A Macroeconomic Framework for Quantifying Systemic Risk

Zhiguo He, University of Chicago and NBER
Arvind Krishnamurthy, Stanford University and NBER

Bank of Canada, August 2017
Financial Crisis in the Model

Note: Capital constraint binds for $e < 0.3957$
Matching Recent Crisis:  \textit{Data(L) and Model(R)}

He and Krishnamurthy (Chicago, Stanford)
Outline

1 Nonlinear macro model of a financial crisis
   ▶ Recent work on financial intermediaries: He-Krishnamurthy, Brunnermeier-Sannikov, Rampini-Viswanathan, Adrian-Boyarchenko, Gertler-Kiyotaki
   ▶ Our approach: occasionally binding constraint; global solution method (similar to Brunnermeier-Sannikov, Adrian-Boyarchenko)
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2 Calibration and Data
   ▶ Nonlinearity in model and data
   ▶ Match conditional moments of the data, conditioning on negative (i.e., recession) states
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2 Calibration and Data
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   ▶ Match conditional moments of the data, conditioning on negative (i.e., recession) states

3 Quantify systemic risk
   ▶ Systemic risk: the state where financial intermediation is widely disrupted to affect real activities severely
     ★ In the model, states where capital constraint binds, crisis state
   ▶ What is the ex-ante (e.g., initial conditions of 2007Q2) likelihood of crisis states? (... low)
   ▶ What makes the probability higher?
   ▶ Economics of stress tests (as opposed to accounting of stress tests)
Agents and Technology

- Two classes of agents: households and bankers
  - Households:
    \[ \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \left( c_t^y \right)^{1-\phi} \left( c_t^h \right)^\phi \, dt \right], \]

- Two types of capital: productive capital \( K_t \) and housing capital \( H \).
  - Fixed supply of housing \( H \equiv 1 \)
  - Price of capital \( q_t \) and price of housing \( P_t \) determined in equilibrium
Agents and Technology

- Two classes of agents: households and bankers
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    \]

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- Production $Y = AK_t$, with $A$ being constant

- Fundamental shocks: stochastic capital quality shock $dZ_t$. TFP shocks
  \[
  \frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t
  \]
Agents and Technology

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- Fundamental shocks: stochastic capital quality shock \( dZ_t \). TFP shocks
  \[ \frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t \]

- Investment/Capital \( i_t \), quadratic adjustment cost
  \[ \Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t \]
  \[ \max_{i_t} q_t i_t K_t - \Phi(i_t, K_t) \Rightarrow i_t = \delta + \frac{q_t - 1}{\kappa} \]
Aggregate Balance Sheet

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_tK_t$  |  Equity $E_t$

Household Sector

Financial Wealth

$W_t = q_tK_t + P_tH$

Housing $P_tH$  |  Debt $W_t - E_t$
Aggregate Balance Sheet

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Housing $P_tH$

Debt $W_t - E_t$

Household Sector

Financial Wealth

$W_t = q_tK_t + P_tH$

$(1 - \lambda)W_t$

$\lambda W_t = "Liquid balances"

benchmark capital structure

He and Krishnamurthy (Chicago, Stanford)

Systemic Risk

Bank of Canada, August 2017
Equity Matters

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

Equity $E_t$

Housing $P_t H$

Debt $W_t - E_t$

Separation of ownership and control

Banker maximizes $E[ROE] - \frac{\gamma}{2} Var[ROE]$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

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benchmark capital structure
### Intermediary Equity Constraint

**Loans to Capital Producers** \( i_t \)

**Intermediary Sector**

- **Capital** \( q_t K_t \)
- **Equity** \( E_t \)
- **Housing** \( P_t H \)
- **Debt** \( W_t - E_t \)

**Aggregate bank capital capacity** \( E_t \)

\[
\frac{dE_t}{E_t} \text{=} \text{ROE}, \text{ ROE is endogenous}
\]

**Household Sector**

- **Financial Wealth**
  \[
  W_t = q_t K_t + P_t H
  \]
  \[
  (1 - \lambda) W_t
  \]
  \[
  \lambda W_t = "\text{Liquid balances}"
  \]

**Separation of ownership and control**

**Banker maximizes**

\[
E[ROE] - \frac{\gamma}{2} \text{Var}[ROE]
\]
Single Bank/Banker Choice of Portfolio and Leverage

<table>
<thead>
<tr>
<th>Capital $q_t k_t$</th>
<th>equity$_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing $P_t h_t$</td>
<td>debt$_t$</td>
</tr>
</tbody>
</table>

Portfolio share in capital: $\alpha^k_t = \frac{q_t k_t}{\text{equity}_t}$

Portfolio share in housing: $\alpha^h_t = \frac{P_t h_t}{\text{equity}_t}$

Borrowing (no constraint): $\text{debt}_t = q_t k_t + P_t h_t - \text{equity}_t = (\alpha^k_t + \alpha^h_t - 1)\text{equity}_t$
Bank Choice of Portfolio and Leverage

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Return on bank equity ROE: $d\tilde{R}_t = \alpha^k_t dR^k_t + \alpha^h_t dR^h_t - (\alpha^k_t + \alpha^h_t - 1) r_t dt$

Banker (log preference) solves: $\max_{\alpha^k_t, \alpha^h_t} \mathbb{E}_t [d\tilde{R}_t - r_t dt] - \frac{\gamma}{2} \text{Var}_t [d\tilde{R}_t]; m \text{ parameter}$
Bank Choice of Portfolio and Leverage

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Properties

- $(k, h)$ scales with equity
- $(k, h)$ increasing in $\mathbb{E}_t[d\tilde{R}_t - r_t dt]$
- $(k, h)$ decreasing in $\text{Var}_t[d\tilde{R}_t]$
General Equilibrium

Intermediary Sector

<table>
<thead>
<tr>
<th>Capital $q_t K_t$</th>
<th>Equity $E_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing $P_t H$</td>
<td>Debt $W_t - E_t$</td>
</tr>
</tbody>
</table>

Household Sector

<table>
<thead>
<tr>
<th>Financial Wealth</th>
</tr>
</thead>
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<tr>
<td>$W_t = q_t K_t + P_t H$</td>
</tr>
</tbody>
</table>

Portfolio share in capital: $\alpha^k_t = \frac{q_t K_t}{E_t} = \frac{q_t K_t}{\min[E_t, (1-\lambda) W_t]}$  

Portfolio share in housing: $\alpha^h_t = \frac{P_t H}{E_t} = \frac{P_t H}{\min[E_t, (1-\lambda) W_t]}$

- Given $E_t$, the equilibrium portfolio shares are pinned down by GE
- But portfolio shares must also be optimally chosen by banks, pinning down prices

$$\max_{\alpha^k_t, \alpha^h_t} \mathbb{E}_t [d\tilde{R}_t - r_t dt] - \frac{\gamma}{2} \text{Var}_t [d\tilde{R}_t]$$

- Asset prices affect real side through investment ($q_t$)
Equity Capital Constraint

- Representative household with $W_t$, split between bonds (at least) $\lambda W_t$ and equity (at most) $(1 - \lambda) W_t$
- Benchmark capital structure: $\lambda W_t$ of Debt, $(1 - \lambda) W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...
Equity Capital Constraint

- Representative household with $W_t$, split between bonds (at least) $\lambda W_t$ and equity (at most) $(1 - \lambda) W_t$
- Benchmark capital structure: $\lambda W_t$ of Debt, $(1 - \lambda) W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...  
- Intermediary equity capital:
  
  $$ E_t = \min [\mathcal{E}_t, (1 - \lambda) W_t] $$

- Suppose a $-10\%$ shock to real estate and price of capital:
- $W_t \downarrow 10\%$ (Household wealth = aggregate wealth)
- Capital capacity: $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = d\tilde{R}_t + \ldots$ and $\mathcal{E}_t \downarrow$ more than $10\%$:
  - Return on equity $= d\tilde{R}_t < -10\%$: equity is levered claim on assets
  - leverage is endogenous in the model
Micro foundation of Capital Constraint

- We develop theory in He-Krishnamurthy (2012, Restud), and applied to MBS market in He-Krishnamurthy (2013, AER)
- Two-agents **endowment** economy, **Households** with wealth $W^h_t$ cannot hold MBS assets but can delegate their money to **Bankers** with wealth $W_t$
- With agency friction, households are only willing to contribute at most $mW_t$ as outside equity capital, so risk-sharing rule cannot fall below $1 : m$
  - "Skin in the game" idea
- When banker’s net worth $W_t$ is low, capital constraint is binding
- Binding capital constraint is a binding **Incentive Compatibility** constraint in delegation/agency contracting problem
  - IC binds after a series of bad shocks where banker’s net worth $W_t$ is low
- Banker’s net worth $W_t$ evolves with fund performance, just like reputation or equity capacity $\epsilon_t$
Equity Dynamics in GE

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_tK_t$

-10%

Housing $P_tH$

-10%

Banker maximizes $E[ROE] - \frac{\gamma}{2} Var[ROE]$

Equity $E_t$

-10% × Lev

Debt $W_t - E_t$

Household Sector

Financial Wealth

$W_t = q_tK_t + P_tH$

(1 - $\lambda$) $W_t$

$\lambda W_t = "Liquid balances"

He and Krishnamurthy (Chicago, Stanford)
Equity Constraint Amplifies Shocks

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

Equity $E_t$

Housing $P_t H$

Debt $W_t - E_t$

Banker maximizes $E[ROE] - \frac{\gamma}{2} Var[ROE]$

Aggregate capital capacity $\mathcal{E}_t$

\[ \frac{d\mathcal{E}_t}{\mathcal{E}_t} = \text{ROE}, \text{ROE is endogenous} \]

Constraint: $E_t \leq \mathcal{E}_t$

No constraint

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

$(1 - \lambda) W_t$

$\lambda W_t = "\text{Liquid balances}"

He and Krishnamurthy (Chicago, Stanford)
## Calibration: Baseline Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Choice</th>
<th>Targets (Unconditional)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Intermediation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma ) Banker risk aversion</td>
<td>2</td>
<td>Average Sharpe ratio</td>
</tr>
<tr>
<td>( \lambda ) Debt ratio</td>
<td>0.75</td>
<td>Average intermediary leverage</td>
</tr>
<tr>
<td>( \eta ) Banker exit rate</td>
<td>13%</td>
<td>Prob. of crisis (model, data = 3%)</td>
</tr>
<tr>
<td>( B ) Entry barrier</td>
<td>6.5</td>
<td>Highest Sharpe ratio</td>
</tr>
<tr>
<td>( \beta ) Entry cost</td>
<td>2.8</td>
<td>Average land price vol (model, data = 14%)</td>
</tr>
<tr>
<td><strong>Panel B: Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma ) Capital quality shock</td>
<td>3%</td>
<td>Consumption volatility (model = 1.4%)</td>
</tr>
<tr>
<td>( \delta ) Depreciation rate</td>
<td>10%</td>
<td>Literature</td>
</tr>
<tr>
<td>( \kappa ) Adjustment cost</td>
<td>3</td>
<td>Literature</td>
</tr>
<tr>
<td>( A ) Productivity</td>
<td>0.133</td>
<td>Average investment-to-capital ratio</td>
</tr>
<tr>
<td><strong>Panel C: Others</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho ) Time discount rate</td>
<td>2%</td>
<td>Literature</td>
</tr>
<tr>
<td>( \phi ) Housing share</td>
<td>0.4</td>
<td>Housing-to-wealth ratio (bank or household)</td>
</tr>
</tbody>
</table>
Results(1): State variable is \( e_t = \mathcal{E}_t / K_t \)

- Capital constraint binds for \( e < 0.3957 \)
- Without the possibility of the capital constraint, all of these lines would be flat. Model dynamics would be i.i.d., with vol=3%
State-dependent Impulse Response: -1% Shock ($= \sigma dZ_t$)
Steady State Distribution

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Nonlinearities in Model and Data

Model:
- Distress states = worst 33% of realizations of $e$ ($e < 0.66$)
- Compute conditional variances, covariances of intermediary equity growth with other key variables

Data:
- Distress states = worst 33% of realizations of (risk premium in) credit spread
  - We use Gilchrist-Zakrajsek (2011) Excess Bond Premium, which we convert to a Sharpe ratio
  - Excess Bond Premium: risk premium of corporate bonds, presumably reflects distress of financial sector
  - Similar results if using NBER recessions
- Compute conditional variances, covariances of intermediary equity growth with other key variables
EBS and Intermediary Equity time series

Intermediary equity: market equity of commercial banks and broker/dealer sectors (SIC codes 6000-6299)
## Distress Classification

<table>
<thead>
<tr>
<th>Distress Periods</th>
<th>NBER Recessions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975Q1 - 1975Q4</td>
<td>11/73 - 3/75</td>
</tr>
<tr>
<td>1982Q3 - 1982Q4</td>
<td>7/81 - 11/82</td>
</tr>
<tr>
<td>1986Q1 - 1987Q1</td>
<td></td>
</tr>
<tr>
<td>1989Q1 - 1990Q1</td>
<td>7/90 - 3/91</td>
</tr>
<tr>
<td>1992Q3 - 1993Q1</td>
<td></td>
</tr>
<tr>
<td>2000Q1 - 2003Q1</td>
<td>3/01 - 11/01</td>
</tr>
<tr>
<td>2007Q4 - 2009Q3</td>
<td>12/07 - 6/09</td>
</tr>
<tr>
<td>2010Q2 - 2010Q4</td>
<td></td>
</tr>
<tr>
<td>2011Q3 - 2013Q1</td>
<td></td>
</tr>
</tbody>
</table>
### Covariances in Data

<table>
<thead>
<tr>
<th></th>
<th>EB</th>
<th>NBER Recession</th>
<th>NBER+, -2Qs</th>
<th>NBER+, Drop Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Distress Periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vol(Eq)</td>
<td>25.73</td>
<td>28.72</td>
<td>27.14</td>
<td>22.11</td>
</tr>
<tr>
<td>vol(I)</td>
<td>7.71</td>
<td>7.24</td>
<td>6.93</td>
<td>4.70</td>
</tr>
<tr>
<td>vol(C)</td>
<td>1.72</td>
<td>1.79</td>
<td>1.83</td>
<td>1.37</td>
</tr>
<tr>
<td>vol(PL)</td>
<td>15.44</td>
<td>15.11</td>
<td>10.51</td>
<td>8.10</td>
</tr>
<tr>
<td>vol(EB)</td>
<td>65.66</td>
<td>107.16</td>
<td>85.04</td>
<td>36.23</td>
</tr>
<tr>
<td>cov(Eq, I)</td>
<td>1.02</td>
<td>1.10</td>
<td>0.60</td>
<td>0.20</td>
</tr>
<tr>
<td>cov(Eq, C)</td>
<td>0.20</td>
<td>0.10</td>
<td>0.07</td>
<td>-0.04</td>
</tr>
<tr>
<td>cov(Eq, PL)</td>
<td>2.38</td>
<td>3.12</td>
<td>1.88</td>
<td>0.11</td>
</tr>
<tr>
<td>cov(Eq, EB)</td>
<td>-8.50</td>
<td>-19.03</td>
<td>-11.32</td>
<td>1.66</td>
</tr>
<tr>
<td><strong>Panel B: Non-distress Periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vol(Eq)</td>
<td>20.54</td>
<td>19.42</td>
<td>18.90</td>
<td>19.15</td>
</tr>
<tr>
<td>vol(I)</td>
<td>5.79</td>
<td>5.92</td>
<td>4.75</td>
<td>4.99</td>
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<tr>
<td>vol(C)</td>
<td>1.24</td>
<td>1.29</td>
<td>1.09</td>
<td>0.91</td>
</tr>
<tr>
<td>vol(PL)</td>
<td>9.45</td>
<td>10.51</td>
<td>10.26</td>
<td>8.63</td>
</tr>
<tr>
<td>vol(EB)</td>
<td>16.56</td>
<td>29.95</td>
<td>29.33</td>
<td>30.95</td>
</tr>
<tr>
<td>cov(Eq, I)</td>
<td>-0.07</td>
<td>-0.06</td>
<td>-0.18</td>
<td>-0.14</td>
</tr>
<tr>
<td>cov(Eq, C)</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>cov(Eq, PL)</td>
<td>-0.43</td>
<td>-0.23</td>
<td>-0.31</td>
<td>-0.59</td>
</tr>
<tr>
<td>cov(Eq, EB)</td>
<td>0.60</td>
<td>0.19</td>
<td>0.02</td>
<td>0.54</td>
</tr>
</tbody>
</table>
Matching State-Dependent Covariances

<table>
<thead>
<tr>
<th></th>
<th>Distress</th>
<th></th>
<th>Non Distress</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Baseline</td>
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<tr>
<td>vol (Eq)</td>
<td>25.73%</td>
<td>21.74</td>
<td>20.54</td>
<td>5.45</td>
</tr>
<tr>
<td>vol (I)</td>
<td>7.71%</td>
<td>6.01</td>
<td>5.79</td>
<td>4.97</td>
</tr>
<tr>
<td>vol (C)</td>
<td>1.72%</td>
<td>5.55</td>
<td>1.24</td>
<td>2.20</td>
</tr>
<tr>
<td>vol (LP)</td>
<td>15.44%</td>
<td>15.16</td>
<td>9.45</td>
<td>7.98</td>
</tr>
<tr>
<td>vol (EB)</td>
<td>66.66%</td>
<td>71.51</td>
<td>16.56</td>
<td>11.67</td>
</tr>
<tr>
<td>cov (Eq, I)</td>
<td>1.02%</td>
<td>0.95</td>
<td>-0.07</td>
<td>0.27</td>
</tr>
<tr>
<td>cov (Eq, C)</td>
<td>0.20%</td>
<td>-0.98</td>
<td>-0.01</td>
<td>-0.09</td>
</tr>
<tr>
<td>cov (Eq, LP)</td>
<td>2.38%</td>
<td>2.86</td>
<td>-0.43</td>
<td>0.43</td>
</tr>
<tr>
<td>cov (Eq, EB)</td>
<td>-8.50%</td>
<td>-8.94</td>
<td>0.60</td>
<td>-0.24</td>
</tr>
</tbody>
</table>

Note: without the capital constraint, all volatilities would be 3%, and have no state dependence.
Based on EBS classification, economy crossed the 33\% boundary \((e = 1.27)\) between 2007Q3 and 2007Q4. Assume \(e = 0.66\) in 2007Q3.

Then choose \((Z_{t+1} - Z_t)\) shocks to match realized intermediary equity series.

<table>
<thead>
<tr>
<th></th>
<th>07QIV</th>
<th>08QI</th>
<th>08QII</th>
<th>08QIII</th>
<th>08QIV</th>
<th>09QI</th>
<th>09QII</th>
<th>09QIII</th>
<th>09QIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>-5.0%</td>
<td>-1.5</td>
<td>-1.5</td>
<td>-0.9</td>
<td>-2.2</td>
<td>-2.6</td>
<td>-2.5</td>
<td>-0.7</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

▶ Total -16.3\%. Capital constraint binds after 08Q3—systemic risk state
▶ In the model (data), land price falls by 47\% (32\%)
▶ In the model (data), investment falls by 23\% (25\%)
Based on EBS classification, we cross the 33% boundary ($e = 0.66$) between 2007Q3 and 2007Q4

What is the likelihood of the constraint binding ("systemic crisis") assuming $e = 0.66$ currently:

- 3.0% in next 1 years
- 16% in next 2 years
- 44% in next 5 years

Small...
Stress testing: “Hidden" Leverage

- Financial sector aggregate leverage fixed at 3.77 in model
- Suppose “hidden" leverage: leverage was 4.10 but agents take as given price functions and returns at leverage=3.77
- Prob. of hitting crisis rises from 16% to 30%!
Stress testing

Key step: Need to map from stress scenario into underlying shock, $dZ_t$.

- Say stress scenario $\Rightarrow$ -30% Return on equity
- Naive partial eqbm: leverage of 4, $\sigma(Z_{t+0.25} - Z_t) = -30/4 = -7.5\%.$
- Feed in $-7.5\%$ shock into the model over one quarter.
- Result: Beginning at $e = 0.66$ in 2007Q3, economy is immediately moved into crisis region.
- Our model helps in figuring out the right shock $dZ_t$

In US stress tests, scenario was over 6 quarters. Feed in shocks quarter-by-quarter, over 6 quarters:

<table>
<thead>
<tr>
<th>Return on Equity</th>
<th>6 QTR Shocks</th>
<th>Prob(Crisis within next 2 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2%</td>
<td>-1.0%</td>
<td>10.9 %</td>
</tr>
<tr>
<td>-5%</td>
<td>-2.3%</td>
<td>19.1%</td>
</tr>
<tr>
<td>-10%</td>
<td>-3.7%</td>
<td>31.97%</td>
</tr>
<tr>
<td>-15%</td>
<td>-5.7%</td>
<td>59.85%</td>
</tr>
<tr>
<td>-25%</td>
<td>-7.5%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>
Summary

- Fully stochastic model of a systemic crisis, with an equity capital constraint on the intermediary sector
- Calibrated model matches differential comovements in distress and non-distress periods for US data
  - Replicate 2007/2008 period with only intermediary capital shocks
- Tool to map macro-stress tests into probability of systemic states: “Macro-VaR"