the “state of the art” in the early development of “liberal education” in fifth-century Greece. One needs to be cautious in speaking about such a development at such a time, which pre-dates the establishment of any independently operating institution devoted to such an educational project. Yet, a speculative reconstruction of the cultural milieu in which mathematical knowledge was growing in both sophistication and in audience can be advanced through the careful analysis of what must be considered (among many other achievements) the foremost example of the practical application of mathematical knowledge in the mid-fifth century: the Parthenon. We believe that such a reconstruction is crucially important for our present moment because investigating the Parthenon as a “vanishing mediator” in the history of liberal education—perhaps, we will suggest, the most important such intermediary—offers unique insights into the state of liberal education today, both with respect to the challenges it faces and with respect to the promises it holds. Specifically, coming to see the Parthenon as a manifestation in material form of the quest to achieve a formal integration of the mathematical arts points to a way in which liberal education has been, and could now be, a vital part of the civic life of a democratic society.

Through the careful consideration of the educational program behind the design of the Parthenon and in its role as a form of civic education, we hope to show that this is the spring of liberal education—the unification of practice and theory that is made possible by a humanistic expression of technical knowledge. We aim to get there by showing how the practical arts (for instance in the design and construction of this example of monumental architecture) are the key mechanism in the birth (and continued life) of liberal education.

Introduction: Thinking the Parthenon and Liberal Arts Education Together
“Pythagoras introduced the quadrivium to Greece.” This traditional understanding—this “creation myth”—of Pythagoras as the first philosopher is attested very early in the classical canon. Though the text is very understated, Plato’s observation that Pythagoras, like Homer, was hailed as a “master of education” points to a then-established view that holds Pythagoras as a model of what Aristotle already refers to as “liberal education.” Our suggestion is that the procedure we will follow in our analysis—testing a formal understanding (in this case of mathematics as an object of theoretical study) against a material object—is precisely the model that Pythagoras introduced as a “model educator,” and the one that inspired the design of the Parthenon. There is very good reason to believe that this “knowledge procedure” belongs to the people responsible for the Parthenon, as well as ourselves.

Part I. Plato on Dialectic and the Problem-Based Study of Mathematics

1. Dialectic and the Mathematical Arts in Republic (9.587b–588a)
The textual basis for our attempted reconstruction of the “constructivism vs. realism” debate about the foundations of mathematics and the nature of mathematical objects of knowledge begins with the place where Plato has Socrates argue that the life of the just man is exactly 729
times more pleasant than the life of the unjust man. In what follows, we will show why we find this seemingly comic and surely not literally intended claim at the moment of greatest drama in the dialogue, the place where Socrates will, once and for all, most decisively answer the central question of the Republic, which is not “What is justice?” but rather “Why be just?” In presenting our account, we offer a perspective on the broader problematic of the relationship between dialectic and mathematics in the dialogues, which we hold to be fundamentally linked with the debate in which Plato was participating about the reality and ideality of mathematical objects.

2. Dialectic and the Mathematical Arts in Timaeus (35b–36c)
In Timaeus, Plato has Timaeus offer Philolaus’s scale (in words virtually identical to the surviving fragments of Philolaus) as the means by which the world is constructed. We will find that fully appreciating Plato’s reception of Philolaus’s construction of the scale entails taking account of both his own appreciation of the Pythagorean interdisciplinary insight into the implications of number theory for harmonics and also his ultimate skepticism that a mathematical approach such as this will ever arrive at the highest truths. We must come to terms with the Timaeus who speaks of a cosmology that cannot in fact be Plato’s own view of the way the form of the good is manifest in the cosmos.

3. Platonic Dialectic, Pythagorean Harmonics, and Liberal Arts Education
Plato’s approach to the place of mathematics in liberal education, on our view, yields two main answers to our biggest questions in approaching the Parthenon as an institution of liberal education. First, Plato thinks that doing mathematics is important, even integral, to cultivating a philosophical disposition, but does not in itself constitute a philosophical disposition. Second, we would best understand the practitioners of the mathematical arts (such as those who worked on the Parthenon) as predecessors who helped articulate the problems that would-be philosophers should try to cope with, but not as models for philosophical practice itself.

Part II. Harmonia and Symmetria of the Parthenon

4. The Parthenon and the Musical Scale
The Parthenon’s design was developed using a rigorous system of continuous proportion (symmetria), most likely based on a triglyph module as unit, a method of construction that became increasingly characteristic of Doric architecture during the later sixth century and the first half of the fifth century. This chapter considers the close relationship between numerical, specifically musical, ratios and the geometry of architectural form in the Parthenon, and in a small number of related Doric monuments from the half century preceding its construction, and attempts to reconstruct the relationship of these architectural problems to contemporaneous developments in music theory and practice. The practical work done on these architectural and musical problems, furthermore, embodied and indeed helped bring about the principal mathematical developments of the fifth century.

5. The Corner Problem
As in the construction of the ancient Greek musical scale, so too in building the Parthenon, considering a geometric object in arithmetic and musical terms as a means of forging an aesthetic (and ontological) whole gave rise to an irreducible tension between magnitude and multitude. That productive tension is decisive in thinking through harmonics (harmonia) in the
Parthenon, i.e., in joining together conflicting elements to create a unity. In Doric architecture, this problem emerges most fully in the need for a harmonious articulation of the building’s corner, and in the multiple small adjustments across the façades of the building that this entails, engaged with unique intensity in the Parthenon.

6. Refinements and the Question of Dialectic
The very issues addressed in the articulation of the Parthenon’s corners perhaps gave rise to the famous “optical refinements” in temples of the early to mid-5th century BCE. In this chapter we argue that these refinements, subtle adjustments to proportionality as well as the introduction of curvature in the building, are made not with optical considerations in mind but rather as a means of addressing and reflecting upon the ontological paradoxes within harmonics as it emerged from working with arithmetic and geometry in close relationship to each other. The problem of integrating the elements of the Parthenon’s construction (based in symmetria) with the overall conception of the building as a unified, organic whole (governed by harmonia) leads not only to the assertion of harmonia’s primacy over symmetria but also to sustained engagement with open-ended questions in a manner that could properly be called dialectical, as it would later be theorized and developed by Plato and the Academy.