Treasury Inconvenience Yields during the COVID-19 Crisis

Zhiguo He  Stefan Nagel  Zhaogang Song

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Abstract

In sharp contrast to most previous crisis episodes, the Treasury market experienced severe stress and illiquidity during the COVID-19 crisis, raising concerns that the safe-haven status of U.S. Treasuries may be eroding. We document large shifts in Treasury ownership and temporary accumulation of Treasury and reverse repo positions on dealer balance sheets during this period. We build a dynamic equilibrium asset pricing model in which dealers subject to regulatory balance sheet constraints intermediate demand/supply shocks from habitat agents and provide repo financing to levered investors. The model predicts that Treasury inconvenience yields, measured as the spread between Treasuries and overnight-index swap rates (OIS), as well as spreads between dealers’ reverse repo and repo rates, should be highly positive during the COVID-19 crisis, which are confirmed in the data. The same model framework, adapted to the institutional setting in 2007-2009, also helps explain the negative Treasury-OIS spread observed during the Great Recession.

JEL Classification: D8, G2

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*He: University of Chicago, Booth School of Business, and NBER, zhiguo.he@chicagobooth.edu; Nagel: University of Chicago, Booth School of Business, and NBER, stefan.nagel@chicagobooth.edu; and Song: The Johns Hopkins Carey Business School, zsong8@jhu.edu. We appreciate helpful comments from Vic Chakrian, Wenxin Du, Darrell Duffie, Michael Fleming, Jay Kahn, Anil Kashyap, Arvind Krishnamurthy, Ricardo Largos, Antoine Martin, and Rene Stulz, as well as participants at the Chicago Finance Workshop, the SaMMF Workshop on Liquidity in Fixed Income Markets, and the Office of Financial Research seminar. We are especially grateful to Grace Hu and Haoyang Liu for the help with data sources, as well as Yiran Fan and Tianshu Lyu for great research assistance.
1 Introduction

U.S. Treasury bonds are generally viewed as some of the most liquid and safe assets in the world. Their safety and liquidity is reflected in a price premium (Longstaff (2004); Krishnamurthy and Vissing-Jorgensen (2012)). During periods of financial market turmoil when prices of risky and illiquid assets fall dramatically due to a flight-to-safety and flight-to-liquidity, the price premium of Treasuries typically rises (Nagel (2016); Adrian, Crump, and Vogt (2019)). More generally, Treasury bonds have had negative beta in recent decades, rising in price when stock prices fall (He, Krishnamurthy, and Milbradt (2019); Baele, Bekaert, Inghelbrecht, and Wei (2019); Campbell, Pflueger, and Viceira (2019)); Cieslak and Vissing-Jorgensen (2020)).

Events in March 2020 during the COVID-19 crisis did not follow this established crisis playbook. Like in many previous periods of financial market turmoil, stock prices fell dramatically, the VIX of implied stock index return volatility spiked, credit spreads widened, the dollar appreciated, and prime money market funds experienced outflows. Yet, in sharp contrast to previous crisis episodes, prices of long-term Treasury securities fell sharply. From March 9 to 23 when the stock market experienced four trading halts, the 10-year Treasury yield increased up to 60 basis points (bps), resulting in a striking and unusual positive correlation between stock and bond returns (see Figure 1). Widening bid-ask spreads and collapsing order book depth indicated market illiquidity in the Treasury bond market (Fleming and Ruela (2020)). In direct response, the Federal Reserve (Fed) first offered essentially unconstrained short-term financing to primary dealers and then quickly began to purchase Treasuries directly in amounts even larger than those during the 2008 crisis.

Why was it different this time? Given the Treasury bond market’s outsized role in the financial system, this stunning deviation from historical correlations in recent decades calls for an explanation. Are the events in March 2020 the canary in the coal mine indicating a fundamental change in the properties of Treasury bonds away from being a negative-beta flight-to-safety target asset? Or can the surprising price movements of Treasury bonds be attributed to market dysfunctionality induced by frictions? Our goal in this paper is to shed light on this question, both theoretically and empirically.

We start by characterizing major features of asset price movements and investor flows during the crucial weeks in March 2020. A simple explanation of the rise in long-term Treasury yields
would be that the enormous fiscal burden of the COVID-19 pandemic triggered a shift in inflation expectations and inflation uncertainty. However, market data suggest that this is unlikely. Prices of inflation-protected bonds (TIPS) fell along with the prices of nominal Treasuries. Inflation-swaps show no increase in risk-neutral inflation expectations. Prices of inflation caps and floors do not provide evidence of an increase in inflation uncertainty either.

An alternative explanation would be that the cyclicality of real interest rates has changed. As Campbell, Sunderam, and Viceira (2017) emphasize, the negative beta of Treasuries in the decades leading up to 2020 partly reflects a positive correlation between stock prices and real interest rates. That March 2020 represents a regime-shift towards a negative correlation cannot be ruled out at this point. But it would be difficult to come up with an economic mechanism that explains this shift. In the post-WWII history examined by Campbell, Sunderam, and Viceira (2017), the only major episode with pro-cyclical bond prices (or, counter-cyclical real interest rates) was the Volcker disinflation of the early 1980s where the rise of real interest rates in a recession was induced by contractionary monetary policy intended to crush inflation. This is clearly not what happened in March 2020.

We therefore turn to an examination of investor flows to understand whether supply and demand balances may have interacted with intermediation frictions to give rise to the unusual price movements in the Treasury market. As shown in Figure 3, during the first quarter of 2020, for-
eign investors (including foreign central banks and investors in tax havens) sold about $270 billion (bn) worth of Treasuries, mutual funds (including Treasury mutual funds and others like corporate bonds and equity funds) sold around $240bn, hedge funds sold more than $30bn, the U.S. Treasury issued about $240bn net, and other investors like pension funds, depository institutions, and insurance companies either sold or purchased a small amount. Much of this supply was temporarily accommodated by broker-dealers, partly through somewhat higher direct holdings (about $50bn), but also indirectly through a massive expansion of $400bn in repo financing that primary dealers provided to levered investors through mid-March. This increase in repo financing also ceased thereafter. The Federal Reserve offered $1.5 trillion funding to primary dealers on March 12, but the take-up was abysmally low.¹ Eventually, towards the end of March, the Federal Reserve came in and purchased $700bn worth of Treasury notes and bonds, and the expansion in dealer balance sheets reverted back. The selling pressure and its eventual accommodation by the Federal Reserve were concentrated in long-term Treasuries. There was little net selling by foreigners and mutual funds in T-bills and the Federal Reserve did not expand their T-bill holdings.

While these are significant shifts in the ownership of Treasuries, it is far from obvious that they could induce substantial increases in Treasury yields in a market that is usually thought to be extremely liquid. Why did financial intermediaries and agile institutional investors financed by intermediaries fail to accommodate this supply more elastically? We argue that balance sheet constraints of dealers played a key role.

We build on the preferred habitat model of Vayanos and Vila (2020) and Greenwood and Vayanos (2014) to understand how a supply shock for long-term Treasuries can affect the term structure of Treasury yields. By modeling risk-averse dealers who intermediate the exogenous demands of habitat investors, the dynamic preferred habitat model delivers market price volatility of Treasuries and their associated risk premia endogenously, consistent with what we have observed during the market turmoil in March 2020.

We extend this model in two important dimensions. First, we allow levered investors (hedge funds) to take positions in Treasuries financed by borrowing from dealers in the repo market, motivated by the aforementioned massive expansion of repo financing in March 2020. Second.

¹For the Fed’s announcement on the repo funding, see https://www.newyorkfed.org/markets/opolicy/operating_policy_200312a. For discussions on the low take-up, see https://www.wsj.com/articles/fed-to-purchase-treasury-securities-to-address-market-disruptions-11584109975.
dealers are subject to a balance sheet constraint, in the spirit of the supplementary leverage ratio (SLR) that was introduced in regulatory reforms following the 2007–09 financial crisis. Importantly, both direct holdings of Treasuries and reverse repo positions that finance levered investments by hedge funds take up balance sheet space.

Dealers therefore demand compensation for the shadow cost of balance sheet expansion, regardless of whether the expansion occurs through direct holdings (with compensation in the form of higher yield) or reverse repos (with compensation in the form of higher reverse repo rates). Consequently, when a massive supply shock of Treasuries results in a rise in yields, the simultaneous rise in reverse repo rates charged by dealers reduces the attractiveness of a levered investment in Treasuries for risk averse hedge funds. The repo friction hurts the demand for Treasury bonds in two ways, first through a more costly current reverse repo funding, and second via the anticipation of a reduced collateral value of Treasuries in future financing. Therefore, dealers’ direct holdings and Treasury yields need to rise even more in order to clear the market. In equilibrium, the yield curve steepens and reverse repo rates rise above the corresponding frictionless risk-free rates.

Empirically, we find support for these predictions. To measure reverse repo rates at which dealers lend to levered investors, we use the General Collateral Finance (GCF) repo rates. Much of the activity in the GCF repo market involves large dealers lending to smaller ones. In this sense, these rates reflect those at which large dealers are willing to lend to levered investors against general Treasury collateral (Baklanova, Copeland, and McCaughrin (2015)). We compare the GCF rate with Triparty repo rates at which large dealers borrow from cash-rich investors like money market funds. Consistent with the balance sheet cost explanation, we find that GCF repo rates substantially exceeded Triparty repo rates at the time when Treasury yields spiked in March 2020.

In the model, the rise in long-term yields has two components. The first component is a heightened risk premium that the expansion of direct Treasury holdings entails, because risk-averse dealers demand compensation for the resulting endogenous market price volatility of Treasuries bonds (due to their interest-rate risk exposure). The second component reflects the inconvenience yield induced by the balance sheet cost due to the SLR constraint. To isolate this second component, we use the dealers’ pricing kernel to price a derivative asset that offers exactly the same cash-flows as physical Treasury bonds, but without the balance sheet cost. We think of this derivative asset as
an overnight index swap (OIS).\(^2\) Practically, the weight imposed by the SLR constraint on interest rate derivative contracts is about two orders of magnitude smaller than the weight on Treasury securities, so zero balance sheet cost is a good approximation.

In line with the model’s predictions, we find that during the two weeks of turmoil, Treasury yields rose substantially above maturity-matched OIS rates, reflecting the inconvenience yield. Other measures of the Treasury convenience yield have been eroding since the Great Recession (Du, Im, and Schreger (2018)). Based on the findings in Krishnamurthy and Vissing-Jorgensen (2012), the rise in the supply of U.S. Treasuries since the Great Recession could also have contributed to a disappearing convenience yield. Viewed from this perspective, the rise of Treasury yields relative to OIS rates in March 2020 is a further extension of this phenomenon (see Klingler and Sundaresan (2020) for evidence prior to the COVID-19 pandemic).

The inconvenience yield of Treasuries during March 2020 is particularly striking in contrast with the financial crisis in 2007–09 (or, the Great Recession). Flight-to-safety and liquidity during the early stages of the financial crisis until mid-2008 pushed Treasury yields significantly below OIS rates, by as much as 50bps. Dealers came into the financial crisis with a short position in Treasuries. Rather than having to absorb a supply of Treasuries like in March 2020, dealers were scrambling to obtain more Treasuries. As a consequence, dealers were willing to lend cash to obtain Treasury collateral, with both triparty and GCF repo rates reaching low levels. The repo spread is low and not significantly positive, consistent with the absence of the SLR-like balance sheet constraints around 2008. In contrast to purchasing Treasuries in March 2020, the Federal Reserve took action in 2008 to increase the supply of Treasuries in the market, for example allowing dealers to obtain Treasuries against non-Treasury collateral in the Term Securities Lending Facility (TSLF). This seems to have alleviated the shortage of Treasuries, leading to a closing of the Treasury-OIS spread.

These empirical patterns in 2007–09 financial crisis (or, the Great Recession) are consistent with our model, too, but under different conditions. If the dealer sector is short in Treasuries, and habitat investors demand more direct holdings of Treasuries, then the Treasury-OIS spread should switch signs compared with March 2020. This is consistent with empirical observations, and our model therefore provides a unified account of the very different Treasury and repo market dislocations in the 2007–09 financial crisis and COVID-19 crisis.

\(^2\)The OIS rate can be interpreted as the risk-neutral expectation of the expected Federal Funds Target Rate.
In summary, the observed movements in Treasury yields and spreads in March 2020 can be rationalized as a consequence of selling pressure that originated from large holders of Treasuries interacted with intermediation frictions, including regulatory constraints such as the SLR. Evidently, the current institutional environment in the Treasury market is such that it cannot absorb large selling pressure without substantial price dislocations or intervention by the Federal Reserve as the market maker of last resort. Indeed, the Fed announced that it would directly purchase Treasuries “to support the smooth functioning of markets” on March 15 and 23, which alleviated market stress (as seen in Figure 1). Consistent with our model particularly, the Fed also announced that it would temporarily exempt Treasuries from the SLR on April 1.\footnote{See \url{https://www.federalreserve.gov/newsevents/pressreleases/monetary20200315a.htm} for the announcement of direct Treasury purchases and \url{https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200401a.htm} for the announcement of the temporary change to the SLR rule.}

Our theory and evidence explain why selling pressure had such a strong price impact based on dealer constraints, but it does not answer the question of what motivated some large holders of U.S. Treasuries to sell in March 2020. The shock that initially triggered a large selling pressure of Treasury bonds during the COVID-19 crisis was reportedly caused by a scramble for cash; e.g., corporate bond mutual funds were actively selling Treasury bonds, especially those who faced severe redemption risk (Ma, Xiao, and Zeng (2020)).\footnote{See \url{https://www.wsj.com/articles/short-term-yields-go-negative-in-scramble-for-cash-11585227369} for some news on scrambling for cash. For example, cash is pursued by corporations for payroll and operation, by foreign central banks for potential fiscal stimuli, and so on. As the prices of Treasury bonds tanked, levered investors like hedge funds who had been taking arbitrage positions (e.g., between the Treasury cash and futures markets) conducted “fire-sales” and amplified the initial shock (Schrimpf, Shin, and Sushko (2020)). See also Di Maggio (2020) for analysis on hedge funds’ selling of Treasury securities in March.} Nor does it answer the question of why other long-term investors stayed away, while dealers, levered investors financed by dealers, and ultimately the Federal Reserve had to absorb this additional supply.

The fact that the Federal Reserve was able to alleviate the dislocations by substantially tilting the maturity structure of U.S. government liabilities away from the long-term securities it purchased towards very short-term liabilities it created (reserves) invites comparisons with emerging markets crises where shortening of maturities by sovereign issuers is a typical response to investors’ concerns about issuers’ ability to repay (Broner, Lorenzoni, and Schmukler (2013)). But since neither inflation nor default risk concerns are apparent in derivatives prices in March 2020, there is little to suggest that concerns about the U.S. fiscal situation are the underlying cause, though we caution that the “safe” asset status of U.S. Treasuries should be disciplined by fundamental fiscal capac-
ities when coordination incentives of market participants play a central role (He, Krishnamurthy, and Milbradt (2019)). Relative to short-term T-bills whose values are largely determined by the near-term promise to repay by the U.S. government, the market prices of long-term Treasuries are endogenous and subject to coordination risk, along the lines of Allen, Morris, and Shin (2006).

**Related literature.** Two related works, Duffie (2020) and Schrimpf, Shin, and Sushko (2020), also provide some evidence on how the Treasury market has been stressed by COVID-19 recently. They mainly focus on raising policy proposals that can potentially make the Treasury market robust to shocks. Other contemporaneous works on the COVID-19 stress of fixed-income markets focus on corporate bonds (Boyarchenko, Kovner, and Shachar (2020), D’Amico, Kurakula, and Lee (2020), Haddad, Moreira, and Muir (2020), Kargar, Lester, Lindsay, Liu, Weill, and Zuniga (2020), Qiu and Nozawa (2020), and O’Hara and Zhou (2020)), agency mortgage-backed securities (Chen, Liu, Sarkar, and Song (2020)), and bond mutual funds (Falato, Goldstein, and Hortacsu (2020); Ma, Xiao, and Zeng (2020)).


We build our model based on Vayanos and Vila (2020) and Greenwood and Vayanos (2014), who study the equilibrium Treasury pricing where a risk-averse intermediary sector absorbs exogenous demand/supply shocks from the habitat agents. We introduce Poisson events which capture temporary (and potentially large) supply shocks in their model, and, more importantly, a levered hedge fund sector that borrows from the dealers via the repo market. The endogenous repo spread (i.e., the difference between the GCF repo lending rates and the Triparty repo borrowing rates) and

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5Augustin, Chernov, Schmid, and Song (2020) entertain the possibility of the default risk by the U.S. government, though it is at odds with sliding inflation expectation reflected by break-even inflation rate as documented in Section 2.1. Ma, Xiao, and Zeng (2020) show that the increase in Treasury yield is robust to adjusting for the US sovereign credit default swap rate.
its connection to the broad demand/supply in the Treasury market are the major contributions of this paper. We further provide strong empirical support for these theoretical predictions.


2 Motivating Evidence and Institutional Background

To set the stage for our main analysis, we offer further evidence on the Treasury market disruption during the COVID-19 crisis, in addition to Figure 1. The evidence motivates us to examine the interaction of supply and demand balances with intermediation frictions as the potential economic mechanism. We also provide relevant institutional background for the development of our modeling in Section 3.

2.1 Motivating Evidence

A simple explanation of the soaring 10-year Treasury yield would be that the expected fiscal burden of the COVID-19 pandemic can trigger a shift in inflation expectations and inflation uncertainty. The left panel of Figure 2 plots weekly time series of two market-based measures of inflation expectations, the 10-year breakeven inflation rate (i.e., the difference between the 10-year constant-maturity nominal yields and TIPS yields) and the 10-year inflation swap rate. Both of them fell throughout the COVID-19 period, especially in March 2020. The right panel plots weekly series of

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6As in Figure 1, we consider three event dates, 1/30/2020 when the World Health Organization declared the outbreak of coronavirus a global health emergency, 3/9/2020 when the S&P 500 index declined by 7%, triggering the first market-wide circuit breaker trading halt, and 3/23/2020 when the Fed announced “its full range of tools” to support the U.S. economy including unlimited purchases of Treasury securities.
measures of inflation uncertainty and the (risk-neutral) probability of a large increase in inflation (of more than 3%), extracted from 5-year inflation caps and floors by the Federal Reserve Bank of Minneapolis. We observe that inflation uncertainty dropped significantly until mid-March, and increased afterwards. The probability of a large increase in inflation has dropped and stayed low throughout April, pointing to concerns about deflation rather than inflation. Finally, investors were not flocking to gold, as gold prices went down as well starting March 9 (based on the Gold Fixing Price in the London Bullion Market).

Figure 2: Inflation Expectation and Uncertainty During the COVID-19 Crisis

Notes: The left panel plots weekly time series of the 10-year breakeven inflation rate (that equals the difference between the 10-year constant-maturity nominal and TIPS yields) and the 10-year inflation swap rate, both in percentage. The right panel plots weekly series of the (risk-neutral) standard deviation and probability of a more than 3% increase in inflation that are estimated using 5-year inflation caps and floors. The sample period is from January 2, 2020 to April 30, 2020.

We have mentioned in the introduction that foreign investors, mutual funds, and hedge funds sold (top left panel of Figure 3),\(^7\) in addition to the net positive issuance by the U.S. Treasury (middle left panel). On the supply side, primary dealers increased their direct holdings somewhat (middle right panel), but most noticeably they massively expanded their repo financing of levered investors during the first quarter and then quickly unwound after March 23 (bottom left panel).

More specifically, during the week from March 9 to 15 when the 10-year Treasury yield increased most sharply (see Figure 1), primary dealers’ direct Treasury holdings remained almost flat, while

\(^7\)Note that the amount of domestic investors flows throughout the first quarter may underestimate the selling amount in March because investors may have purchased Treasury securities on net in January and February like foreign investors. Moreover, the sample of hedge funds only contain domestic ones. According to Barth and Kahn (2020), the selling amount of hedge funds ranges from $90bn to $120bn.
Notes: This figure plots the monthly changes in holdings of long-term Treasury securities of foreign investors (top left panel), quarterly changes in holdings of Treasury securities of domestic investors (top right panel), monthly net issuance amounts of Treasury notes and bonds (middle left panel), weekly amounts of primary dealers’ net positions in Treasury securities (middle right panel), weekly amounts of primary dealers’ gross reverse repo balance (bottom left panel), and weekly amounts of the Federal Reserve’s Treasury holdings (bottom right panel) The units are all billions of U.S. dollars. Data source: the FR2004 data collected by the Federal Reserve Bank of New York, the SIFMA reports, the Financial Accounts of the United States (Z.1) provided by the Federal Reserve, the Treasury International Capital system, and the Federal Reserve’s H.4.1 release.
their reverse repo lending increased by about another $120bn. The increase in reverse repo lending ceased thereafter, suggesting a somewhat tight balance sheet constraint faced by dealers; in fact, the Fed offered $1.5 trillion repo funding to primary dealers on March 12 (i.e., they were able to access the funding at low repo rate), but the taking was abysmally low. Eventually, starting from March 15, the Federal Reserve came in and purchased $700 billion worth of Treasury notes and bonds (bottom right panel), and the expansion in dealer balance sheets reverted back. The selling pressure and its eventual accommodation by the Federal Reserve were concentrated in long-term Treasuries, as there was little net selling by foreigners in T-bills and the Federal Reserve did not expand their holdings of T-bills.

In sum, examinations of asset price movements and investor flows during the crucial weeks in March 2020 suggest that shifts of fundamental risks are unlikely to have caused the unusual price movements in the Treasury market. Instead, the large shifts in the supply of Treasuries, primary dealers’ balance sheet movements, and the nature of the Federal Reserve’s interventions point to an economic interpretation based on financial constraints. To understand the relevant frictions, we briefly introduce some institutional background.

2.2 Institutional Background

In this section, we first outline institutional features of the Treasury and repo markets, especially the important role of dealers as intermediaries in both markets. We then discuss the key regulations that were introduced in the U.S. since the 2007–09 financial crisis and their potential impacts on the Treasury market and dealers.\(^8\)

2.2.1 The Treasury and Repo Markets

The U.S. Treasury market is one of the largest fixed-income markets with an outstanding balance held by the public of about $17 trillion as of March 2020. The major component is coupon-bearing Treasuries, about $12.5 trillion, while T-bills comprise the second largest fraction, about $2.6 trillion.

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The secondary cash market of Treasuries maintains an average daily total trading volume of about $575bn over the period of August 2017 to July 2018, of which coupon Treasuries and T-bills account for the largest bulk, about 82% and 15%, respectively. About 70% of the trading volume is concentrated in on-the-run securities, which are most recently auctioned of a given tenor, while the rest is in off-the-run securities, which consist of all the previously issued securities. While on-the-run securities are mainly traded on electronic exchange-like platforms, off-the-run Treasuries, which account for 95% of outstanding securities (Clark, Cameron, and Mann (2016)), mostly trade over-the-counter with broker-dealers as counterparties.

Broker-dealers, especially the primary dealers who are trading counterparties of the Federal Reserve Bank of New York in its implementation of monetary policy (Song and Zhu (2018)), are important participants in both the primary and secondary markets of Treasuries. In particular, primary dealers are expected to participate in all issuance auctions of Treasuries, and have traditionally been the predominant purchasers at these auctions. Dealers are also key intermediaries in the Treasury cash market, accounting for about 75% of all transactions. In fact, all client transactions, which account for half of the total $575bn daily trading volume, go through dealers.

When intermediating Treasury trades of clients, dealers need to use their balance sheet to hold inventories, like in classical market-making models with non-Walrasian dealers (e.g., Amihud and Mendelson (1980), Ho and Stoll (1981)). In our model, carrying inventory entails risk for which competitive risk-averse dealers simply demand risk premium.

In addition to selling them outright in the Treasury cash market, investors often post Treasuries as collateral to borrow cash on a short-term basis, particularly in the repo market. The U.S. repo market is comprised of two segments, triparty repo and bilateral repo. A triparty repo involves a third party known as a clearing bank who provides clearing and settlement services such as keeping the repo on its books and ensuring the execution according to repo terms. Differently, in bilateral

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9 The rest are Treasury Inflation-Protected Securities (TIPS), floating rate notes (FRNs), and Separate Trading of Registered Interest and Principal Securities (STRIPS). The outstanding balance is obtained from the SIFMA (https://www.sifma.org/resources/research/us-fixed-income-issuance-and-outstanding/).

10 The summary of trading volume can be found at https://libertystreeteconomics.newyorkfed.org/2018/09/unlocking-the-treasury-market-through-trace.html. Off-the-run securities were hit hardest by the COVID-19 market disruption, though on-the-run securities were also notably affected (Fleming and Ruela (2020)).

11 The so-called principal trading firms that specialize in electronic and automated intermediation only participate in the inter-dealer segment.

repo, the clearing and settlement are managed by each counterparty’s custodian bank. Furthermore, within the triparty repo market, a special General Collateral Financing Repo Service (GCF Repo) allows securities dealers registered with the Fixed Income Clearing Corporation (FICC) as netting members to trade repos among themselves. That is, the GCF repo is mainly an inter-dealer market, while in the non-GCF triparty repo market (referred to as triparty repo hereafter), broad cash lenders including money market mutual funds (MMFs), banks, and securities lenders lend cash to dealers.\textsuperscript{13}

As in the cash Treasury market, broker-dealers are also key intermediaries in the repo market, transmitting funds from lenders to borrowers who cannot directly deal with each other for certain reasons.\textsuperscript{14} In particular, large dealers borrow cash in the triparty market from cash lenders and lend to small dealers in the GCF market. Large dealers also borrow cash in the triparty market and lend to levered investors especially hedge funds in bilateral repo markets.\textsuperscript{15} Hence, both the GCF and bilateral repo markets are used by levered investors to finance their cash Treasury positions, where the funds are transmitted by large dealers from the triparty market.

Dealers also often use (mainly bilateral) repo market to intermediate security sourcing and facilitating short-selling.\textsuperscript{16} Although securities-driven repo often targets specific Treasuries, termed as “special repo” by Duffie (1996), it is also used widely to source general securities when Treasuries are in shortage as a whole. In either case, short-selling incurs costs of searching for securities and failing to deliver, which can lead to large market disruptions (Duffie, Garleanu, and Pedersen (2002), Fleming and Garbade (2004), and Garbade, Keane, Logan, Stokes, and Wolgemuth (2010)). The implementation of the Fed’s TSLF program from March 2008 to June 2009 that lent Treasuries to dealers against non-Treasury collateral was designed to alleviate such costs (Fleming, Hrung, and Keane (2009)).

\textsuperscript{13}In addition, investors mostly use Treasury notes and bonds as collateral for repo financing, at least in tri-party repo markets. For example, Hu, Pan, and Wang (2019) show that only about 6\% of the repo collateral between dealers and MMFs consist of T-bills.

\textsuperscript{14}Dealers also obtain funding for their market-making inventories in the triparty repo market by posting Treasuries as collateral assets. This is effectively similar to the strategy of levered investors like hedge funds who finance their proprietary portfolios through repo funding. The hedge fund sector in our model includes this strategy of dealers (see Section 3 for details).

\textsuperscript{15}In recent years, repo transactions cleared through the FICC Delivery-versus-Payment (DVP) repo service have increased, which facilitate cash flow from the triparty repo market to hedge funds. See https://www.jpmorgan.com/global/research/sponsored-repo for details.

\textsuperscript{16}For example, dealers can lend cash to providers of Treasury securities, and use the received collateral assets to establish short positions in the cash Treasury market or cover short sales.
2.2.2 Post-Crisis Regulations

Despite the widely accepted safety status of Treasury securities, the high volatility of Treasuries during March 2020 may have posed large price risk and affected the willingness of risk-averse dealers to take them into inventory. However, such risk is negligible for dealers’ repo intermediation activities where Treasuries are serving as collateral, so the apparent limits to a further expansion of primary dealers’ repo financing provision discussed above points to balance sheet constraints. The balance sheet constraints that particularly interest us are those associated with regulatory reforms imposed on financial institutions as a response to the 2007–09 financial crisis (Duffie (2018)). Among them, the most relevant for the Treasury market is the SLR.

To strengthen the resilience of the global banking system in the wake of 2007–09 financial crisis, the Basel III regulatory framework proposed a new leverage ratio rule as a backstop to risk-based capital regulation, while U.S. regulators proposed the SLR in 2012 and finalized the rule of the “enhanced” SLR in April 2014. In general, the leverage ratio is computed as the Tier I capital divided by total leverage exposure irrespective of its riskiness, which is distinct from the conventional risk-weighted-asset (RWA) capital requirement. The total leverage exposure includes both on-balance-sheet assets and off-balance-sheet exposures to derivatives. The Basel Committee proposed a 3% minimum leverage ratio, while U.S. regulators require global systemically important institutions (G-SIBs) to maintain an SLR of at least 5% on a consolidated basis and at least 6% for their depository subsidiaries. The denominator of the SLR was finalized in September 2014, and from 2015, G-SIBs and other large banking institutions are required to make public disclosures related to the SLR. It finally took effect mostly in January 2018.

The leverage exposure in the SLR includes the total notional of all cash and repo transactions, regardless of which securities are used as collateral, and so it affects bank dealers’ intermediation activities in both cash and repo markets of Treasury securities. As argued by Duffie (2018), “the SLR increases ‘rental cost’ for the space on a bank’s balance sheet.” Our model incorporates this balance sheet cost. That the Federal Reserve responded to the Treasury market disruption with a temporary exemption of Treasuries from the SLR on April 1, 2020 is consistent with this balance sheet cost being a relevant friction.

 Compared with the constraint on Treasury cash and repo positions, the constraint imposed by
the SLR on standard interest-rate derivatives is minor. Specifically, the exposures to derivatives are calculated based on the Current Exposure Method, consisting of the Current Exposure (CE) and Potential Future Exposure (PFE); for standard interest-rate derivatives like vanilla Libor swaps and OIS, the constraint imposed by SLR on interest-rate derivatives is about two orders of magnitude smaller than that on the cash Treasury positions.\footnote{Since Libor swaps and OIS are centrally cleared, the CE is effectively zero because the variation margin is posted on a daily basis. The PFE is defined using a combination of net and gross risk exposures, equal to $PFE = 0.4 \times A_{\text{gross}} + 0.6 \times NGR \times A_{\text{gross}}$, where $A_{\text{gross}}$ is the adjusted gross notional equal to gross notional multiplied by a maximum of 1.5%, and $NGR$ is the net-to-gross ratio equal to the net current mark-to-market value and gross current mark-to-market value. For details, see Polk (2014).}

In addition, we briefly discuss two other regulations—the Liquidity Coverage Ratio (LCR) and Volcker rule—that have been progressively put into effect since 2014. The objective of the LCR is to ensure that potential outflows over a 30-day period are sufficiently covered by cash and high-quality liquid assets (HQLAs),\footnote{Specifically, it assumes that dealers may lose all of their collateralized funding with terms of less than 30 days and hence stipulates that they need to hold sufficient cash and HQLAs to cover this loss of funding. A companion liquidity regulation is the net stable funding ratio (NSFR) designed to limit maturity transformation. It requires sufficient stable funding, equity or long-term debt, to cover assets over a one-year horizon.} but this constraint is almost irrelevant in our analysis because cash and Treasuries are treated equivalently as HQLAs.\footnote{Similarly, the LCR is unlikely to have constrained banks from purchasing Treasuries using cash either. Further, a large amount of cash flow into banks during the market stress, which actually alleviates the LCR constraints.} The Volcker rule prohibits proprietary trading of banks (or financial institutions with access to FDIC insurance or the Federal Reserve’s discount window) that are financed by low-cost deposits of the affiliated bank branch. However, the Volcker rule exempts government securities and so it is unlikely to have constrained dealers in the Treasury market turmoil.

3 The Model

We now show within a model how supply shocks can interact with intermediation frictions to give rise to the observed Treasury market disruptions. Moreover, the model provides additional empirical predictions about spreads between different repo rates, swap rates, and Treasury yields that we examine subsequently.

The model in this section is an extension of Greenwood and Vayanos (2014); in their setting, preferred habitat agents trade with arbitrageurs. We separate the arbitrageurs in Greenwood and Vayanos (2014) into hedge funds and dealers, and we introduce a repo market in which the former...
group borrows from the latter. Throughout, we use lowercase letters to denote an individual agent’s choices while uppercase letters denote aggregate quantities.

3.1 Aggregate Shocks and Assets

There are two sources of aggregate risk in this dynamic model, following Greenwood and Vayanos (2014). The first is the stochastic evolution of the short-term interest rate $r_t$ as an Ornstein–Uhlenbeck process

$$dr_t = \kappa (\tau - r_t) dt + \sigma dZ_t,$$

where $\{Z_t : 0 \leq t < \infty\}$ is a standard Brownian motion, $\kappa$ is the mean-reversion parameter, and $\sigma$ is the volatility of the short rate.

The second aggregate shock is a Treasury demand/supply shock $\tilde{\beta}_t$, which follows a Markov chain $\tilde{\beta}_t \in \{0, \beta\}$. The jump intensity from $\tilde{\beta}_t = 0$ ($\tilde{\beta}_t = \beta$) to $\tilde{\beta}_t = \beta$ ($\tilde{\beta}_t = 0$) is denoted by $\xi_0$ ($\xi_\beta$). We interpret $\tilde{\beta}_t = 0$ ($\tilde{\beta}_t = \beta$) as the normal (stress) state. Our model can capture both demand and supply shocks depending on the sign of $\beta$.

We consider a continuum of zero-coupon Treasury bonds which mature at the tenor $\tau \in [0, T]$. Denote by $P_t(\tau)$ their endogenous price to be solved for in equilibrium.

3.2 Habitat Agents

There is an exogenous demand/supply shock from habitat agents, so that their holding of bonds with tenor $\tau$ is

$$H_t(\tau) = -\theta(\tau) \tilde{\beta}_t,$$  \hspace{1cm} (1)

where $\theta(\tau) \geq 0$ captures the exposure to the shock. The case of $\beta > 0$ corresponds to an exogenous supply shock to the economy, while the case of $\beta < 0$ it represents a demand shock. We think of these habitat agents as representing insurance companies, pension funds, and/or foreign central banks.\footnote{In the context of the COVID-19 crisis, these habitat agents also include hedge funds who were heavily engaging in cash-futures basis trade and hence were forced to delever after significant losses following the Treasury market turmoil in March 2020.}
Let $\Theta \equiv \int_0^T \theta (\tau) \, d\tau$. Depending on applications, we later specialize the function $\theta (\cdot)$ either to

$$
\theta (\tau) = \begin{cases} 
1, & \text{for } \tau > \hat{\tau}, \\
0, & \text{otherwise},
\end{cases}
$$

so that the (negative) demand shock hits the long-end of the curve, or $\theta (\tau) = 1$ for all $\tau \in [0, T]$ so that the demand shock applies to the entire curve.

### 3.3 Hedge Funds and Repo

A unit measure of hedge funds in this economy can borrow from dealers in the repo market to exploit the investment opportunity created by aggregate demand/supply shocks $\tilde{\beta}$. When a hedge fund borrows from a dealer by pledging Treasury bonds as collateral, she needs to pay an endogenous repo financing rate of $R_t$.

Following Greenwood and Vayanos (2014) we assume that each hedge fund solves the following instantaneous mean-variance objective at time $t$:

$$
\max_{q^h_t(\tau) \geq 0} \mathbb{E}_t \left[ dG^h_t \right] - \frac{1}{2\rho_h} \text{Var}_t \left[ dG^h_t \right],
$$

where $dG^h_t$ is the hedge fund’s trading gain, and $\rho_h > 0$ is the hedge fund’s risk-bearing capacity (or risk tolerance, the inverse of their absolute risk-aversion). Given the repo financing cost $R_t$ which will be determined in equilibrium, the dynamics of her wealth is given by

$$
dG^h_t = \int_0^T q^h_t (\tau) \underbrace{\left( \frac{dP_t (\tau)}{P_t (\tau)} - R_t \, dt \right)}_{\text{trading profit}} \, d\tau.
$$

For each $\tau \in [0, T]$, the hedge fund’s repo demand $q^h_t (\tau)$ depends on the repo financing cost $R_t$. The higher the $R_t$, the lower the demand; and this price-dependent repo demand is one of our key contributions relative to Greenwood and Vayanos (2014).
3.4 The Dealer Sector

A unit measure of risk-averse dealers absorb the residual Treasury supply/demand shocks and provide overnight repo funding to the hedge fund sector. We will use “he” to refer to the dealer while “she” to refer to the hedge fund.

3.4.1 Dealer’s problem

Following Greenwood and Vayanos (2014) we assume that each dealer solves the following instantaneous mean-variance objective at time $t$:

$$\max_{x_t(\tau), q^d_t(\tau) \geq 0} \mathbb{E}_t [dw^d_t] - \frac{1}{2\rho_d} \text{Var}_t [dw^d_t],$$

(5)

where $\rho_d > 0$ is the dealer’s risk-bearing capacity, and the dynamics of his wealth, $dw^d_t - w^d_t r_t dt$, is given by

$$\int_0^T x_t(\tau) \left( \frac{dP^r_t}{P^r_t} - r_t dt - \frac{\Lambda_t dt}{B/S \text{ cost}} \right) d\tau + \int_0^T q^d_t(\tau) \left( \frac{\Delta_t}{\text{repo wedge}} - \frac{\Lambda_t}{B/S \text{ cost}} \right) d\tau$$

(6)

Here, $x_t(\tau)$ is the direct holding of bond $\tau$ (in terms of dollars, which could be negative) and $q^d_t \geq 0$ is the dealer’s reverse repo position (i.e., repo funding provided to hedge funds), which correspond to the dealers’ inventory in the Treasury cash market and intermediation amount in the repo market, respectively, that we discussed in Section 2.

In Eq. (6), we have defined

$$\Delta_t \equiv R_t - r_t,$$

(7)

which captures the spread between the collateralized lending rate $R_t$ in the repo market, and the risk-free borrowing short rate $r_t$. Here, lending/borrowing is from the perspective of dealers, and we do not specify in more detail the funding markets in which dealers borrow at rate $r_t$. Empirically, in the context of COVID-19 crisis in March 2020, we proxy for $r_t$ with the repo rate from the triparty market whose funding flows are mainly from cash lenders like money market funds to large dealers. We proxy for $R_t$ with the repo rate from the GCF market whose funding flows are mainly from large dealers to smaller dealers. The wedge between GCF and triparty repo rates therefore captures the
spread between the rates at which large dealers lend and borrow in collateralized funding markets. This GCF-triparty repo wedge is a riskless profit earned by the dealer sector, corresponding to $\Delta_t$ in our model.

### 3.4.2 Balance sheet cost

Compared with Greenwood and Vayanos (2014), we study the repo market in which dealers provide repo services $q^d_t(\tau)$ in (6), in addition to their portfolio choice $x_t(\tau)$. Furthermore, dealers face an additional balance sheet cost—denoted by $\Lambda_t$—in their portfolio choices, as shown in (6). The marginal cost $\Lambda_t$ hits the direct holdings $x_t(\tau)$ and repo services $q^d_t(\tau)$ linearly and each dealer takes it as given.

The balance sheet cost $\Lambda_t$ depends on the aggregate holdings only; one can think of a frictionless inter-dealer market which equalizes these costs across dealers. For each tenor $\tau$, the aggregate bond holdings in the dealer sector is $X_t(\tau)$, and the aggregate reverse repo $Q^d_t(\tau) \geq 0$. When $X_t(\tau) \geq 0$, then the accounting is straightforward. The balance sheet occupied by tenor-$\tau$ bonds, denoted by $B_t(\tau)$, is simply

$$B_t(\tau) = X_t(\tau) + Q^d_t(\tau).$$

Integrating over $\tau \in [0, T]$, the balance sheet size of the entire dealer sector is

$$B_t \equiv X_t + Q^d_t,$$

with $Q^d_t \equiv \int_0^T Q^d_t(\tau) \, d\tau$ and $X_t \equiv \int_0^T X_t(\tau) \, d\tau$. Of course, the equilibrium direct dealer holdings $X_t(\tau)$ and its sign will depend on the demand shock $\tilde{\beta}_t$. In Section 4 where we specialize our model to the COVID-19 crisis with $\tilde{\beta}_t > 0$, we show that $X_t(\tau) > 0$ and hence (8) gives the dealer balance sheet size.

We assume that the marginal balance sheet cost $\Lambda_t \equiv \Lambda(B_t)$ is linear in the balance sheet size $B_t$:

$$\Lambda_t = \lambda B_t, \text{ with } \lambda > 0.$$

In words, the dealer is bearing a marginal cost from taking on an extra dollar of Treasuries (whether directly held by the dealer or indirectly by financing hedge funds’ positions) onto the balance sheet,
and this cost is increasing in the aggregate balance sheet size $B_t$. As explained in the introduction, the balance sheet cost captures the Supplementary Leverage Ratio (SLR) constraint combined with costly external equity financing (with an upward sloping equity supply curve, say as in He and Krishnamurthy (2012, 2013)).

For completeness we briefly discuss the case of $X_t(\tau) < 0$, i.e., when the dealer is short selling some tenor-$\tau$ bonds. The exact balance sheet size then depends on how the dealer performs the short selling. One such way of short-selling, that is linked to repo transactions, has the dealer first borrow bonds $Q^d_t(\tau)$ via repo, and then sell some of them (i.e., $|X_t(\tau)| < Q^d_t(\tau)$). In this hypothetical case, the dealer’s liability side books both the short sale and his triparty repo market borrowing, and $B_t(\tau) = Q^d_t(\tau)$.\(^{21}\)

### 3.5 Equilibrium

We focus on symmetric equilibrium in which individual agents (hedge funds and dealers) are employing the same strategy as their own groups, respectively. In aggregate, for tenor $\tau$, the dealer sector has $X_t(\tau) = x_t(\tau)$ amount of direct Treasury holdings and provides an amount $Q^d_t(\tau) = q^d_t(\tau) \geq 0$ of reverse repo.

We follow Greenwood and Vayanos (2014) by normalizing the aggregate bond supply for each tenor $\tau \in [0, T]$ to be zero. The equilibrium is defined in the standard way:

**Definition 1.** A (symmetric) equilibrium is a collection of quantities \( \{q^h_t(\tau), Q^h_t(\tau)\} \) by hedge funds, \( \{x_t(\tau), X_t(\tau), q^d_t(\tau), Q^d_t(\tau)\} \) by dealers, and prices \( \{P_t(\tau), \Lambda_t, \Phi_t(\tau)\} \), such that

1. Each hedge fund solves the problem in (3);
2. Each dealer solves the problem in (5);
3. Allocations are symmetric and consistent: $q^h_t(\tau) = Q^h_t(\tau)$, $x_t(\tau) = X_t(\tau)$, and $q^d_t(\tau) = Q^d_t(\tau)$;

\(^{21}\)As an example, suppose that a dealer engages in $q^d = 3$ repo lending, and at the same time holds a short position $x = -1$ by having (short) sold in the Treasury market. This implies that the dealer first takes 3 dollars worth of bonds that he receives through reverse repo; he then passes 2 dollars’ worth of bonds as collateral in the Triparty repo market (where the collateral would have rested without being available to anyone else for purchase), and sells the remaining 1 dollar of bonds to the Treasury market. On the dealer’s balance sheet, the dealer’s obligation to return the borrowed and short-sold Treasuries to the habitat agents is recorded as a liability of 1 as “Financial Instruments Sold but not yet Purchased.” Adding the 2 dollars of liability in the form of Triparty repo, the balance sheet size of this dealer is 3, which is his total repo position. If the dealer has $x = -4$, then he needs to engage in an extra naked short-selling of 1, and the total dealer balance sheet would be $4 = q^d + \max(0, x + q^d)$. 

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4. Both Treasury and repo markets clear for \( \tau \in [0, T] \), i.e.,

\[
0 = H_t (\tau) + Q^h_t (\tau) + X_t (\tau),
\]

\[
0 = Q^h_t (\tau) - Q^d_t (\tau).
\]

Since \( Q^h_t (\tau) = Q^d_t (\tau) \) in equilibrium, we will use \( Q \) to denote them whenever there is no risk of confusion. Figure 4 illustrates the model setting with \( X > 0 \). There is an aggregate bond (risk) supply to be borne by dealers via direct holdings \( X \) and by hedge funds via repo \( Q \). But as shown, dealers are serving as the counterparty for repo transactions \( Q \), which occupy their balance sheet—hence a balance sheet size of \( B = Q + X \). We will come back to Figure 4 in Section 4.1.5 to offer a full illustration of how these balance sheet mechanics affect equilibrium quantities and pricing.

4 Model Solution and Implications

We first consider a special case of our model to illustrate its economic mechanism in detail. We then show that this case is sufficiently rich to deliver interesting empirical patterns during the Treasury market breakdown when the pandemic hit the U.S. in mid-March 2020. Finally we modify our model to shed light on the Treasury market movement during 2007-09 financial crisis.

4.1 Model Solution and Mechanism

Consider the case of \( \beta > 0 \) which entails a positive supply shock of Treasuries to be absorbed by dealers and hedge funds, as we observe in the COVID-19 crisis.

4.1.1 Equilibrium balance sheet size and repo spread

Given the exogenous aggregate supply shock \( H_t (\tau) = -\theta (\tau) \tilde{\beta}_t < 0 \), we will show shortly that in equilibrium

\[
X_t (\tau) > 0, \text{ and } Q_t (\tau) \geq 0 \text{ for all } \tau \in [0, T].
\]

As explained in Section 3.4.2, under (10) the dealer’s balance sheet size for bond \( \tau \) is \( B_t (\tau) = Q_t (\tau) + X_t (\tau) \). This is particularly convenient, because market clearing \( H (\tau) + Q_t (\tau) + X_t (\tau) = 0 \) in (9) implies that the equilibrium balance sheet size is independent of the equilibrium repo size.
\[ Q_t(\tau): \]
\[ B_t(\tau) = -H_t(\tau) \geq 0. \]

To see this, imagine the hedge fund sector buys one dollar worth of bonds from the dealer; but the financing has to come from the dealer sector via the repo market. While this reduces the risk the dealer has to bear, it does not relax the dealer sector’s balance sheet.

We now use (1) to calculate the equilibrium aggregate balance sheet (recall \( \Theta = \int_0^T \theta(\tau) \, d\tau \))

\[ B_t = \int_0^T B_t(\tau) \, d\tau = \tilde{\beta}_t \Theta. \]  \hspace{1cm} (11)

Because the dealer’s objective is linear with a marginal benefit of \( \Delta_t - \Lambda_t \), in equilibrium this marginal benefit must be zero and hence\(^{22}\)

\[ \Delta_t = \Lambda_t = \lambda B_t > 0. \]  \hspace{1cm} (12)

Per unit of Treasury bond, the dealer’s holding cost is the balance sheet cost \( \Lambda_t \) while the hedge fund’s holding cost is the repo financing wedge \( \Delta_t \). In equilibrium they must be the same. This has important implications for our model, as we show next.

**4.1.2 Optimal risk sharing within the intermediary sector**

Because both dealers and hedge funds have the same cost in holding the bonds, they face exactly the same problem. More specifically, plugging in \( \Delta_t = \Lambda_t \), one can show that the wealth dynamics of a dealer (6) and a hedge fund (4) are identical. They differ in their risk-bearing capacity \( \rho_d \) and \( \rho_h \) only, and the standard asset pricing insight implies optimal risk sharing in equilibrium:

\[ X_t(\tau) = -\frac{\rho_d}{\rho_d + \rho_h} H_t(\tau), \]  \hspace{1cm} (13)

\[ Q_t(\tau) = -\frac{\rho_h}{\rho_d + \rho_h} H_t(\tau). \]  \hspace{1cm} (14)

\(^{22}\)We will show soon that the hedge fund (and hence the dealer) is taking an interior repo position in equilibrium (14).
4.1.3 Euler equation and equilibrium asset pricing

The optimal risk sharing given in (13) pins down the endogenous equilibrium pricing kernel. Equilibrium bond prices can be obtained by extending the method in Greenwood and Vayanos (2014) with Poisson jumps.

Denote \( P_{\hat{\beta}t} \equiv P_{\hat{\beta}t}(\tau) \) and guess that the equilibrium prices take the form

\[
P_{\hat{\beta}t} = \exp \left[ - \left( A(\tau) r_t + C_{\hat{\beta}}(\tau) \right) \right], \text{with } \hat{\beta}t \in \{0, \beta\}
\]

where \( A(\tau), C_0(\tau), \text{ and } C_{\beta}(\tau) \) are endogenous functions of \( \tau \). For illustration, consider the stress state (i.e., \( \hat{\beta}t = \beta \) so that the economy has been hit by a supply shock), and denote by \( \mu_{\hat{\beta}t} \) the drift of the bond-\( \tau \) return \( \mu_{\hat{\beta}t} \). One can derive the dealer’s Euler equation as his first-order condition:

\[
\mu_{\hat{\beta}t} - r_t - \Lambda_{\hat{\beta}t} = A(\tau) \frac{\sigma^2}{\rho_d} \int_0^T X_{\hat{\beta}t}(u) A(u) \, du + \rho_d \int_0^T \left\{ X_{\hat{\beta}t}(u) \left( e^{C_{\beta}(\tau)-C_0(\tau)} - 1 \right) \right\} \, du,
\]

where we have invoked the dealer’s equilibrium holdings \( x_{\hat{\beta}t}(\tau) = X_{\hat{\beta}t}(\tau) \).

As the standard Euler equation for a risk-averse dealer, the left-hand side of Eq. (16) gives the bond \( \tau \)'s expected effective excess return, net of the balance sheet cost. In equilibrium it equals the risk premium that compensates the dealer for bearing the Brownian interest rate risk \( \sigma dZ_t \) and Poisson demand risk \( \hat{\beta} \), as shown on the right-hand side of (16). Appendix A gives detailed derivations and numerical methods to solve for the Treasury prices \( A(\tau), C_0(\tau), \text{ and } C_{\beta}(\tau) \), endogenous price volatility, and risk premia in our model.

Equation (16) also makes it clear that the dealer’s equilibrium portfolio \( \{X_t(\tau) : \tau \in [0, T]\} \) given by (13) prices each bond with tenor \( \tau \), as demand shocks affect the dealer’s equilibrium holdings and hence his pricing kernel, just like in Greenwood and Vayanos (2014). Dealers who absorb an increase in the supply of long-term bonds bear more interest rate risk in their portfolio, and hence in equilibrium require all bonds—both long-term and short-term—to offer higher expected

\footnote{One can also write the Euler equation from the entire intermediary sector’s perspective: it absorbs the whole supply shock \(-H_t(\tau)\) but with an effective risk bearing capacity of \( \rho_t \equiv \rho_d + \rho_h \).}
returns in excess of the short rate. Different from Greenwood and Vayanos (2014), in our model the dealer also demands an extra premium to compensate for the additional balance sheet cost $\Lambda_{\beta t}$; we will show shortly that this term drives the Treasury-OIS spread.

### 4.1.4 Shadow price for OIS curves

One of the key empirical objectives in our paper is to track changes in the (in-)convenience yield of Treasuries during the COVID-19 crisis episode. To this end, we seek a maturity-matched benchmark for comparison that isolates the (in-)convenience yield. In our model, we can define as benchmark a derivative asset with the exact same cash flows as physical Treasury bonds, but free from balance sheet concerns. Empirically, we proxy for the yield of this derivative asset with overnight index swap (OIS) rates. In practice, the weight imposed by the SLR constraint on interest rate derivative contracts is not exactly zero, but it is about two orders of magnitude smaller than the weight on Treasury securities.\(^{24}\)

Suppose that dealers in our model are quoting prices for OIS contracts, which are zero-net supply in equilibrium. Denote by $P_{OIS, \tau}^{\beta t}$ the price of an OIS contract with tenor $\tau$, which takes the following functional form (one can show that $A(\tau) = \frac{1-e^{-\kappa \tau}}{\kappa}$ as in Section 4.1.3)

$$P_{OIS, \tau}^{\beta t} = \exp \left[- \left( A(\tau) r_t + C_{OIS}^{\beta t}(\tau) \right) \right], \text{with } \tilde{\beta}_t \in \{0, \beta\}. $$

(17)

In contrast to (16), the drift of $dP_{OIS, \tau}^{\beta t}/dt$ must satisfy the following standard Euler equation but without the balance sheet cost $\Lambda_{\beta t}$:

$$\mu_{OIS, \tau}^{\beta t} - r_t = A(\tau) \frac{\sigma^2}{\rho_d} \int_0^T X_{\beta t}(u) A(u) du + \left( e^{C_0(\tau) - C_{\beta}(\tau)} - 1 \right) \frac{\xi_B}{\rho_d} \int_0^T \left\{ X_{\beta t}(u) \left( e^{C_{\beta}(\tau) - C_0(\tau)} - 1 \right) du \right\}. $$

(18)

One can solve for $\{C_0^{OIS}(\cdot), C_{\beta}^{OIS}(\cdot)\}$ following the same technique as in Section 4.1.3 (for details, see Appendix A.3).

The Treasury-OIS spread at tenor-$\tau$ roughly captures the Treasury’s average extra holding cost $\Lambda_t$ during the remaining time-to-maturity $\tau$, discounted by the equilibrium pricing kernel. Hence

\(^{24}\)The Potential Future Exposure of derivative contracts equals the effective notional principal amount, times the add-on factor. The add-on factor for interest rate derivatives is 0 for bonds with remaining maturity of one-year or less; 0.5% over 1 to 5 years; and 1.5% over 5 years.
Figure 4: Model Schematic Diagram

Dealers’ Balance Sheet $B = Q + X = -H$

$B \uparrow$ drives up B/S cost $\Lambda = \lambda B \uparrow$

Hedge fund demand via Repo, decreasing in $\Delta = \Lambda \uparrow$

Dealers’ direct holding $\Lambda(\uparrow)$ and pricing kernel

$Q \uparrow$ $X(\uparrow)$

Optimal Risk Sharing

Hedge funds’ Risk Exposure $Q$ Dealers’ Risk Exposure $X$

$\tilde{\beta}$ shock to Habitat agents’ holdings $-H \uparrow$

Notes: A schematic representation of the model when the economy suffers from a supply shock $\tilde{\beta} > 0$ from habitat agents. Increasing one-to-one to absorb the supply shock, the dealers’ balance sheet (with a size $B = -H$) accommodates not only dealers’ direct holdings ($X$) but also repo financing ($Q$) from hedge funds.

through the lens of our model, Treasury-OIS spreads observed in the empirical data capture the Treasury (in)convenience yields.

4.1.5 Summary and model mechanism

The following proposition summarizes what we have shown.

**Proposition 1.** Consider the scenario of a potential supply shock, i.e., $\beta > 0$. Given the habitat agents’ demand $H_t(\tau) = -\theta(\tau) \tilde{\beta}_t$, the equilibrium is characterized by:

1. The dealer’s balance sheet size is $B_t = \tilde{\beta}_t \Theta$, and the repo spread equals to balance sheet cost $\Delta_t = \Lambda_t = \lambda B_t = \lambda \Theta \tilde{\beta}_t$;

2. The dealer sector holds $X_t(\tau) = \frac{\rho_d}{\rho_d + \rho_h} \theta(\tau) \tilde{\beta}_t$ directly and hedge funds hold $Q_t(\tau) = \frac{\rho_h}{\rho_d + \rho_h} \theta(\tau) \tilde{\beta}_t$ via repo financing;

3. The Treasury (OIS) prices $P^T_{\tilde{\beta}_t} (P^{OIS,\tau}_{\tilde{\beta}_t})$ in Eq. (15) (Eq. (17)) solves the ODE system of $\{C_0(\cdot), C_\beta(\cdot)\}$ $\{C^{OIS}_0(\cdot), C^{OIS}_\beta(\cdot)\}$ in Eq. (32) (Eq. (33)) given in Appendix A.3.
Figure 4 illustrates the workings of our model; the mechanism applies to tenor-$\tau$ bonds as well as the entire maturity spectrum. After a supply shock hits habitat agents ($\tilde{\beta}_t$ jumps from 0 to $\beta > 0$), the dealers’ balance sheet $B = -H$ expands due to market clearing. The greater the (negative) demand shock size $\beta$, the larger the dealers’ balance sheet size, and hence the higher the balance sheet cost $\Lambda = \lambda B$. Fundamentally, this balance sheet cost is tied to the scarcity of intermediary capital (He and Krishnamurthy (2012, 2013)) and/or debt overhang (Andersen, Duffie, and Song (2019)).

Hedge funds step in to absorb the supply shock from habitat agents via collateralized repo financing from dealers. Since reverse repo takes up space on the dealers’ balance sheet, in equilibrium dealers pass the balance sheet cost $\Lambda$ through to hedge funds via the repo spread $\Delta$ on a one-to-one basis, adversely affecting the repo demand $Q$ from the hedge fund sector. In equilibrium, both dealers and hedge funds achieve the optimal risk sharing.

The dealers’ direct holding $X$ pins down their pricing kernel and hence equilibrium bond prices via their Euler equation (16). As noted in Figure 4, the balance sheet cost $\Lambda$, as the holding cost or inconvenience yield per unit of Treasury bonds, also enters in the Euler equation (16). In contrast, the OIS curve, which does not entail the balance sheet cost, is driven by the dealers’ pricing kernel only.

4.2 Treasury Market Breakdown in the COVID-19 Crisis in 2020

We now specialize our model to a supply shock to long-term Treasuries, as motivated by the quantity evidence in Section 2.1. We then present further supporting evidence on asset pricing during the COVID-19 crisis.

4.2.1 Model implications: Treasury inconvenience yield

Consider $\theta(\tau) = 1_{\{\tau>\hat{\tau}\}}$ and $\beta > 0$, so that long-term Treasuries are being sold by habitat agents. We then have $\Theta = \int_0^T \theta(\tau) d\tau = T - \hat{\tau}$ in all the equilibrium objects in Proposition 1. In our illustrative numerical example, we set $\hat{\tau} = 5$.

Figure 5 Panel A plots the equilibrium yield curves at the normal (stress) state $\tilde{\beta} = 0$ ($\tilde{\beta} = \beta > 0$). The yield at the long-end rises when hedge funds and dealers are absorbing the aggregate supply...
Notes: Panel A (left) plots model-implied Treasury yield curves in both states (normal $\tilde{\beta} = 0$ and stressed $\tilde{\beta} = \beta$); Panel B (right) plots model-implied GCF-Triparty Repo spreads and Treasury–OIS spreads. The model captures a supply shock for Treasury bonds from habitat agents so that $\beta > 0$, just like in 2020 COVID-19 crisis. Parameters: $\bar{r} = 0.055$, $\kappa = 0.201$, $\rho_h = \rho_d = 1/57$, $\sigma = 0.017$, $\xi_0 = 0.1$, $\xi_{\beta} = 0.4$, $\lambda = 0.01$, and $r_t = 0$.

Importantly, we observe that the yield curve steepens in the stressed state, consistent with yield curve movements in March 2020 (see Figure 1).

Figure 5 Panel B plots the equilibrium GCF-triparty repo spread $\Delta$ (left axis) and 10-year Treasury-OIS spread (right axis) in the stress state, both as a function of the supply shock size $\beta$. The equilibrium repo spread $\Delta$ equals the balance sheet cost $\Lambda = \lambda \Theta \beta$, which is linear in the supply shock size $\beta$ in the stress state. This represents the (state-dependent) inconvenience yield of Treasuries, driving a positive implied 10-year Treasury-OIS spread. Both GCF-Triparty repo spread and Treasury-OIS spread are positively related to the balance sheet cost, and these theoretical predictions will be confirmed in the data, as shown in the next section.

4.2.2 Empirical evidence of Treasury inconvenience yield

Figure 6 plots daily series of the 10-year and 3-month Treasury-OIS spreads (in the left panel) during the COVID-19 crisis. Consistent with the model prediction (Figure 5 Panel B), the 10-year Treasury-OIS spread is indeed positive. Moreover, it fell before 3/9/2020, consistent with a standard flight-to-safety to long-term Treasuries, but shot up afterwards amid a selling pressure shock.\footnote{As in Greenwood and Vayanos (2014), the equilibrium yield at the short-end also rises because all Treasury bonds are priced by the same marginal investors.}
by investors who scrambled for cash. On 3/15 right before the Fed announced direct purchases of Treasuries, the 10-year Treasury-OIS spread jumped up by about 30bps, consistent with an increase of \( \Lambda_t \) in our model. In contrast, the 3-month Treasury-OIS spread dropped on 3/15, consistent with the supply shock being concentrated at the long end. The 10-year Treasury-OIS spread began to ease afterwards, likely because the Fed’s direct purchases weakened the supply shock.

Figure 6: Treasury-OIS and GCF-Triparty Repo Spreads during the COVID-19 Crisis

![Figure 6: Treasury-OIS and GCF-Triparty Repo Spreads during the COVID-19 Crisis](image)

Notes: This figure plots daily series of the 10-year and 3-month Treasury-OIS spreads (left panel) and of the GCF-Triparty repo spread (right panel), from January 1, 2020 to April 30, 2020.

The right panel of Figure 6 plots daily series of the GCF-triparty repo spread during the COVID-19 crisis. Consistent with the model prediction (Figure 5 Panel B), the repo spread is mostly positive. Indicating a surging balance sheet cost \( \Lambda_t \) associated with the supply shock, the repo spread spiked up as high as 60bps during the two-week period from 3/9 to 3/23. Holders of long-term Treasury securities found it hard to finance these positions in the repo market.

Together with the results in Section 2.1, we show that large sales of long-term Treasuries drove up both the long-term Treasury yield and the repo spread sharply during the COVID-19 crisis. Primary dealers only absorbed a small amount of this supply through direct purchases. Dealers’ provision of repo funding to levered investors was limited, too, likely because of the SLR constraint. Direct purchases by the Federal Reserve eventually absorbed the supply, relieving the strain on dealers’ balance sheets.
4.3 Excess Treasury Demand and 2007–09 Financial Crisis

While our focus in this paper is on the COVID-19 crisis in 2020, we argue that the essential element of our model also helps us understand the movements of yields and yield spreads during the 2007–09 financial crisis. This serves as a useful additional validation of our framework.

We posit that in the early stages of the 2007–09 financial crisis, there was a positive demand shock for Treasuries (in contrast to a supply shock in March 2020), and the intermediary sector short-sold some bonds to meet this excess demand. Since this is an episode prior to the SLR regulation, the analysis in this section ignores the balance sheet cost (it is trivial to add back the balance sheet cost). Also, as there is no clear term structure pattern in the 2007–09 financial crisis, we eliminate the tenor-dependence of demand shocks by setting $\theta(\tau) = 1$ (so $\Theta = T$) and drop $\tau$ whenever appropriate.

4.3.1 Excess Treasury Demand and Naked Short-Selling

When Treasury demand surges, $\tilde{\beta}_t = \beta < 0$ and $H_t > 0$. Following the same logic as in Section 4.1.2, it is reasonable to conjecture that, in equilibrium, dealers as well as the hedge fund sector are short selling bonds to absorb this excess demand. To accommodate this case with empirically realistic limits on short selling, we assume that naked short-selling is costly. The short-selling costs can include potential reputation loss or regulatory penalty when the dealer fails to deliver the short-sold bonds.\(^\text{26}\)

For each tenor $\tau$, denote by $n^d_t(\tau) \equiv \max\left(-x_t(\tau) - q^d_t(\tau), 0\right) \geq 0$ the (absolute) amount of naked short-selling that the dealer is engaged in, with a marginal cost $\Gamma_t \geq 0$ (to be specified shortly). The dealer’s wealth dynamics $dw^d_t - r_t w^d_t dt$ now equal

$$\int_0^T x_t(\tau) \left(\frac{dP^\tau_T}{P^\tau_t} - r_t d\tau\right) d\tau + \int_0^T \left[q^d_t(\tau) \Delta_t - n^d_t(\tau) \Gamma_t\right] d\tau dt \tag{19}$$

s.t. $q^d_t(\tau) \geq 0 \tag{20}$

\(^{26}\)Fleming and Garbade (2005) discuss costs of failing to deliver, including foregone interest, counterparty credit risk, labor costs, and worsened customer relations. We could equivalently introduce a securities lending market in which dealers engage in a search for Treasury bonds to borrow from some habitat agents, and then sell to other habitat agents who demand these bonds, as long as these security lending activities entail some cost (for instance, it is difficult to locate the securities).
The dealer is facing the exact same problem as before in (6), except one difference: now he can short-sell \( n_t \) dollars of bonds at a cost of \( n_t^d \Gamma_t \). Because the dealer can borrow securities via reverse repo (and then short-sell), the dealer’s naked short-selling equals \( n_t^d = -x_t - q_t^d \) if it is positive; otherwise, there is no naked short-selling with \( n_t^d = 0 \).

We assume that hedge funds are endowed with the same technology in naked short-selling as dealers. Given her short-selling amount of bonds \( n_t^h(\tau) \equiv \max\left( -q_t^h(\tau), 0 \right) \), the hedge fund’s wealth dynamics \( dw_t^h = w_t^hr_t dt \) read

\[
\int_0^T q_t^h(\tau) \left( \frac{dP_t^r}{P_t} - \left( R_t1_{q_t^h(\tau) \geq 0} + r_t1_{q_t^h(\tau) < 0} \right) dt \right) d\tau - \int_0^T n_t^h(\tau) \Gamma_t d\tau dt.
\]

Here, by holding a short position \( q_t^h(\tau) < 0 \), the hedge fund a) earns the short-term risk-less rate \( r_t \) on the sale proceeds \(-q_t^h(\tau) \) (of the bond); b) is exposed to the bond’s risky return \( dP_t^r/P_t^r \); and c) pays a short-selling cost \( \Gamma_t \).

Suppose that in equilibrium the hedge fund sector is offering \( N_t^h \) amount of naked short-selling. To close the model, we assume that the naked short-selling cost \( \Gamma_t \) is increasing in the equilibrium aggregate naked short selling \( N_t \equiv N_t^h + N_t^d \). For simplicity we assume that

\[ \Gamma_t \equiv \Gamma(N_t) = \gamma N_t \text{ for some } \gamma > 0. \]

### 4.3.2 Equilibrium characterization and implications

A positive naked short-selling amount \( N_t > 0 \) must occur in equilibrium, because market clearing implies that

\[ N_t = -Q_t - X_t = H_t = -\tilde{\beta}_t > 0. \]

\(^{27}\)For instance, \( x_t(\tau) = -1.5 \), \( q_t^d(\tau) = 1 \), and \( n_t^d(\tau) = 0.5 \) imply that the dealer short-sells 1 unit of Treasury via repo, and short-sells another 0.5 through either naked positions or security lending. For more institutional details of reverse repo and short-selling and their implication on balance sheet size, see footnote 21.
As in Section 4.1.2, in equilibrium both hedge funds and dealers are absorbing the habitat agents’ demand in proportion to their risk-bearing capacities, respectively:

\[ X_t = -N_t^d = \frac{\rho_h}{\rho_h + \rho_d} \tilde{\beta}_t, \text{ and } Q_t = -N_t^h = \frac{\rho_d}{\rho_h + \rho_d} \tilde{\beta}_t. \]

It follows that there is a strictly positive cost of naked short-selling \( \Gamma_t = \gamma N_t > 0 \) in equilibrium, which pins down the shadow price of repo financing \( \Delta_t \), despite a zero volume on reverse repo in equilibrium.\(^{28}\) To see this, each dealer is solving the following problem

\[
\max_{q_t^d \geq 0, n_t^d > 0} -n_t^d \cdot \Gamma_t + q_t^d \cdot \Delta_t,
\]

with a linear constraint \( n_t^d = -x_t - q_t^d \). Because dealers are free to obtain one unit of security via repo and then sell it short to cut naked short-selling (and save \( \Gamma_t > 0 \)), the equilibrium indifference

\(^{28}\)The repo market can only be used to finance long positions of levered investors in our model, but both dealers and hedge funds are short selling.
condition for dealers with \( n_t^d = N_t^d > 0 \) implies that:\(^{29}\)

\[
\Delta_t = -\Gamma_t = -\gamma N_t = \gamma T \tilde{\beta}_t < 0. \tag{21}
\]

Following a demand shock on Treasury bonds, the equilibrium collateralized borrowing/lending rate falls below the risk-free rate, as \( \Delta_t = R_t - r_t < 0 \). But dealers are desperate to borrow and short-sell Treasury bonds to satisfy the habitat agents’ surging demand. This mechanism is similar to Treasury being “special” in Duffie (1996). We then derive the equilibrium bond pricing as in Section 4.1.5, recognizing that now the holding cost of an additional Treasury bond is negative with \( \Delta_t = \gamma T \tilde{\beta}_t < 0 \)—i.e., a convenience yield.

Figure 7 Panel A (left) plots the equilibrium yield curves at the normal (stressed) state \( \tilde{\beta} = 0 \) (\( \tilde{\beta} = \beta < 0 \)). In contrast to the supply shock studied in Section 4.2, the demand shock \( \tilde{\beta} = \beta < 0 \) pushes down the entire yield curve. Because dealers (hedge funds) are in a short position in equilibrium, long-term bonds provide a hedging benefit for the marginal investor and hence demand a lower premium. This explains a downward sloping yield curve in Panel A. Figure 7 Panel B plots the convenience yield \( \Delta \) (left axis) and 10-year Treasury-OIS spread (right axis) after the demand shock, both as a function of the shock size \( \beta < 0 \). As a widely documented empirical regularity in the safe asset literature, a Treasury convenience yield in the context of 2007-09 financial crisis explains the negative implied 10-year Treasury-OIS spread (we will come back to this point shortly).

We conclude this subsection by discussing the model implication on repo spreads, highlighting one crucial conceptual difference between supply and demand shocks. In the supply-shock case studied in Section 4.2, a good proxy for Treasury (in)convenience yield \( \Delta = R_t - r_t \) is the repo spread, i.e., the wedge between the dealers’ GCF lending rate and their triparty borrowing rate; the latter is close to the risk-free rate \( r_t \) given the abundance of collateral). In contrast, when Treasury bonds are in shortage in the 2007-09 financial crisis, the repo rate \( R_t \) can still be proxied by the GCF (or bilateral repo) rate, but it is inappropriate to use triparty rate—which is still a collateralized borrowing rate—to proxy for the risk-free rate \( r_t \). Our analysis hence clarifies that

---

\(^{29}\)We have assumed away tenor-dependent demand shocks in this section, by setting \( \theta (\tau) = 1 \) for all \( \tau \in [0, T] \). For general \( \theta (\tau) \), in equilibrium \( N_t (\tau) = H_t (\tau) \) as well as the cost of naked short-selling \( \gamma N_t (\tau) \) will be \( \tau \)-dependent. This would be inconsistent with our General Collateral (GC) repo market setting, in which the repo wedge \( \Delta_t \) is uniform across all tenor \( \tau \). The special repo market studied in Duffie (1996) precisely captures this tenor-\( \tau \) dependent demand, and would be an interesting direction for future research.
the repo GCF-Triparty wedge is ultimately driven by balance sheet (SLR-type) constraints, and should be non-positive in the 2007-09 financial crisis.

### 4.3.3 Treasury convenience yield in 2007–09 financial crisis

Turning to empirical evidence, Figure 8 plots weekly series of the holdings of Treasuries by foreign investors, Fed, and primary dealers, as well as the repo amounts of primary dealers, from January 1, 2007 to December 31, 2008. The three event dates considered are 7/31/2007 when Bear Stearns liquidated two of its subprime hedge funds, 9/15/2008 when Lehman Brothers filed for bankruptcy, and 11/25/2008 when the Fed announced the direct purchases of agency MBS. We observe a flight-to-safety to long-term Treasuries in the 2007–09 crisis, in sharp contrast to their selling pressure during the COVID-19 crisis (Figure 3). In particular, the top left panel shows a large increase (by about $350bn) in foreign investors’ holdings of long-term Treasuries since July 31, 2007, while the top right panel shows a decrease in the Fed’s holdings. In stark contrast to March 2020, the Fed reduced its holdings of Treasuries to help accommodate the surging demand for safety. Primary dealers were net short in Treasuries, especially coupon Treasuries, and were on the net lending side in the repo market. That is, they maintained short positions in their market-making portfolios of Treasuries and often borrowed Treasuries in the repo market, consistent with our theory presented above. Their net short cash positions and net repo lending amounts trended lower towards the end of 2008, suggesting an easing of the shortage of Treasuries.

The excess demand of Treasuries would imply a positive shorting cost $\Gamma_t$, affecting the Treasury-OIS spread in our model (Figure 7 Panel B). The left panel of Figure 9 plots daily series of the 10-year Treasury-OIS spread from January 2007 to December 2008, which stayed below zero mostly and reached as low as -50bps. That is, amid a flight-to-safety to long-term Treasuries (with decreasing 10-year Treasury yield shown in the right panel), the large shorting cost pushed the Treasury-OIS spread to negative levels, consistent with our model. The Treasury-OIS spread then moved up notably after the Fed’s TSLF program was introduced in March 2008, which likely eased the shortage of Treasuries and dampened the shorting cost $\Gamma_t$. We also observe that the 3-month Treasury-OIS spread stays slightly more negative, showing that the excess demand for T-bills is even stronger.

Regarding repo market, daily series of triparty repo rates are not readily available. Daily GCF
Figure 8: **Investor Flows and Positions of Treasuries during the 2007–09 Financial Crisis**

Notes: The top left panel plots monthly series of the changes in holdings of long-term Treasuries by foreign investors. The top right panel plots weekly series of the amount of the Fed’s Treasury holdings. The bottom left panel plots weekly series of the primary dealers’ net positions of Treasuries. The bottom right panel plots weekly series of the primary dealers’ gross repo amount, gross reverse repo amount, and net reverse repo amount. The units are all in billions of U.S. dollars. The sample period is from January 1, 2007 to December 31, 2008. Data source: the FR2004 data collected by the Federal Reserve Bank of New York, the Treasury International Capital system, and the Federal Reserve’s H.4.1 release.

repo rates extend back to 2005, but we find large month-end drops of GCF repo rates in this period, making it difficult to draw reliable inference. That being said, using the month-end series of triparty repo rates available through fillings of money market funds with the SEC (Krishnamurthy, Nagel, and Orlov (2014); Hu, Pan, and Wang (2019)), we do find a negative GCF-Triparty repo spread, consistent with the absence of SLR-like balance sheet constraints around 2008.
Notes: This figure plots daily series of the 3-month and 10-year Treasury-OIS spreads (left panel), as well as the constant maturity Treasury yields of 3-month and 10-year maturities (right panel), from January 1, 2007 to December 31, 2008.

5 Regression Analysis of Dealer Constraints and Costs

The empirical evidence presented so far follows an event-study approach, and has revealed that (a) the shock to the demand of long-term Treasuries is negative in the COVID-19 crisis and positive in the 2007–09 financial crisis, (b) dealers incur a balance sheet cost in holding long-term Treasuries and intermediating repos in the COVID-19 crisis, and (c) dealers incur a cost in short-selling Treasuries in the 2007–09 crisis. In this section, we conduct a formal regression analysis using the full sample from January 2006 to April 2020 as further supporting evidence to our model.

5.1 Dealer Constraints and Costs across Sub-periods

Guided by our theoretical framework in Section 4, we first break up the full sample period according to variations of dealers’ balance sheet cost and shorting cost ($\Lambda_t$ and $\Gamma_t$ in the model, respectively). The post-crisis implementation of SLR that drives dealers’ balance sheet cost phased in progressively from 2012 onward (see Section 2.2.2). We define pre-SLR and post-SLR as before and after September, 2014, respectively, when the denominator of the SLR was finalized and banks learned how binding the rule would be. We expect the balance sheet cost $\Lambda_t$ to be effectively positive post-SLR, but either negligible or minor pre-SLR.\footnote{Though not in effect officially until 2018, banks could have had reputation concerns due to the disclosures and also begin to prepare for the final adoption long before, which would make the SLR exert influence much earlier.} Moreover, according to the NBER Business Cycle...
classifications, the Great Recession (GR) ended in June 2009, so we define pre-GR and post-GR as the periods before and after June 2009, respectively. Given that primary dealers entered the GR with large net short positions of Treasuries, we expect the shorting cost $\Gamma_t$ to be positive, though likely abated significantly around March 2008–June 2009 during which the Fed allowed dealers to obtain Treasuries against non-Treasury collateral in the TSLF program (Fleming, Hrung, and Keane (2010)). We hence define pre-TSLF and post-TSLF as the periods before and after March 2008, respectively.

In sum, our sample period consists of four segments roughly together the respective interpretation of our model: (1) pre-TSLF, when $\Gamma_t$ is positive and $\Lambda_t$ is zero, leading to a positive repo spread and positive Treasury-OIS spread; (2) post-TSLF and pre-GR, when $\Gamma_t$ is reduced substantially but still positive and $\Lambda_t$ is zero, (3) post-GR and pre-SLR when both $\Gamma_t$ and $\Lambda_t$ are zero, and (4) post-SLR when $\Gamma_t$ is zero and $\Lambda_t$ is positive, implying a negative Treasury-OIS spread.

5.2 Empirical Results

The left panel of Figure 10 reports the average daily GCF-Triparty repo spread, which is only over the last two sub-periods due to limitation of daily triparty repo rates. Consistent with our
Table 1: **Repo Spread Regressions**

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Post-SLR</td>
<td>Pre-SLR</td>
<td>All</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.038***</td>
<td>0.021***</td>
<td>0.051***</td>
<td>0.051***</td>
</tr>
<tr>
<td></td>
<td>(10.907)</td>
<td>(5.116)</td>
<td>(127.982)</td>
<td>(127.982)</td>
</tr>
<tr>
<td>(D_{Post-SLR})</td>
<td>0.068***</td>
<td>-0.018***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8.673)</td>
<td>(-4.291)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D_{QuarterEnd})</td>
<td>0.202**</td>
<td>0.039**</td>
<td>0.039**</td>
<td>0.164*</td>
</tr>
<tr>
<td></td>
<td>(2.310)</td>
<td>(2.285)</td>
<td>(2.285)</td>
<td>(1.835)</td>
</tr>
<tr>
<td>(D_{QuarterEnd} \times D_{Post-SLR})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>1,930</td>
<td>1,412</td>
<td>518</td>
<td>1,930</td>
</tr>
<tr>
<td>Adj (R^2)</td>
<td>0.142</td>
<td>0.253</td>
<td>0.323</td>
<td>0.360</td>
</tr>
<tr>
<td>Quarter FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Column (1) reports regressions of the daily GCF-Triparty repo spread on dummy variables for the sub-periods over the whole sample period of August 1, 2012 to April 30, 2020. Columns (2) and (3) report regressions of the GCF-Triparty repo spread on the dummy for quarter ends (\(D_{QuarterEnd}\)) for post-SLR and pre-SLR samples, respectively, while column (4) reports regressions on \(D_{Post-SLR}\), \(D_{QuarterEnd}\), and their interaction term for the whole sample. Quarterly fixed effects are included in all regressions except that in column (1). The \(t\)-statistics based on standard errors clustered at calendar quarters are reported in parentheses. The significance levels are represented by *\(p<0.1\), **\(p<0.05\), ***\(p<0.01\).

characterizations of these sub-periods, the GCF-triparty repo spread is only slightly positive post-GR, about 4 bps before the SLR formally phased in, and then increased to about 11 bps afterwards. Column (1) of Table 1 reports regressions of the repo spread on the sub-period dummies to formally test the significance of its change:

\[
GCF\text{-Triparty}_t = \alpha + \beta D_{Post-SLR} + \varepsilon_t, \tag{22}
\]

where the full sample of August 1, 2012 to April 30, 2020 is used (recall that the daily triparty repo rate series become available only since August 2012). The significantly positive coefficient on \(D_{Post-SLR}\) confirms the significance of the incremental changes.

Regarding the balance sheet cost \(\Gamma_t\), the effect of the SLR rule on the dealers’ balance sheet is likely to come in over an extended period of time rather than as an immediate effect. Our post-SLR dummy in column (1) of Table 1 captures this average effect over this extended period, but it could be confounded by other factors. We explore a variation based on quarter-end effects to mitigate this concern and further quantify the potential effect of the SLR on the repo spread. Foreign bank dealers’ repo intermediation activities contract at quarter ends when snapshots of
Table 2: Treasury-OIS Spread Regressions

<table>
<thead>
<tr>
<th></th>
<th>2006 – 2020</th>
<th>2012 –2020</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-SLR</td>
<td>Post-SLR</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.433***</td>
<td>0.167***</td>
</tr>
<tr>
<td></td>
<td>(-28.410)</td>
<td>(8.010)</td>
</tr>
<tr>
<td>$D_{Post-TSLF}$</td>
<td>0.341***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.506)</td>
<td></td>
</tr>
<tr>
<td>$D_{Post-GR}$</td>
<td>0.266***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.735)</td>
<td></td>
</tr>
<tr>
<td>$D_{Post-SLR}$</td>
<td>0.139***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.889)</td>
<td></td>
</tr>
<tr>
<td>GCF-Triparty</td>
<td>0.163</td>
<td>0.407*</td>
</tr>
<tr>
<td></td>
<td>(0.495)</td>
<td>(1.821)</td>
</tr>
<tr>
<td>Obs</td>
<td>3,565</td>
<td>413</td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td>0.835</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Notes: The first column reports regressions of the daily 10-year Treasury-OIS spread on dummy variables for sub-periods over the full sample period of January 2006 to April 2020. The last two columns report regressions of the daily Treasury-OIS spread on the daily GCF-Triparty repo spread for the sample of pre-SLR and post-SLR, respectively, with the full sample period as from August 2012 to April 2020. The t-statistics based on standard errors clustered at calendar quarters are reported in parentheses. The significance levels are represented by * $p<0.1$, ** $p<0.05$, *** $p<0.01$.

their balance sheet are used to calculate leverage ratio (Duffie (2018)). In consequence, when $\Lambda_t$ turns positive post-SLR, U.S. dealers receive higher demand for repo intermediation, resulting effectively in a higher $\Lambda_t$ on quarter ends than other days. This quarter-end effect should be much weaker pre-SLR.

Table 1 report regressions of the repo spread on quarter-end dummies ($D_{QuarterEnd}$); with quarter fixed effects, the coefficient on $D_{QuarterEnd}$ captures the difference of the repo spread between the quarter-end and other days within the same quarter. The difference is about 20 bps post-SLR (Column 2) but only 4 bps pre-SLR (Column 4); Column (4) further confirms that the pre- and post-SLR difference in quarter-end effects is statistically significant by investigating the interaction term of $D_{QuarterEnd}$ and $D_{Post-SLR}$.

Turning to the Treasury-OIS spread, the right panel of Figure 10 reports the average daily Treasury-OIS spread over the four sub-periods. Again, consistent with our theory, the average Treasury-OIS spread was as low as about -40 bps during the pre-TSLF period. The Fed’s TSLF program shrunk this spread greatly to -9 bps, and it turned positive post-GR. It then reached about 16 bps before the SLR phased in and climbed to about 34 bps afterwards. The first column
of Table 2 reports a regression of the 10-year Treasury-OIS spread similar to (22) but with dummies for all sub-periods. The coefficients measure the incremental change of a sub-period relative to the previous sub-period (so to obtain the average level of Treasury-OIS spread for the post-SLR period as in Figure 10, we need take the sum of the intercept and all the dummy coefficients). The significantly positive dummy coefficients imply that balance sheet constraints are becoming more and more binding since the 2007–09 crisis.

The last two columns of Table 2 report contemporaneous time-series regressions of the 10-year Treasury–OIS spread on the repo (GCF-Triparty) spread for pre-SLR and post-SLR sub-periods, respectively. The regression coefficient is low and insignificantly different from zero pre-SLR because the balance sheet cost $\Lambda_t$ are likely to be negligible. Post-SLR, instead, the regression coefficient is significantly positive because both GCF–Triparty and Treasury–OIS spreads contain a positive $\Lambda_t$ (which is potentially time-varying).

6 Conclusion

In sharp contrast to most previous crisis episodes, the Treasury market experienced severe stress and illiquidity in March 2020 during the COVID-19 pandemic, raising concerns that the safe-haven status of U.S. Treasuries could be eroding. We document that some large owners of Treasuries substantially reduced their holdings during March 2020 and the intermediary sector struggled to absorb this supply shock.

To understand the inelastic response of the intermediary sector, we build a model in which balance sheet constraints of dealers and supply shocks from habitat agents interact with each other, affecting equilibrium Treasury yields. A novel element of our model is to introduce repo financing as an important part of dealers’ intermediation activities, through which levered investors obtain leverage. Both direct holdings of Treasuries and reverse repo positions of dealers are subject to a balance sheet constraint related to regulation reforms since the 2007–09 crisis such as the SLR. Consistent with model implications, the spread between the Treasury yield and OIS rate and the spread between dealers’ reverse repo and repo rates are both highly positive in the COVID-19 crisis. Over the whole sample of 2006–2020, both the Treasury-OIS and Triparty-GCF spreads increased after 2015 when the regulatory reforms phased in, with their correlation turning significantly positive.
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Appendix

A Euler Equations for Treasury Pricing

We derive the ODE system for \( \{ C_0 (\cdot), C_\beta (\cdot) \} \) based on the dealer’s Euler equation first. We then offer a detailed numerical procedure to solve the ODE system, and then give the results on the equilibrium OIS curves.

A.1 Euler Equation and ODE System

Guess that

\[
\begin{align*}
P_{\beta t}^\tau &= \exp [-(A(\tau) r_t + C_0(\tau))] \\
P_{\beta t}^\tau &= \exp [-(A(\tau) r_t + C_\beta(\tau))]
\end{align*}
\]

and recall the yield is defined as 
\[ y_t^*= -\frac{\log P_{\beta t}^\tau}{\tau} = \frac{A(\tau) r_t + C_\beta(\tau)}{\tau}. \]

Return dynamics. One can show that (where \( dN_t \) denotes the Poisson shock with intensity \( \xi_\beta \))

\[
\begin{align*}
\frac{dP_{0t}^\tau}{P_{0t}^\tau} &= \left[ A'(\tau) r_t + A(\tau) \kappa (r_t - \tau) + C_0'(\tau) + \frac{\sigma^2}{2} A^2(\tau) \right] dt + \left\{ e^{C_0(\tau)-C_\beta(\tau)} - 1 \right\} dN_t - A(\tau) \sigma dZ_t, \\
\frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} &= \left[ A'(\tau) r_t + A(\tau) \kappa (r_t - \tau) + C_\beta'(\tau) + \frac{\sigma^2}{2} A^2(\tau) \right] dt + \left\{ e^{C_\beta(\tau)-C_0(\tau)} - 1 \right\} dN_t - A(\tau) \sigma dZ_t.
\end{align*}
\]

(23)

We perform the calculation for \( \frac{dP_{0t}^\tau}{P_{0t}^\tau}, \frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} \) follows similarly. First,

\[
\begin{align*}
dP_{\beta t}^\tau &= \exp [-(A(\tau) r_t + C_\beta(\tau))]
\left[ A'(\tau) r_t dt + A(\tau) \kappa (r_t - \tau) dt + C_\beta'(\tau) dt - A(\tau) \sigma dZ_t \right] \\
&+ \exp [-(A(\tau) r_t + C_\beta(\tau))]
\frac{1}{2} A^2(\tau) \sigma^2 dt + \left[ e^{-(A(\tau) r_t + C_0(\tau))} - e^{-(A(\tau) r_t + C_\beta(\tau))} \right] dN_t,
\end{align*}
\]

(24)
which implies that
\[
\frac{dP_{\beta t}}{P_{\beta t}} = A'(\tau) \, r_t \, dt + A(\tau) \, \kappa (r_t - \tau) \, dt + C_{\beta}'(\tau) \, dt + \frac{A^2(\tau) \, \sigma^2}{2} \, dt - A(\tau) \, \sigma \, dZ_t + \left\{ e^{C_{\beta}(\tau) - C_0(\tau)} - 1 \right\} \, dN_t.
\]

It is easy to calculate that \( \mathbb{E}_t \left[ \frac{dP_{\beta t}}{P_{\beta t}} \right] = \mu_{\beta t} \, dt \) with
\[
\mu_{\beta t} \equiv A'(\tau) \, r_t + A(\tau) \, \kappa (r_t - \tau) + C_{\beta}'(\tau) + \frac{1}{2} A^2(\tau) \, \sigma^2 + \xi_{\beta} \left( e^{C_{\beta}(\tau) - C_0(\tau)} - 1 \right).
\]

**Euler equations.** As before let us focus on the distress state. The arbitrager is maximizing the mean-variance objective over
\[
\int_0^T x_{\beta t}^\tau \left( \frac{dP_{\beta t}}{P_{\beta t}} - r_t \, dt - \Lambda_{0t} \, dt \right) \, d\tau.
\]

Given (23), we have
\[
\mathbb{E}_t \left[ \int_0^T x_{\beta t}^\tau \left( \frac{dP_{\beta t}}{P_{\beta t}} - r_t \, dt - \Lambda_{0t} \, dt \right) \, d\tau \right] = \int_0^T x_{\beta t}^\tau (\mu_{\beta t} - r_t - \Lambda_{0t}) \, d\tau \, dt
\]
\[
\mathbb{V} \mathbb{A} r_t \left[ \int_0^T x_{\beta t}^\tau \left( \frac{dP_{\beta t}}{P_{\beta t}} - r_t \, dt - \Lambda_{0t} \, dt \right) \, d\tau \right] = \left( \int_0^T \left\{ x_{\beta t}^\tau (e^{C_{\beta}(\tau) - C_0(\tau)} - 1) \, d\tau \right\} \right)^2 \xi_{0t} \, dt + \left( \int_0^T x_{\beta t}^\tau A(\tau) \, d\tau \right)^2 \sigma^2 \, dt
\]

Therefore the dealer’s FOC with respect to \( x_{\beta t}^\tau \) is (we omit \( dt \) from now on)
\[
\mu_{\beta t} - r_t - \Lambda_{\beta t} = A(\tau) \frac{\sigma^2}{\rho_d} \int_0^T x_{\beta t}^\tau A(u) \, du + \left( e^{C_{\beta}(\tau) - C_0(\tau)} - 1 \right) \cdot \frac{\xi_{\beta}}{\rho_d} \int_0^T \left\{ x_{\beta t}^\tau (e^{C_{\beta}(u) - C_0(u)} - 1) \, du \right\}.
\]

Expanding \( \mu_{\beta t} \) in (25) and collecting terms we have
\[
(A'(\tau) - 1) \, r_t - \Lambda_{\beta t} + A(\tau) \, \kappa (r_t - \tau) + C_{\beta}'(\tau) + \frac{1}{2} A^2(\tau) \, \sigma^2 \, r_t + \xi_{\beta} \left( e^{C_{\beta}(u) - C_0(u)} - 1 \right) = A(\tau) \cdot \frac{\sigma^2}{\rho_d} \left[ \int_0^T x_{\beta t}^\tau A(u) \, du \right] + \left( e^{C_{\beta}(u) - C_0(u)} - 1 \right) \cdot \frac{\xi_{0}}{\rho_d} \int_0^T \left\{ x_{\beta t}^\tau (e^{C_{\beta}(u) - C_0(u)} - 1) \, d\tau \right\}.
\]

Because this equation holds for all \( r_t \) we must have \( A(\tau) \kappa + A'(\tau) = 1 \); with initial condition \( A(0) = 0 \) we have
\[
A(\tau) = \frac{1 - e^{-\kappa \tau}}{\kappa}.
\]

(26)
Moving on to the function $C_0(\cdot)$, and collecting terms, we have

$$C'_{\beta}(\tau) + \xi_{\beta} \left( e^{C_0(u)-C_0} - 1 \right) \left( 1 - \frac{1}{\rho_d} \int_0^T \left\{ x_{\beta t}^u \left( e^{C_0(u)-C_0} - 1 \right) du \right\} \right)$$

$$= A(\tau) \cdot \frac{\sigma^2}{\rho_d} \int_0^T x_{\beta t}^u A(u) du + \Lambda_{\beta} + A(\tau) \kappa\tau - \frac{1}{2} A^2(\tau) \sigma^2.$$

This is the equation when $\tilde{\beta}_t = \beta$.

For the normal state $\beta_t = 0$ we have

$$\frac{dP_0^t}{P_0^t} = \mu_0^t dt + \left\{ e^{C_0(\tau)-C_0} - 1 \right\} d\hat{N}_t - A(\tau) \sigma dZ_t$$

where

$$\mu_0^t \equiv A' (\tau) \lambda + A(\tau) \kappa (\tau - \overline{\tau}) + C_0'(\tau) + \frac{1}{2} A^2(\tau) \sigma^2 + \xi_0 \left( e^{C_0(\tau)-C_0} - 1 \right).$$

And the ODE for $C_0(\cdot)$ based on the dealer’s Euler equation at the stress state can be derived analogously:

$$C_0'(\tau) + \xi_0 \left( e^{C_0(\tau)-C_0} - 1 \right) \left( 1 - \frac{1}{\rho_d} \int_0^T \left\{ x_{t\beta}^u \left( e^{C_0(\tau)-C_0} - 1 \right) du \right\} \right)$$

$$= A(\tau) \cdot \frac{\sigma^2}{\rho_d} \int_0^T x_{t\beta}^u A(u) du + \Lambda_{\beta} + A(\tau) \kappa\tau - \frac{1}{2} A^2(\tau) \sigma^2.$$

We hence have arrived at the ODE system (27) and (28) for \{C_0(\cdot), C_{\beta}(\cdot)\}, with boundary conditions

$$C_0(0) = C_{\beta}(0) = 0, C_0'(0) = \Lambda_0 = \lambda B_0, C_{\beta}'(0) = \Lambda_{\beta} = \lambda B_{\beta}.$$

### A.2 Numerical Methods

This section outlines the numerical procedure in solving for (27) and (28). Denote

$$D_0(\tau) \equiv A(\tau) \cdot \frac{\sigma^2}{\rho_d} \int_0^T x_{t0}^u A(u) du + \Lambda_0 + A(\tau) \kappa\tau - \frac{1}{2} A^2(\tau) \sigma^2,$$

$$D_{\beta}(\tau) \equiv A(\tau) \cdot \frac{\sigma^2}{\rho_d} \int_0^T x_{t\beta}^u A(u) du + \Lambda_{\beta} + A(\tau) \kappa\tau - \frac{1}{2} A^2(\tau) \sigma^2.$$
which have been solved in closed-form given \( x_0^0 \) and \( A(\tau) = \frac{1-e^{-\kappa \tau}}{\kappa} \) (recall Eq. (26)).

The numerical procedure is as follows.

1. Start with \( K_0^{(0)} = K_\beta^{(0)} = 1 \).

2. With \( \left( K_0^{(n)}, K_\beta^{(n)} \right) \), define

\[
\hat{D}_0 (\tau) \equiv D_0 (\tau) + K_0^{(n)}, \quad \hat{D}_\beta (\tau) \equiv D_\beta (\tau) + K_\beta^{(n)}.
\]

We now obtain the solution by solving the ODE system (29)

\[
\begin{cases}
C'_0 (\tau) + K_0^{(n)} e^{C_0(\tau) - C_\beta(\tau)} = \hat{D}_0 (\tau) \\
C'_\beta (\tau) + K_\beta^{(n)} e^{C_\beta(\tau) - C_0(\tau)} = \hat{D}_\beta (\tau)
\end{cases}
\]

with the initial conditions

\[
C_0 (0) = C_\beta (0) = C'_0 (0) = C'_\beta (0) = 0,
\]

by following these steps.

(a) First of all, from the first equation in (29) we have

\[
e^{C_0(\tau) - C_\beta(\tau)} = \frac{\hat{D}_0 (\tau) - C'_0 (\tau)}{K_0^{(n)}}.
\]

Taking derivative with respect to \( \tau \) on both sides of the first equation in (29), and plugging in \( e^{C_0(\tau) - C_\beta(\tau)} \), we get

\[
C''_0 (\tau) + \left( \hat{D}_0 (\tau) - C'_0 (\tau) \right) \left( C'_0 (\tau) - C'_\beta (\tau) \right) = \hat{D}'_0 (\tau).
\]

(b) Now, using \( C'_\beta (\tau) = \hat{D}_\beta (\tau) - K_\beta^{(n)} e^{C_\beta(\tau) - C_0(\tau)} \) from the second equation, and plugging in (30), one can get,

\[
C''_0 (\tau) + \left( \hat{D}_0 (\tau) - C'_0 (\tau) \right) \left( C'_0 (\tau) - \hat{D}_\beta (\tau) \right) + K_\beta^{(n)} K_0^{(n)} = \hat{D}'_0 (\tau).
\]
(c) Letting $z(\tau) \equiv C_0'(\tau)$, we have a Riccati equation for $z(\tau)$

$$z'(\tau) = \left(\hat{D}_0'(\tau) + \hat{D}_\beta(\tau) \hat{D}_0(\tau) - K_0^{(n)} K_0^{(n)}\right) - \left(\hat{D}_\beta(\tau) + \hat{D}_0(\tau)\right) z(\tau) + z^2(\tau)$$

with a boundary condition $z(0) = 0$. Once we numerically solve for $z(\tau)$, we can calculate $C_0(\tau) = \int_0^\tau z(u) \, du$ and derive $C_\beta(\tau)$ according to Eq. (30).

3. Calculate $K_0^{(n+1)}, K_\beta^{(n+1)}$ based on the new solution using (31):

$$\begin{align*}
K_0^{(n+1)} &\equiv \xi_0 \left(1 - \frac{1}{\rho_d} \int_0^T \left\{ x_0^u \left(e^{C_0(u)-C_\beta_0(u)-1}\right) du \right\} \right) \\
K_\beta^{(n+1)} &\equiv \xi_\beta \left(1 - \frac{1}{\rho_d} \int_0^T \left\{ x_\beta^u \left(e^{C_\beta(u)-C_0(u)-1}\right) du \right\} \right),
\end{align*}$$

(31)

4. If $\left\| (K_0^{(n+1)} - K_0^{(n)}, K_\beta^{(n+1)} - K_\beta^{(n)}) \right\| < \epsilon$ then terminate. Otherwise set $n = n + 1$ and go to Step 2.

A.3 Derivation of Equilibrium OIS Curves

We guess and verify that the equilibrium OIS prices take the following forms with $A(\tau) = \frac{1-e^{-\kappa \tau}}{\kappa}$:

$$\begin{align*}
P_{0t}^{OIS,\tau} &= \exp \left[- \left(A(\tau) r_t + C_0^{OIS}(\tau)\right)\right] \\
P_{\beta t}^{OIS,\tau} &= \exp \left[- \left(A(\tau) r_t + C_\beta^{OIS}(\tau)\right)\right]
\end{align*}$$

where $\{C_0^{OIS}(\cdot), C_\beta^{OIS}(\cdot)\}$ are to be determined endogenously. Note that the Treasury-OIS spread at tenor $\tau$, denoted by $\Delta y^\tau$, then can be calculated as

$$\Delta y^\tau = \frac{\ln P^\tau - \ln P_{\beta t}^{OIS,\tau}}{\tau} = \frac{C^{OIS}(\tau) - C(\tau)}{\tau}.$$
Based on similar derivations as in Section A.1, we arrive at

\[
C_0^{OIS'}(\tau) + \xi_0 \left( \exp \left( C_0^{OIS}(\tau) - C_0^{OIS}(\tau) \right) - 1 \right) \left( 1 - \frac{1}{\rho_d} \int_0^T x_{t0}^u \left( \exp \left( C_0^{OIS}(\tau) - C_0^{OIS}(\tau) \right) - 1 \right) du \right) = A(\tau) \kappa T - \frac{A^2(\tau)}{2} \sigma^2 + \frac{A(\tau) \tau^2}{\rho_d} \int_0^T x_{t0}^u A(u) du
\]

(32)

\[
C_\beta^{OIS'}(\tau) + \xi_\beta \left( \exp \left( C_\beta^{OIS}(\tau) - C_0^{OIS}(\tau) \right) - 1 \right) \left( 1 - \frac{1}{\rho_d} \int_0^T x_{t\beta}^u \left( \exp \left( C_\beta^{OIS}(\tau) - C_0^{OIS}(\tau) \right) - 1 \right) du \right) = A(\tau) \kappa T - \frac{A^2(\tau)}{2} \sigma^2 + \frac{A(\tau) \tau^2}{\rho_d} \int_0^T x_{t\beta}^u A(u) du
\]

(33)

with initial conditions \( C_0^{OIS}(0) = C_0^{OIS'}(0) = C_\beta^{OIS}(0) = C_\beta^{OIS'}(0) = 0 \).

B Data and Additional Evidence

In this appendix, we first provide details of the data and variables used in empirical analysis. We then present two sets of additional empirical evidence, one highlighting the flight-to-cash nature of the COVID-19 shock and the other breaking down primary dealers’ repo positions into different tenor buckets.

B.1 Data

We obtain daily series of constant-maturity Treasury (CMT) yields from the H.15 reports of the Federal Reserve, which are equal to the coupon rates on par bonds. We obtain daily series of overnight index swap (OIS) rates from Bloomberg. The OIS is a fully collateralized interest rate swap contract that exchanges a constant cash flow against a flow of floating payment indexed to the geometric average of the daily effective federal funds rate. OIS contracts with maturities of up to one year have only one final payment, while cash payments for those with maturities of over one year are made quarterly. Hence, OIS rates are effectively zero-coupon yields for maturities of up to one year and par yields for maturities of over one year, both comparable to the CMT yields. We then take the difference between the CMT yield and the maturity-matched OIS rate as the Treasury-OIS spread.
We use overnight repo rates of Treasury securities of both the tri-party market and the GCF market. Daily series of GCF repo rates are provided by the Depository Trust & Clearing Corporation (DTCC), available starting from 2005, calculated as the average interest rate across repo transactions weighted by volume within a day.\textsuperscript{31} The tri-party repo rates are from multiple sources. First, we obtain daily series of the Tri-party General Collateral Rate (TGCR) computed by the Federal Reserve Bank of New York, available from August 22, 2014. The TGCR is calculated as the volume-weighted median of transaction-level tri-party repo data, excluding GCF Repo transactions and transactions to which the Federal Reserve is a counterparty.\textsuperscript{32} Second, we obtain daily series of tri-party repo rates from August 1, 2012 to August 21, 2014 calculated by the Bank of New York Mellon, the largest of the two clearing banks. These repo rates are also calculated as volume-weighted medians on each business day for new, overnight repo trades with U.S. Treasuries (excluding Strips) as collateral assets.\textsuperscript{33} Third, for November 2010–July 2012, we obtain monthly series of tri-party repo rates using the overnight tri-party repo trades between MMFs and dealers reported in the N-MFP filings with the SEC, similar to Hu, Pan, and Wang (2019). Specifically, we calculate the volume-weighted medians of all overnight tri-party repo trades on the last business day of each month. Fourth, for October 2006 – April 2010, we use the month-end value-weighted average overnight repo rates (weighted by notional amounts) constructed in Krishnamurthy, Nagel, and Orlov (2014) based on quarterly N-CSR, N-CSRS, and N-Q filings with SEC. MMFs file these reports at different month-ends throughout each quarter, so monthly series of repo rates can be calculated.

The weekly series of primary dealers’ net positions and financing amounts of Treasury securities are obtained from the FR2004 data collected by the Federal Reserve Bank of New York. The data are reported on a weekly basis, as of the close of business each Wednesday.\textsuperscript{34} The reported series are netted and aggregated across all primary dealers, available for four categories, including T-bills, T-notes, and T-bonds.

\textsuperscript{31}The series can be downloaded at \url{http://www.dtcc.com/charts/dtcc-gcf-repo-index#download}.
\textsuperscript{32}The TGCR is one of the three overnight repo rates provided by the Federal Reserve Bank of New York as important reference rates to financial markets, together with the Broad General Collateral Rate (BGCR) and the Secured Overnight Financing Rate (SOFR). The calculation of BGCR includes all trades used in the calculation of TGCR plus the GCF Repo transactions. The calculation of SOFR includes all trades used in the calculation of BGCR plus the bilateral Treasury repo transactions cleared through the Delivery-versus-Payment (DVP) service offered by the FICC but filtered to remove a portion of transactions considered “specials.” For further details, see \url{https://www.newyorkfed.org/markets/treasury-repo-reference-rates-information}.
\textsuperscript{33}The data can be found at \url{https://repoindex.bnymellon.com/repoindex/}.
\textsuperscript{34}For details, see \url{https://www.newyorkfed.org/markets/gsds/search}. 
coupon-bearing nominal securities (coupons), Treasury inflation-protected securities (TIPS), and Floating Rate Notes (FRNs) that began to be issued in January 2014 and reported from 2015. The net positions include both spot cash positions and Treasury derivatives like futures (Fleming and Rosenberg (2007)).

Regarding financing amounts, the FR2004 data separate repo from other financing activities like security lending contracts from April 3, 2013, but repo and securities lending contracts are blended together earlier. Hence, we use the amounts of repo and reverse repo from April 3, 2013 onward, but total financing amounts of cash in and cash out before, which will be referred to as repo and reverse repo for convenience. Note that the financing contracts are defined from the perspective of dealers, so dealers borrow cash through repo and lend cash out through reverse repo. For both repo and reverse repo, the amounts are available for overnight and term contracts separately. We mainly focus on the total amount by adding the overnight and term financing amounts together, but will briefly discuss the breakdown in the next section.

The daily VIX series are obtained from the Chicago Board Options Exchange (CBOE). We obtain daily series of constant-maturity yields of TIPS also from the H.15 reports of the Federal Reserve. We compute the breakeven inflation rate as the difference between the CMT nominal yield and TIPS yield of the same maturity, which is a market-based measure of expected inflation. We also obtain daily series of inflation swap rates from Bloomberg as an alternative measure of expected inflation. To measure the uncertainty and tail event probability of inflation, we obtain weekly series of the standard deviation and the probability of a large increase in inflation (of more than 3%) based on inflation density estimates using 5-year inflation caps and floors.35

We obtain the amounts of Treasury holdings and issuance from various sources. First, the monthly series of net issuance amount (gross minus retirement) of Treasury notes and bonds are obtained from the SIFMA.36 Second, quarterly series of flows into Treasury securities of pension funds, mutual funds, insurance companies, and hedge funds are obtained from the Financial Accounts of the United States – Z.1, provided by the Federal Reserve.37 The amounts for mutual funds exclude T-bills, while those for others include them. We use the quarterly change in market

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36The series can be found at https://www.sifma.org/resources/research/us-marketable-treasury-issuance-outstanding-and-interest-rates/.
37The series can be found at https://www.federalreserve.gov/releases/z1/20200921/html/default.htm.
values of Treasury holdings for hedge funds, but the flow series for insurance companies, pension funds, and mutual funds adjust for capital gains. None are seasonally adjusted. To give a sense of the magnitude of the capital gains adjustment, the level change is around $240bn while the flow is around $200bn, for mutual funds in 2020 Q1. Third, monthly series of foreign net purchases of U.S. long-term Treasury securities are obtained from the Treasury International Capital (TIC) system. We group them into four categories: Europe, Asia, Caribbean (including a lot popular tax haven countries), and Other (including all other countries and international organizations). The nonmarketable Treasuries are excluded. Fourth, weekly series of the total face value of U.S. Treasury securities held by the Federal Reserve are provided in their H.4.1 release. We group these series into T-bills, nominal coupons, and TIPS.

We obtain weekly series of total net assets (TNA) of prime, government, and tax-exempt money market funds from the ICI. We obtain weekly seasonally adjusted series of the commercial and industrial loans (C&I), cash assets, treasury and agency securities, fed funds sold and cash lent out through reverse repo, and deposits, of all commercial banks, provided in the H.8 release of the Federal Reserve.

B.2 Additional Empirical Evidence

We provide two sets of additional empirical evidence. First, Figure B1 presents series of fund flows of MMFs and banks. The top panels show that there are negative net flows out of prime MMFs and positive net flows into government MMFs, while the middle panels show that bank deposits increased significantly, in both the COVID-19 and 2007–09 crises. The bottom panels show that banks allocate the incoming deposits into cash assets significantly and into long-term Treasuries and agency securities slightly. All these features are characteristic of flight-to-safety and flight-to-liquidity.

Second, Figure B2 provides a breakdown of primary dealers’ repo and reverse repo amounts into overnight and term contracts. The net reverse repo amounts show that primary dealers conduct

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38 The series can be found at https://www.treasury.gov/resource-center/data-chart-center/tic/Pages/ticsec.aspx.
39 The series can be found at https://www.federalreserve.gov/releases/h41/.
40 These include total federal funds sold to, and reverse repos with, commercial banks, brokers and dealers, and others, including the Federal Home Loan Banks.
41 The series can be found at https://www.federalreserve.gov/releases/h8/.
maturity transformation in their Treasury repo intermediation, borrowing overnight and lending term funds in both the COVID-19 and 2007–09 crises. Yet, the net borrowing amount through overnight repo is much larger in 2020 than in 2007–09, while the net lending amount through term reverse repo is similar in these two time periods. That is, primary dealers become more of a net cash borrower in the repo market post-crisis.
Figure B1: Fund Flows of MMFs and Banks During the COVID-19 and 2007–09 Crises

Notes: The top panels plot weekly series of the flows of money market funds (MMFs). The middle panels plot weekly series of the amounts of deposits of commercial banks. The bottom panels plot weekly series of the amounts of commercial and industrial loans (C&I), cash assets, Treasuries and agency securities, and funds lent in the federal funds and repo markets, on the asset side of commercial banks. The units are all in billions of U.S. dollars. The sample period is from January 1, 2007 to December 31, 2008 for the left panels, and from January 1, 2020 to April 30, 2020 for the right panels.
Figure B2: Breakdown of Primary Dealers’ Treasury Repo Positions

Notes: This figure plots weekly series of the primary dealers’ gross repo amount, gross reverse repo amount, and net reverse repo amount for the overnight (left panels) and term (right panels) contracts. The sample period is from January 1, 2020 to April 30, 2020 for the top panels, and from January 1, 2007 to December 31, 2008 for the bottom panels. The units are all in billions of U.S. dollars.