Treasury inconvenience yields during the COVID-19 crisis

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\textbf{ABSTRACT}

In sharp contrast to previous crisis episodes, the Treasury market experienced severe stress and illiquidity during the COVID-19 crisis, raising concerns that the safe-haven status of US Treasuries may be eroding. We document large shifts in Treasury ownership and temporary accumulation of Treasury and reverse repo positions on dealer balance sheets during this period. We build a dynamic equilibrium asset pricing model in which dealers subject to regulatory balance sheet constraints intermediate demand/supply shocks from habitat agents and provide repo financing to levered investors. The model predicts that Treasury inconvenience yields, measured as the spread between Treasuries and overnight-index swap rates (OIS), as well as spreads between dealers' reverse repo and repo rates, should be highly positive during the COVID-19 crisis, as is confirmed in the data. The same model framework, adapted to the institutional setting in 2007–2009, can also explain the negative Treasury-OIS spread observed during the Great Recession.

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1. Introduction

Treasury bonds issued by the US government are generally viewed as one of the most important liquid and safe assets in the world, and accordingly command a price premium (Longstaff, 2004; Krishnamurthy and Vissing-Jorgensen, 2012). During periods of financial market turmoil when prices of risky and illiquid assets fall dramatically due to a flight-to-safety and flight-to-liquidity, the price premium of Treasuries typically rises (Nagel, 2016; Adrian et al., 2019). More generally, Treasury bonds have had negative beta in recent decades, rising in price when stock prices fall (Baele et al., 2019; Campbell et al., 2019; He et al., 2019b; Cieslak and Vissing-Jorgensen, 2020).

Events in March 2020 during the COVID-19 crisis did not follow this established crisis playbook. As in many previous periods of financial market turmoil, stock prices fell dramatically, the VIX spiked, credit spreads widened, the US dollar appreciated, and prime money market funds experienced outflows. Yet, in sharp contrast to previous crisis episodes, the prices of long-term Treasury securities fell sharply. From March 9 to 23, when the stock market experienced four trading halts, the 10-year Treasury yield increased up to 60 bps, resulting in a striking and unusual positive correlation between stock and bond returns.
Fig. 1. Treasury yields and VIX during the COVID-19 crisis. This figure shows the daily series of the constant maturity Treasury (CMT) yields in percent, of 3-month and 10-year maturities (left panel), and of the VIX (right panel), from January 1, 2020 to June 30, 2020.

(see Fig. 1). Widening bid-ask spreads and collapsing order book depth indicated market illiquidity in the Treasury bond market (Fleming and Ruela, 2020). In direct response, the Federal Reserve (Fed) first offered essentially unconstrained short-term financing to primary dealers and then quickly began to purchase Treasuries directly in amounts even larger than those during the 2007–2009 crisis.

Why was it different this time? Given the Treasury market’s outsized role in the financial system, this stunning deviation from historical correlations in recent decades calls for an explanation. Are the events of March 2020 the canary in the coal mine, indicating a fundamental change in the properties of Treasury bonds away from being a negative-beta flight-to-safety target asset? Or could the surprising price movements of Treasury bonds be attributed to market dysfunctionality induced by frictions? Our goal in this paper is to provide insight on this question, both theoretically and empirically.

We start by characterizing the major features of asset price movements and investor flows during the crucial weeks in March 2020. A simple explanation of the rise in long-term Treasury yields would be that the enormous fiscal burden of the COVID-19 pandemic triggered a shift in inflation expectations and inflation uncertainty. However, market data suggest that this is unlikely. The prices of Treasury inflation-protected bonds (TIPS) fell along with the prices of nominal Treasuries. Inflation-swaps show no increase in risk-neutral inflation expectations. The prices of inflation caps and floors likewise do not point to an increase in inflation uncertainty.

An alternative explanation would be that the cyclical of real interest rates has changed. As Campbell et al. (2017) emphasize, the negative beta of Treasuries in the decades leading up to 2020 partly reflects a positive correlation between stock prices and real interest rates. That March 2020 represents a regime-shift towards a negative correlation (and hence procyclical bond prices) cannot be ruled out completely, but it seems difficult to come up with an economic mechanism for such a shift at this point. In the post-WWII history examined by Campbell et al. (2017), the only major episode with procyclical bond prices (or, countercyclical real interest rates) was the Volcker disinflation of the early 1980s where the rise of real interest rates in a recession was induced by contractionary monetary policy intended to crush inflation. This is clearly not what happened in March 2020.

We therefore turn to an examination of investor flows to understand whether supply and demand balances may have interacted with intermediation frictions to give rise to the unusual price movements in the Treasury market. During the first quarter of 2020, foreign investors (including foreign central banks and investors in tax havens) sold about $270 billion worth of Treasuries; mutual funds (including Treasury mutual funds and others like corporate bonds and equity funds) sold around $240 billion; hedge funds sold more than $30 billion; the U.S. Treasury issued about $240 billion net; and other investors like pension funds, depository institutions, and insurance companies either sold or purchased a small amount (see Section 2.1 for details). Much of this supply was temporarily accommodated by broker-dealers, partly through somewhat higher direct holdings (about $50 billion), but also indirectly through a massive expansion of $400 billion in repo financing that primary dealers provided to levered investors through mid-March. Yet, this increase in repo financing ceased in the height of the March 2020 market stress. The Fed then offered $1.5 trillion of funding to primary dealers on March 12, but the take-up was abysmally low. Eventually, towards the end of March, the Fed came in and purchased $700 billion worth of Treasury notes and bonds, and the expansion in dealer balance sheets reverted back subsequently. Both the selling pressure and its eventual accommodation by the Fed were concentrated in long-term Treasuries (see also He and Krishnamurthy (2020)).

While these are significant shifts in the ownership of Treasuries, it is far from obvious that they could induce substantial increases in Treasury yields in a market usually thought to be extremely liquid and deep. Why did fi-
nancial intermediaries and agile institutional investors financed by intermediaries fail to accommodate this supply more elastically? We argue that the balance sheet constraints of dealers played a key role.

We build on the preferred habitat model of Greenwood and Vayanos (2014) and Vayanos and Vila (2020) to understand how a supply shock for long-term Treasuries can affect the term structure of Treasury yields. By modeling risk-averse dealers who intermediate the exogenous demands of habitat investors, the dynamic preferred habitat model has potential to deliver the market price volatility of Treasuries and their associated risk premia endogenously. To explain the market turmoil in March 2020, we extend this model in two important dimensions. First, we allow levered investors (hedge funds) to take positions in Treasuries financed by borrowing from dealers in the repo market, motivated by the aforementioned massive expansion of repo financing in March 2020. Second, we introduce constraints to dealers’ balance sheets, in the spirit of the supplementary leverage ratio (SLR) as an important regulatory reform following the 2007–2009 financial crisis. Importantly, both the direct holdings of Treasuries and reverse repo positions take up dealers’ balance sheet space and are subject to constraints.

The model therefore implies that dealers demand compensation for the shadow cost of balance sheet expansion, regardless of whether the expansion occurs through direct holdings (with compensation in the form of higher yield) or reverse repos (with compensation in the form of higher reverse repo rates). Consequentially, when a massive supply shock of Treasuries results in a rise in yields, the simultaneous rise in reverse repo rates charged by dealers reduces the attractiveness of a levered investment in Treasuries for (risk-averse) hedge funds. More specifically, repo friction hurts the demand for Treasury bonds from levered investors in two ways. First, investors face more costly current reverse repo funding. Second, investors who are concerned about future liquidity shocks anticipate a reduced collateral value of Treasuries in future financing. In other words, a disrupted repo market limits investors’ ability to use Treasury bonds to raise short-term funding and hence severs the link between long-term Treasury bond yields and short-term interest rates. Hence, dealers’ direct holdings and Treasury yields need to rise even more to clear the market. In equilibrium, the yield curve steepens and reverse repo rates rise above the corresponding frictionless risk-free rates.

Empirically, we find support for these predictions. To measure reverse repo rates at which dealers lend to levered investors, we use the general collateral finance (GCF) repo rates. Much of the activity in the GCF repo market involves large dealers lending to smaller ones. In this sense, these rates reflect those at which large dealers are willing to lend to levered investors against general Treasury collateral (Baklanova et al., 2015). We compare the GCF rate with tri-party repo rates at which large dealers borrow from cash-rich investors like money market funds. Consistent with the balance sheet cost explanation, we find that GCF repo rates substantially exceeded tri-party repo rates when Treasury yields spiked in March 2020.

In the model, the rise in long-term yields has two components. The first component is a heightened risk premium entailed by the expansion of direct Treasury holdings, because risk-averse dealers demand compensation for the resulting endogenous market price volatility of Treasuries bonds (due to their interest rate–risk exposure). The second component reflects the inconvenience yield induced by the balance sheet cost due to the SLR constraint. To isolate this second component, we use the dealers’ pricing kernel to price a derivative asset that offers exactly the same cash flows as physical Treasury bonds, but without the balance sheet cost. We think of this derivative asset as an overnight index swap (OIS).2 Practically, the weight imposed by the SLR constraint on interest rate derivative contracts is about two orders of magnitude smaller than the weight on Treasury securities, so zero balance sheet cost is a good approximation.

In line with the model’s predictions, we find that during the two weeks of the COVID-19 turmoil, Treasury yields rose substantially above maturity-matched OIS rates, pointing to an inconvenience yield. Based on the findings in Krishnamurthy and Vissing-Jorgensen (2012), the rise in the supply of US Treasuries since the Great Recession could also have contributed to a disappearing convenience yield. Viewed from this perspective, the rise of Treasury yields relative to OIS rates in March 2020 is a further extension of this phenomenon (see Klingler and Sundaresan (2020) for evidence prior to the COVID-19 pandemic). Indeed, other measures of the Treasury convenience yield have also been eroding since the Great Recession (Du et al., 2018).

The inconvenience yield of Treasuries during March 2020 is particularly striking in contrast to the financial crisis in 2007–2009. Flight-to-safety and liquidity from the early stages of that financial crisis until mid-2008 pushed Treasury yields significantly below OIS rates, by as much as 50 bps. Dealers came into the financial crisis with a short position in Treasuries. Rather than having to absorb a supply of Treasuries as in March 2020, dealers scrambled to obtain more Treasuries. As a consequence, dealers were willing to lend cash to obtain Treasury collateral, with both tri-party and GCF repo rates reaching low levels. The repo spread is low and not significantly positive, consistent with the absence of the SLR-like balance sheet constraints around 2008. In contrast to purchasing Treasuries in March 2020, the Federal Reserve took actions in 2008 to increase the supply of Treasuries in the market, for example allowing dealers to obtain Treasuries against non-Treasury collateral in the Term Securities Lending Facility (TSLF). This seems to have alleviated the shortage of Treasuries, leading to a closing of the Treasury-OIS spread.

These empirical patterns in the 2007–2009 financial crisis are consistent with our model too, but under an opposite shock: habitat investors demand more direct holdings of Treasuries. Accordingly, the dealer sector should be short in Treasuries, and the Treasury-OIS spread should switch signs compared with March 2020, which are predictions that are consistent with empirical observations.

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2 The OIS rate can be interpreted as the risk-neutral expectation of the expected federal funds target rate.
Hence, our model provides a unified account of the very different Treasury and repo market dislocations in the 2007–2009 financial crisis and the COVID-19 crisis.

In summary, the observed movements in Treasury yields and spreads in March 2020 can be rationalized as a consequence of the interaction between the selling pressure that originated from large holders of Treasuries and intermediate frictions that include regulatory constraints such as the SLR on dealers. Evidently, the current institutional environment in the Treasury market is such that it cannot absorb large selling pressure without substantial price dislocations or interventions by the Fed as the market maker of last resort. Indeed, the Fed announced that it would directly purchase Treasuries “to support the smooth functioning of markets” on March 15 and 23, which alleviated market stress (as seen in Fig. 1). Consistent with our model particularly, the Fed also announced that it would temporarily exempt Treasuries from the SLR on April 1.3

Our theory and evidence explain why selling pressure had such a strong price impact based on dealer constraints, but it does not account for what motivated some large holders of US Treasuries to sell in March 2020. The shock that initially triggered a large selling pressure on Treasury bonds during the COVID-19 crisis was reportedly caused by a scramble for cash. For example, corporate bond mutual funds were actively selling Treasury bonds, especially those that faced secondary redemption risk (Ma et al., 2020). Nor does it account for why other long-term investors stayed away, while dealers, levered investors financed by dealers, and ultimately the Fed had to absorb this additional supply.

The fact that the Fed was able to alleviate the dislocations by substantially tilting the maturity structure of US government liabilities away from the long-term securities it purchased towards very short-term liabilities it created (reserves) invites comparisons with emerging market crises where the shortening of maturities by sovereign issuers is a typical response to investors’ concerns about issuers’ ability to repay (Broner et al., 2013). But since neither inflation, nor default risk concerns are apparent in derivatives prices in March 2020, there is little to suggest that concerns about the US fiscal situation are the underlying cause. But we caution that the “safe” asset status of US Treasuries should be disciplined by fundamental fiscal capacities when coordination incentives of market participants play a central role (He et al., 2019b). Rel-ative to short-term T-bills whose values are largely determined by the near-term promise to repay by the US government, the market prices of long-term Treasuries are endogenous and subject to coordination risk, along the lines of Allen et al. (2006).

Three related works, Duffie (2020), Schrimpf et al. (2020), and Vissing-Jorgensen (2020) also provide some evidence on how the Treasury market has been stressed by COVID-19 shocks; the first two papers focus on raising policy proposals that can potentially make the Treasury market robust to shocks, while the third paper argues that after soaring during the week of March 9–15, 2021, yields were causally driven down by subsequent Fed purchases. He and Krishnamurthy (2020) summarize these works (and our paper) and emphasize that the March 2020 Treasury market disruption only occurred on the long end of the maturity spectrum. Other contemporaneous works on the COVID-19 stress of fixed-income markets include Boyarchenko et al. (2020), D’Amico et al. (2020), Haddad et al. (2020), Kargar et al. (2020), Qiu and Nozawa (2020), and O’Hara and Zhou (2020) on corporate bonds, Chen et al. (2021) on agency mortgage-backed securities, and Falato et al. (2020) and Ma et al. (2020) on bond mutual funds.

An expanding literature examines safe assets’ supply, demand, and convenience premium, including early studies by Bansal and Coleman (1996), Duffee (1996), and Longstaff (2004), and more recent ones by Krishnamurthy and Vissing-Jorgensen (2012), Bansal et al. (2011), Xie (2012), Sunderam (2014), Nagel (2016), Greenwood et al. (2015), Du et al. (2018), Fleckenstein and Longstaff (2018), Kapciszky et al. (2019), Jiang et al. (2019), and He and Song (2021), among others. See Gorton (2017) and Caballero et al. (2017) for broad surveys on safe assets, and Lagos et al. (2017) for the macro/money search-based studies on these issues.

Our model is based on Greenwood and Vayanos (2014) and Vayanos and Vila (2020), who study the equilibrium Treasury pricing where a risk-averse intermediary sector absorbs exogenous demand/supply shocks from the habitat agents. We introduce Poisson events that capture temporary (and potentially large) supply shocks, and more importantly, a levered hedge fund sector that borrows from the dealers via the repo market. The endogenous repo spread (i.e., the difference between the GCF repo lending rates and the tri-party repo borrowing rates) and its connection to the broad demand/supply in the Treasury market are the major contributions of this paper. We also provide strong empirical support for the theoretical predictions.

Our paper is also related to studies of the collateral market of safe assets, including Duffie (1996), Duffie et al. (2002), Gorton and Metrick (2012), Krishnamurthy et al. (2014), Bartolini et al. (2011), Hu et al. (2019), Song and Zhu (2019), and Lenel et al. (2019).
Our paper contributes to the literature on intermediary-based asset pricing à la He and Krishnamurthy (2013), especially studies that highlight constraints on dealers such as He et al. (2017), Klingler and Sundaresan (2019), Jermann (2019), Fleckenstein and Longstaff (2020), Boyarchenko et al. (2018), and He et al. (2019a). See He and Krishnamurthy (2018) for a recent survey (and Gromb and Vayanos (2010) for a survey on limits-of-arbitrage).

2. Motivating evidence and institutional background

To set the stage for our main analysis, we offer further evidence on the Treasury market disruption during the COVID-19 crisis, in addition to Fig. 1. The evidence motivates us to examine the interaction of supply and demand balances with intermediation frictions as the potential economic mechanism. Additional relevant institutional background for the development of our modeling is also provided in Section 3, while details of the data sources and measures are provided in the Appendix.

2.1. Motivating evidence

A simple explanation of the soaring 10-year Treasury yield would be that the expected fiscal burden of the COVID-19 pandemic can trigger a shift in inflation expectations and inflation uncertainty. The left panel of Fig. 2 shows the weekly time series of two market-based measures of inflation expectations: the 10-year breakeven inflation rate (i.e., the difference between the 10-year constant-maturity nominal yields and TIPS yields) and the 10-year inflation swap rate. However, both of them fell throughout the COVID-19 period, especially in March 2020. The right panel shows the weekly series of measures of the inflation uncertainty and the (risk-neutral) probability of a large increase in inflation (of more than 3%), extracted from 5-year inflation caps and floors by the Federal Reserve Bank of Minneapolis. We observe that inflation uncertainty dropped significantly until mid-March, and increased afterwards. The probability of a large increase in inflation dropped and stayed low throughout April, pointing to concerns about deflation rather than inflation. Finally, investors were not flocking to gold, as gold prices went down as well starting March 9 (based on the Gold Fixing Price in the London Bullion Market).

As mentioned in the Introduction, foreign investors, mutual funds, and hedge funds sold (top left panel of Fig. 3), in addition to the net positive issuance by the US Treasury (middle left panel). On the supply side, primary dealers increased their direct holdings somewhat (middle right panel), but most noticeably they greatly expanded their repo financing of levered investors during the first quarter and then quickly unwound after March 23 (bottom left panel).

More specifically, during the week from March 9 to 15 when the 10-year Treasury yield increased most sharply (see Fig. 1), primary dealers’ direct Treasury holdings remained almost flat, while their reverse repo lending increased by about $120 billion. The increase in reverse repo lending ceased thereafter, suggesting a somewhat tight balance sheet constraint faced by dealers; in fact, the Fed offered $1.5 trillion of repo funding to primary dealers on March 12 (i.e., they were able to access the funding at a low repo rate), but the taking was abysmally low. Eventually, starting from March 15, the Fed came in and pur-

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6 As in Fig. 1, we consider three event dates: January 30, 2020 when the World Health Organization declared the outbreak of coronavirus a global health emergency; March 9, 2020 when the S&P 500 index declined by 7%, triggering the first market-wide circuit breaker trading halt; and March 23, 2020 when the Fed announced “its full range of tools” to support the US economy including unlimited purchases of Treasury securities.

7 Note that the amount of domestic investors flows throughout the first quarter may underestimate the selling amount in March because investors may have purchased Treasury securities on balance in January and February like foreign investors. Moreover, the sample of hedge funds only includes domestic ones. According to Barth and Kahn (2020), the selling amount of hedge funds ranges from $90 billion to $120 billion.
Fig. 3. Investor flows and positions of Treasuries during the COVID-19 crisis This figure shows the monthly changes in holdings of long-term Treasury securities of foreign investors (top left panel), quarterly changes in holdings of domestic investors’ Treasury securities (top right panel), monthly net issuance amounts of Treasury notes and bonds (middle left panel), weekly amounts of primary dealers’ net positions in Treasury securities (middle right panel), weekly amounts of primary dealers’ gross reverse repo balance (bottom left panel), and weekly amounts of the Federal Reserve’s Treasury holdings (bottom right panel). The units are all billions of US dollars. Data source: the FR2004 data collected by the Federal Reserve Bank of New York, the SIFMA reports, the Financial Accounts of the US (Z.1) provided by the Federal Reserve, the Treasury International Capital system, and the Federal Reserve’s H.4.1 release.

We also look into the bidding behaviors of auction participants in issuance auctions of long-term Treasury securities that occurred from January to April 2020.\textsuperscript{8} In par-

8 We thank the anonymous referee for the suggestion of exploring the bidding behaviors of primary dealers who are required to participate in Treasury auctions during this crisis episode.

purchased $700 billion of Treasury notes and bonds (bottom right panel), and the expansion in dealer balance sheets reverted back. The selling pressure and its eventual accommodation by the Federal Reserve were concentrated in long-term Treasuries, as there was little net selling by foreigners in T-bills and the Fed did not expand their holdings of T-bills.
ticular, for each auction, we calculate the ratio of the bidding amount to the offering amount, known as the bid-to-cover ratio, for primary dealers and all other auction bidders, respectively. We then take the average of the bid-to-cover ratios across auctions for three subsamples in 2020: January 1–March 8, March 9–23, and March 24–April 28. From Fig. 4, the bid-to-cover ratio of auction bidders other than primary dealers is particularly low from March 9 to 23 relative to both the preceding and subsequent periods. In contrast, for primary dealers who are required to participate in these issuance auctions especially when the demand is low, the bid-to-cover ratio did not drop—if anything, it slightly increased—entering the period of March 9 to 23. This finding confirms that habitat investors have weak demand for long-term Treasury securities and primary dealers stepped in to take them.9

In sum, examinations of asset price movements and investor flows during the crucial weeks in March 2020 suggest that shifts of fundamental risks are unlikely to have caused the unusual price movements in the Treasury market. Instead, the large shifts in the supply of Treasuries, primary dealers’ balance sheet movements, and the nature of the Federal Reserve’s interventions point to an economic interpretation based on financial constraints. To understand the relevant frictions, we briefly introduce some institutional background.

2.2. Institutional background

In this section, we first outline the institutional features of the Treasury and repo markets, especially the important role of dealers as intermediaries in both markets. We then discuss the key regulations that were introduced in the US since the 2007–2009 financial crisis and their potential impacts on the Treasury market and dealers.10

2.2.1. The Treasury and repo markets

The US Treasury market is one of the largest fixed income markets with an outstanding balance held by the public of about $17 trillion as of March 2020. The largest component is coupon-bearing Treasuries, about $12.5 trillion, while T-bills comprise the second largest portion, about $2.6 trillion.11 The secondary cash market of Treasuries maintains an average daily total trading volume of about $575 billion over based on the data from the period of August 2017 to July 2018, of which coupon Treasuries and T-bills account for the largest percentages, about 82% and 15%, respectively. About 70% of the trading volume is concentrated in on-the-run securities, which are the most recently auctioned securities of a given tenor, while the rest are in off-the-run securities, which consist of all the previously issued securities.12 While on-the-run securities are mainly traded on electronic exchange-like platforms, off-the-run Treasuries, which account for 95% of outstanding securities (Clark et al., 2016), mostly trade over-the-counter with broker-dealers as counterparties.

Broker-dealers, especially the primary dealers who are trading counterparties of the Federal Reserve Bank

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9 We also calculate the ratio of the accepted amount to bidding amount, termed as “accept-to-bid” ratio, which can capture the aggressiveness in bidding prices. We find no significant difference regarding their accept-to-bid ratios across primary dealers and other auction participants. Combining this with the information on bid-to-cover ratios, we conclude that primary dealers and other auction participants had similar pricing behaviors, despite the significant difference in their demanded quantities. In addition, we examine the changes in the issuance yields and find that the spread of issuance yield to OIS rate spiked during the COVID-19 crisis, consistent with our analysis in Section 4.2.2 (see Fig. 7).

10 We keep the details to a minimum, and refer to Fleming (1997), Fleming and Garbade (2004), and Baklanova et al. (2015) for additional details on Treasury and repo markets, Fleming and Rosenberg (2007) on dealers, and Duffie (2018) and Boyarchenko et al. (2018) on post-crisis regulations.

11 The rest are Treasury inflation protected securities (TIPS), floating rate notes (FRNs), and separate trading of registered interest and principal securities (STRIPS). The outstanding balance is obtained from the SIFMA (https://www.sifma.org/resources/research/us-fixed-income-issuance-and-outstanding/).

12 A summary of the trading volume can be found at https://libertystreeteconomics.newyorkfed.org/2018/09/unlocking-the-treasury-market-through-trace.html. Off-the-run securities were hit hardest by the COVID-19 market disruption, though on-the-run securities were also notably affected (Fleming and Ruela, 2020).
of New York in its implementation of monetary policy (Song and Zhu, 2018), are important participants in both the primary and secondary markets of Treasuries. In particular, primary dealers are expected to participate in all issuance auctions of Treasuries, and have traditionally been the predominant purchasers at these auctions. Dealers are also key intermediaries in the Treasury cash market, accounting for about 75% of all transactions. In fact, all client transactions, which account for half of the total $575 billion daily trading volume, go through dealers. When intermediating the Treasury trades of clients, dealers need to use their balance sheet to hold inventories, like in classical market-making models with non-Walrasian dealers (e.g., Amihud and Mendelson (1980), Ho and Stoll (1981)). In our model, carrying inventory entails risk for which competitive risk-averse dealers simply demand a risk premium.

In addition to selling them outright in the Treasury cash market, investors often post Treasuries as collateral to borrow cash on a short-term basis, particularly in the repo market. The US repo market is comprised of two segments: tri-party repo and bilateral repo. A tri-party repo involves a third party known as a clearing bank that provides clearing and settlement services, such as keeping the repo on its books and ensuring the execution according to repo terms. Conversely, in a bilateral repo, the clearing and settlement are managed by each counterparty’s custodian bank. Furthermore, within the tri-party repo market, a special general collateral financing repo service (GCF Repo) allows securities dealers registered with the Fixed Income Clearing Corporation (FICC) as netting members to trade repos among themselves. That is, the GCF repo is mainly an interdealer market, while in the non-GCF tri-party repo market (referred to as tri-party repo hereafter), broad cash lenders include money market mutual funds (MMFs), banks, and securities lenders that lend cash to dealers.

As in the cash Treasury market, broker-dealers are also key intermediaries in the repo market, transmitting funds from lenders to borrowers who cannot directly deal with each other for certain reasons. In particular, large dealers borrow cash in the tri-party market and lend to small dealers in the GCF market. Large dealers also borrow cash in the tri-party market and lend to levered investors especially hedge funds in bilateral repo markets. Hence, both the GCF and bilateral repo markets are used by levered investors to finance their cash Treasury positions, where the funds are transmitted by large dealers from the tri-party market.

Dealers also often use (mainly bilateral) repo market to intermediate security sourcing and facilitating short-selling. Although a securities-driven repo often targets specific Treasuries, termed as “special repo” by Duffie (1996), it is also used widely to source general securities when there are a shortage of Treasuries overall. In either case, short-selling incurs costs of searching for securities and failing to deliver, which can lead to large market disruptions (see Duffie et al. (2002), Fleming and Garbade (2004) and Garbade et al. (2010) for more discussions). The implementation of the Fed’s TSLF program from March 2008 to June 2009 in which Treasuries were lent to dealers against non-Treasury collateral was designed to alleviate such costs (Fleming et al., 2009).

2.2.2. Post-crisis regulations

Despite the widely accepted safe status of Treasury securities, the high volatility of Treasuries during March 2020 may have posed a large price risk and affected the willingness of risk-averse dealers to take them into inventory. However, such risk is negligible for dealers’ repo intermediation activities where Treasuries serve as collateral, so the apparent limits to a further expansion of primary dealers’ repo financing provision discussed above point to balance sheet constraints. The balance sheet constraints that particularly interest us are those associated with regulatory reforms imposed on financial institutions as a response to the 2007–2009 financial crisis. Among them, the most relevant for the Treasury market is the SLR (Duffie, 2018).

To strengthen the resilience of the global banking system in the wake of 2007–2009 financial crisis, the Basel III regulatory framework proposed a new leverage ratio rule as a backstop to risk-based capital regulation, while US regulators proposed the SLR in 2012 and finalized the rule of the “enhanced” SLR in April 2014. In general, the leverage ratio is computed as the Tier 1 capital divided by total leverage exposure irrespective of its riskiness, which is distinct from the conventional risk-weighted-asset (RWA) capital requirement. The total leverage exposure includes both on-balance-sheet assets and off-balance-sheet exposures to derivatives. The Basel Committee proposed a 3% minimum leverage ratio, while US regulators require global systemically important institutions (G-SIBs) to maintain an SLR of at least 5% on a consolidated basis and at least 6% for their depository subsidiaries. The denominator of the SLR was finalized in September 2014, and from 2015, G-SIBs and other large banking institutions have been required to make public disclosures related to the SLR. The final implementations were mostly finished in January 2018.

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13 The so-called principal trading firms that specialize in electronic and automated intermediation only participate in the interdealer segment.

14 In the US, tri-party repo services have been offered by the Bank of New York Mellon and JP Morgan Chase. The latter announced a plan to exit this business in 2019 (https://www.wsj.com/articles/j-p-morgan-to-exit-part-of-its-government-secuirties-business-1469135462).

15 In addition, investors mostly use Treasury notes and bonds as collateral for repo financing, at least in tri-party repo markets. For example, Hu et al. (2019) show that only about 6% of the repo collateral between dealers and MMFs consist of T-bills.

16 Dealers also obtain funding for their market-making inventories in the triparty repo market by posting Treasuries as collateral assets. In practice, this is similar to the strategy of levered investors like hedge funds that finance their proprietary portfolios through repo funding. The hedge fund sector in our model includes this strategy of dealers (see Section 3 for details).

17 In recent years, repo transactions cleared through the FICC delivery-versus-payment (DVP) repo service have increased; this service facilitates cash flow from the triparty repo market to hedge funds. See https://www.jpmorgan.com/global/research/sponsored-repo for details.

18 For example, dealers can lend cash to providers of Treasury securities and use the received collateral assets to establish short positions in the cash Treasury market or cover short sales.
The leverage exposure in the SLR includes the total notional of all cash and repo transactions, regardless of which securities are used as collateral, so it affects bank dealers’ intermediation activities in both the cash and repo markets of Treasury securities. As argued by Duffie (2018), “the SLR increases ‘rental cost’ for the space on a bank’s balance sheet.” Our model incorporates this balance sheet cost. That the Fed responded to the Treasury market disruption with a temporary exemption of Treasuries from the SLR on April 1, 2020, is consistent with this balance sheet cost being a relevant friction.

Compared with the constraint on Treasury cash and repo positions, the constraint imposed by the SLR on standard interest rate derivatives is minor. Specifically, the exposures to derivatives are calculated based on the current exposure method, consisting of the current exposure (CE) and potential future exposure (PFE); for standard interest rate derivatives like vanilla Libor swaps and OIS, the constraint imposed by SLR on interest rate derivatives is about two orders of magnitude smaller than that on the cash Treasury positions.19

In addition, we briefly discuss two other regulations—the liquidity coverage ratio (LCR) and Volcker rule—that have been progressively put into effect since 2014. The objective of the LCR is to ensure that potential outflows over a 30-day period are sufficiently covered by cash and high-quality liquid assets (HQLAs);20 but this constraint is almost irrelevant in our analysis because cash and Treasuries are treated equivalently as HQLAs.21 The Volcker rule prohibits proprietary trading by banks (or financial institutions with access to FDIC insurance or the Federal Reserve’s discount window) that are financed by the low-cost deposits of the affiliated bank branch. However, the Volcker rule exempts government securities and so it is unlikely to have constrained dealers in the Treasury market turmoil.

3. The model

We now show within a model how supply shocks can interact with intermediation frictions to give rise to the observed Treasury market disruptions. Moreover, the model provides additional empirical predictions about spreads between different repo rates, swap rates, and Treasury yields that we subsequently examine.

The model in this section is an extension of Greenwood and Vayanos (2014); in their setting, preferred habitat agents trade with arbitrageurs. The novel elements we add include separating the arbitrageurs into hedge funds and dealers and introducing a repo market in which the former group borrows from the latter. Throughout, we use lowercase letters to denote an individual agent’s choices and uppercase letters to denote aggregate quantities.

3.1. Aggregate shocks and assets

There are two sources of aggregate risk in this dynamic model, following Greenwood and Vayanos (2014). The first is the stochastic evolution of the short-term interest rate \( r_t \), as an Ornstein-Uhlenbeck process:

\[
dr_t = \kappa (r - r_t) dt + \sigma dZ_t,
\]

where \( \{Z_t : 0 \leq t < \infty \} \) is a standard Brownian motion, \( \kappa \) is the mean-reversion parameter, and \( \sigma \) is the volatility of the short rate.

The second aggregate shock is a Treasury demand/supply shock \( \beta_t \), which follows a Markov chain \( \beta_t \in \{0, \beta\} \). The jump intensity from \( \beta_t = 0 \) (\( \beta_t = \beta \)) to \( \beta_t = \beta \) (\( \beta_t = 0 \)) is denoted by \( \xi_0 (\xi_\beta) \). We interpret \( \beta_t = 0 \) (\( \beta_t = \beta \)) as the normal (stress) state. Our model can capture both demand and supply shocks depending on the sign of \( \beta \).

We consider a continuum of zero-coupon Treasury bonds that mature at the tenor \( \tau \in [0, T] \). Denote by \( P_t(\tau) \) their endogenous price to be solved for in equilibrium.

3.2. Habitat agents

There is an exogenous demand/supply shock from habitat agents, so that their holding of bonds with tenor \( \tau \) is:

\[
H_t(\tau) = -\theta(\tau)\beta_t,
\]

where \( \theta(\tau) \geq 0 \) captures the exposure to the shock. The case of \( \beta > 0 \) corresponds to an exogenous supply shock to the economy, while the case of \( \beta < 0 \) represents a demand shock. We think of these habitat agents as representing insurance companies, pension funds, and/or foreign central banks.22

Let \( \theta = \int_0^\tau \theta(\tau)d\tau \). Depending on applications, we later specialize the function \( \theta(\tau) \) either to the piecewise function:

\[
\theta(\tau) = \begin{cases} 1, & \text{for } \tau > \hat{\tau}, \\ 0, & \text{otherwise,} \end{cases}
\]

so that the (negative) demand shock hits the long-end of the curve, or to \( \theta(\tau) = 1 \) for all \( \tau \in [0, T] \) so that the demand shock applies to the entire curve.

---

19 Since Libor swaps and OIS are centrally cleared, the CE is effectively zero because the variation margin is posted on a daily basis. The PFE is defined using a combination of the net and gross risk exposures, equal to \( PFE = 0.4 \times A_{\text{notional}} + 0.6 \times NGR \times A_{\text{notional}} \), where \( A_{\text{notional}} \) is the adjusted gross notional equal to the gross notional multiplied by a maximum of 1.5%, and \( NGR \) is the net-to-gross ratio equal to the net current mark-to-market value and gross current mark-to-market value. For details, see Polk (2014).

20 Specifically, it assumes that dealers may lose all of their collateralized funding with terms of less than 30 days and hence stipulates that they need to hold sufficient cash and HQLAs to cover this loss of funding. A companion liquidity regulation is the net stable funding ratio (NSFR) designed to limit maturity transformation. It requires sufficient stable funding, equity or long-term debt, to cover assets over a one-year horizon.

21 Similarly, the LCR is unlikely to have constrained banks from purchasing Treasuries using cash. Further, a large amount of cash flows into banks during the market stress, which actually alleviates the LCR constraints.

22 In the context of the COVID-19 crisis, these habitat agents also include a subset of hedge funds that were heavily engaged in cash-futures basis trades and hence were forced to delever after significant losses following the Treasury market turmoil in March 2020.
3.3. Hedge funds and repo

A unit measure of hedge funds in this economy can borrow from dealers in the repo market to exploit the investment opportunity created by aggregate demand/supply shocks $\beta$. When a hedge fund borrows from a dealer by pledging Treasury bonds as collateral, the fund needs to pay an endogenous repo financing rate of $R_t$. We will use “he” to refer to the dealer while “she” refers to the hedge fund.

Following Greenwood and Vayanos (2014), we assume that each hedge fund solves the following instantaneous mean-variance objective at time $t$:

$$\max_{q^f(t) \geq 0} E_t \left[ dw^f_t \right] - \frac{1}{2 \rho_h} \text{Var}_t \left[ dw^f_t \right],$$

where $dw^f_t$ is the hedge fund’s change in wealth, and $\rho_h > 0$ is the hedge fund’s risk-bearing capacity (or risk tolerance, the inverse of their absolute risk-aversion). Given the repo financing cost $R_t$, which will be determined in equilibrium, the dynamics of her wealth $dw^f_t - r_t w^f_t \, dt$ are given by:

$$\int_0^T q^f(t) \, \text{hedging holding, repo demand} \left( \frac{dP(t)}{R(t)} - R_t \, dt \right) \, d\tau.$$  \hspace{1cm} (4)

For each $\tau \in [0, T]$, the hedge fund’s repo demand $q^f(t)$ depends on the repo financing cost $R_t$. The higher the $R_t$, the lower the demand; and this price-dependent repo demand is one of our key contributions relative to Greenwood and Vayanos (2014).

3.4. The dealer sector

A unit measure of risk-averse dealers absorbs the residual Treasury supply/demand shocks and provides overnight repo funding to the hedge fund sector.

3.4.1. Dealer’s problem

Following Greenwood and Vayanos (2014), we assume that each dealer solves the following instantaneous mean-variance objective at time $t$:

$$\max_{x_t(\tau), q^d(\tau) \geq 0} E_t \left[ dw^d_t \right] - \frac{1}{2 \rho_d} \text{Var}_t \left[ dw^d_t \right],$$

where $\rho_d > 0$ is the dealer’s risk-bearing capacity, and the dynamics of the dealer’s wealth, $dw^d_t - w^d_t r_t \, dt$, is given by:

$$\int_0^T x_t(\tau) \, \text{direct holdings} \left( \frac{dP(t)}{R(t)} - r_t \, dt - \frac{\Delta_t}{1+B/S} \right) \, d\tau,$$

$$\int_0^T q^d(\tau) \, \text{repo} \left( \frac{\Delta_t}{1+B/S} - \frac{\Delta_t}{1+B/S} \right) \, d\tau. \hspace{1cm} (6)$$

Here, $x_t(\tau)$ is the direct holding of bond $\tau$ (in terms of dollars, which could be negative) and $q^d_\tau \geq 0$ is the reverse repo position, which corresponds to the dealer’s inventory in the Treasury cash market and intermediation amount in the repo market, respectively, that we discussed in Section 2. For clarity of exposition, when we discuss the implications of repo transactions on the dealer’s balance sheet cost, we impose $q^d_\tau \geq 0$, i.e., dealers are engaging in reverse repo to provide financing to hedge funds.

In Eq. (6), we have computed the repo wedge as:

$$\Delta_t \equiv R_t - r_t,$$

which captures the spread between the collateralized lending rate $R_t$ in the repo market, and the risk-free borrowing short rate $r_t$. Here, lending/borrowing is from the perspective of dealers, and we do not specify in more detail the funding markets in which dealers borrow at rate $r_t$. Empirically, in the context of the COVID-19 crisis in March 2020, we proxy for $r_t$ with the repo rate from the tri-party market whose funding flows are mainly from cash lenders like MMFs to large dealers. We proxy $R_t$ with the repo rate from the GCF market whose funding flows are mainly from large dealers to smaller dealers. The wedge between GCF and tri-party repo rates therefore captures the spread between the rates at which large dealers lend and borrow in collateralized funding markets. This GCF-tri-party repo wedge is a riskless profit earned by the dealer sector, corresponding to $\Delta_t$ in our model.

3.4.2. Balance sheet cost

Compared with Greenwood and Vayanos (2014), we study the repo market in which dealers provide repo services $q^d_\tau(\tau)$ in (6), in addition to their portfolio choices $x_t(\tau)$. Furthermore, dealers face an additional balance sheet cost—denoted by $\Lambda_t$—in their portfolio choices, as shown in (6). The marginal cost $\Lambda_t$ hits the direct holdings $x_t(\tau)$ and repo services $q^d_\tau(\tau)$ linearly and each dealer takes it as given.

The balance sheet cost $\Lambda_t$ depends on aggregate holdings only: one can think of a frictionless interdealer market, which equalizes these costs across dealers. For each tenor $\tau$, the aggregate bond holding in the dealer sector is $X_t(\tau)$, and the aggregate reverse repo is $Q^d_\tau(\tau) \geq 0$. When $X_t(\tau) \geq 0$, then the accounting is straightforward. The balance sheet occupied by tenor-$\tau$ bonds, denoted by $B_t(\tau)$, is simply:

$$B_t(\tau) = X_t(\tau) + Q^d_\tau(\tau).$$

Integrating over $\tau \in [0, T]$, the balance sheet size of the entire dealer sector is:

$$B_t = X_t + Q^d_t,$$

with $Q^d_t = \int_0^T Q^d_\tau(\tau) \, d\tau$ and $X_t = \int_0^T X_t(\tau) \, d\tau$. Of course, the equilibrium direct dealer holdings $X_t(\tau)$ and its sign will depend on the demand shock $\beta_t$. In Section 4, where we tailor our model to the COVID-19 crisis with $\beta_t > 0$, we show that $X_t(\tau) > 0$ and hence (8) gives the dealer balance sheet size.

We assume that the marginal balance sheet cost $\Lambda_t = \Lambda(B_t)$ is linear in the balance sheet size $B_t$:

$$\Lambda_t = \lambda B_t, \text{ with } \lambda > 0.$$
To put this into words, the dealer bears a marginal cost from taking an extra dollar of Treasuries (whether the dealer directly holds it or finances the holding by hedge funds) onto the balance sheet, and this cost is increasing in the aggregate balance sheet size \( B \). This balance sheet cost captures the SLR constraint, in addition to costly external equity financing with an upward sloping equity supply curve, say, as in He and Krishnamurthy (2012, 2013).

For completeness, we briefly discuss the case of \( X_t(\tau) < 0 \), i.e., when the dealer is short selling some tenor-\( \tau \) bonds. The exact balance sheet size then depends on how the dealer performs the short-selling. One such way of short selling, which is linked to repo transactions, has the dealer first borrowing bonds \( Q^d_t(\tau) \) via repo, and then selling some of them (i.e., \( [X_t(\tau)] < Q^d_t(\tau) \)). In this hypothetical case, the dealer’s liability side books both the short sale and his tri-party repo market borrowing, and \( B_t(\tau) = Q^d_t(\tau) \).

3.5. Equilibrium

We focus on the symmetric equilibrium in which individual agents (hedge funds and dealers) employ the same strategy as their own groups. In aggregate, for tenor \( \tau \), the dealer sector has \( X_t(\tau) = X_t(\tau) \) amount of direct Treasury holdings and provides an amount \( Q^d_t(\tau) = Q^d_t(\tau) \geq 0 \) of reverse repo.

We follow Greenwood and Vayanos (2014) by normalizing the aggregate bond supply for each tenor \( \tau \) to be zero. The equilibrium is defined in the standard way:

**Definition 1.** A (symmetric) equilibrium is a collection of quantities \( \{q^h_t(\tau), Q_t^d(\tau)\} \) by hedge funds, \( \{X_t(\tau), Q_t^d(\tau), Q_t^d(\tau)\} \) by dealers, and prices \( \{\lambda_t(\tau), \Lambda_t(\tau), \Phi_t(\tau)\} \), such that:

1. each hedge fund solves the problem in (3),
2. each dealer solves the problem in (5),
3. allocations are symmetric and consistent: \( q^h_t(\tau) = Q_t^d(\tau), X_t(\tau) = X_t(\tau), \) and \( Q_t^d(\tau) = Q_t^d(\tau) \),
4. and both Treasury and repo markets clear for \( \tau \in [0, T] \),

\[
0 = H_t(\tau) + Q_t^d(\tau) + X_t(\tau),
0 = Q_t^h(\tau) - Q_t^d(\tau).
\]

Since \( Q_t^h(\tau) = Q_t^d(\tau) \) in equilibrium, we will use \( Q \) to denote them whenever there is no risk of confusion. Fig. 5 illustrates the model setting with \( X > 0 \). There is an aggregate bond (risk) supply to be borne by dealers via

\[q^d(t) = X^t_t + \frac{\delta X^t_t}{\delta H^t_t} H^t_t = X^t_t - H^t_t \]

As an example, suppose that a dealer engages in \( q^d = 3 \) repo lending, and at the same time holds a short position \( x = -1 \) by having (short) sold in the Treasury market. This implies that the dealer first takes \( 3 \) worth of bonds that he receives through reverse repo; he then pays \( 2 \) worth of bonds as collateral in the Triparty repo market (where the collateral would have rested without being available to anyone else for purchase), and sells the remaining \( 1 \) of bonds to the Treasury market. On the dealer’s balance sheet, the dealer’s obligation to return the borrowed and short-sold Treasuries to the habitat agents is recorded as a liability of \( 1 \) as “financial instruments sold but not yet purchased.” Adding the \( 2 \) of liability in the form of tri-party repo, the balance sheet size of this dealer is \( 3 \), which is his total repo position. If the dealer has \( x = -4 \), then he needs to engage in an extra naked short selling of \( 1 \), and the total dealer balance sheet would be \( 4 = q^d + \max(0, x + q^d) \).
We now use Eq. (1) to calculate the equilibrium aggregate balance sheet:

\[ B_t = \int_0^T B_t(\tau) \, d\tau = \Theta \tilde{B}_t, \quad (11) \]

where we recall that \( \Theta = \int_0^\tau \theta(\tau) \, d\tau \).

Because the dealer’s objective is linear with a marginal benefit of \( \Delta_t - \lambda_t \), in equilibrium this marginal benefit must be zero and hence,\(^24\)

\[ \Delta_t = \lambda_t = \lambda_B t = \lambda \Theta \tilde{B}_t \geq 0. \quad (12) \]

Per unit of Treasury bond, the dealer’s holding cost is the balance sheet cost \( \lambda_t \) while the hedge fund’s holding cost is the repo financing wedge \( \Delta_t \). In equilibrium, they must be the same. This has important implications for our model, as we show next.

4.1.2. Optimal risk sharing within the intermediary sector

Because both dealers and hedge funds have the same cost in holding the bonds, they face exactly the same problem. More specifically, plugging in \( \Delta_t = \lambda_t \), one can show that the wealth dynamics of a dealer \((6)\) and a hedge fund \((4)\) are identical. They differ in their risk-bearing capacity \( \rho_d \) and \( \rho_h \), and the standard asset pricing insight implies optimal risk sharing in equilibrium:\(^25\)

\[ X_t(\tau) = -\frac{\rho_d}{\rho_d + \rho_h} H_t(\tau), \quad (13) \]

\[ Q_t(\tau) = -\frac{\rho_h}{\rho_d + \rho_h} H_t(\tau). \quad (14) \]

4.1.3. Euler equation and equilibrium pricing

The optimal risk sharing given in Eq. (13) pins down the endogenous equilibrium pricing kernel. Equilibrium bond prices can be obtained by extending the method in Greenwood and Vayanos (2014) with Poisson jumps.

Denote \( P_{\tilde{B}_t}^t \equiv P_{\tilde{B}_t}^t(\tau) \) and guess that the equilibrium prices take the form:

\[ P_{\tilde{B}_t}^t = \exp[-(A(\tau) \tau + C_\beta(\tau))], \quad \tilde{B}_t \in (0, \beta), \quad (15) \]

where \( A(\tau), C_0(\tau) \), and \( C_\beta(\tau) \) are endogenous functions of \( \tau \). For illustration, consider the stress state \( \tilde{B}_t = \beta \) so that the economy has been hit by a supply shock. Denote by \( \mu_{\tilde{B}_t}^\tau \), the drift of the bond-\( \tau \) return and \( \Lambda_{\tilde{B}t} \) the balance sheet cost in this state. One can derive the dealer’s Euler equation as his first-order condition:

\[ \mu_{\tilde{B}_t}^\tau - \tau_i - \Lambda_{\tilde{B}_t} \]

\[ = A(\tau) \sigma^2 \frac{\rho_h}{\rho_d + \rho_h} \int_0^T X_{\tilde{B}_t}(u) A(u) du \]

\[ + \left( e^{\epsilon(\tau)} - C_\beta(\tau) - 1 \right) \frac{\rho_h}{\rho_d + \rho_h} \int_0^T X_{\tilde{B}_t}(u) \left( e^{\epsilon(\tau)} - C_\beta(\tau) - 1 \right) du, \]

\[ \text{risk premium for Brownian interest rate risk } \sigma d\zeta \]

\[ + \left( e^{\epsilon(\tau)} - C_\beta(\tau) - 1 \right) \frac{\rho_h}{\rho_d + \rho_h} \int_0^T X_{\tilde{B}_t}(u) \left( e^{\epsilon(\tau)} - C_\beta(\tau) - 1 \right) du, \]

\[ \text{risk premium for Poisson demand risk } \beta \]

\[ \begin{align*}
0 + \int_0^T Q_t(\tau) \Delta_t \cdot d\tau & - \frac{\lambda}{2} \left( \int_0^T Q_t(\tau) \Delta_t \cdot d\tau \right)^2 \\
& \quad \text{quadratic risk cost } \frac{\lambda}{2} (\beta)^2.
\end{align*} \]

where we have invoked the dealer’s equilibrium holdings \( X^\beta_t(\tau) = X^\beta_t(\tau) \).\(^26\)

As the standard Euler equation for a risk-averse dealer, the left-hand side of Eq. (16) gives the bond \( \tau \)’s expected effective excess return net of the balance sheet cost. In equilibrium, it equals the risk premium that compensates the dealer for bearing the Brownian interest rate risk \( \sigma d\zeta \) and Poisson demand risk \( \beta \), as shown on the right-hand side of Eq. 16. The Internet Appendix gives detailed derivations and numerical methods to solve for the Treasury prices \( A(\tau), C_0(\tau), \) and \( C_\beta(\tau) \), endogenous price volatility, and risk premia in our model.

Eq. (16) also makes it clear that the dealer’s equilibrium portfolio \( X^\beta_t(\tau), \tau \in [0, T] \) given by Eq. (13) prices each bond with tenor \( \tau \), as demand shocks affect the dealer’s equilibrium holdings and hence his pricing kernel, just like in Greenwood and Vayanos (2014). Dealers who absorb an increase in the supply of long-term bonds bear more interest rate risk in their portfolio, and hence in equilibrium require all bonds—both long-term and short-term—to offer higher expected returns in excess of the short rate. Unlike Greenwood and Vayanos (2014), in our model the dealer also demands an extra premium to compensate for the additional balance sheet cost \( \Lambda_{\tilde{B}t} \); we show below that this term drives the Treasury-OIS spread.

Remark. Our model builds on a representative dealer sector where all dealers are homogeneous. There are several straightforward extensions that accommodate heterogeneous dealers, as long as we maintain an instantaneous mean-variance objective in the spirit of Eq. (5).

1. Suppose that dealers differ in their risk aversion, so that each dealer’s risk-bearing capacity is \( \rho_d^d \). It is straightforward to show that the model is isomorphic to our homogeneous dealer sector model with an aggregate risk bearing capacity \( \rho^d = \int \rho^d d\tau > 0 \). (The same result applies to hedge funds.)

2. We have so far assumed that each dealer faces the same balance sheet cost \( \Lambda_t \), that depends on the aggregate balance sheet \( B_t \); implicitly, this assumption is justified by a frictionless interdealer market that equalizes these costs across dealers. With frictions, each dealer’s balance sheet cost could be heterogeneous and typically depends on his individual holdings, denoted by \( b(\tau) = \int (x^d_t(\tau) + q^d_t(\tau)) \, d\tau \). One tractable way to introduce heterogeneity is to assume a balance sheet cost that is quadratic in the dealer holdings \( b(\tau) \), with a dealer-specific cost parameter \( \lambda_i > 0 \). The dealer’s wealth dynamics \( dw_t = \bar{w}_t \, d\tau \) then are:

\[ \int_0^T x_t^d(\tau) \left( \frac{dp^d}{\rho^d} - r^p \right) \, d\tau + \int_0^T q_t^d(\tau) \Delta_t \cdot d\tau - \frac{\lambda_i}{2} \left( \int_0^T x_t^d(\tau) + q_t^d(\tau) \, d\tau \right)^2 \]

\[ \text{quadratic B/S cost } \frac{\lambda_i}{2} (\tau)^2. \]
We further assume that each dealer’s balance sheet cost $\lambda^i$ is inversely related to his risk bearing capacity $\rho^i_d$, so that:

$$\lambda^i = \frac{M}{\rho^i_d}, \quad \text{for some constant } M > 0.$$  

In this special case, the model is again isomorphic to our homogeneous dealer sector model with $\rho^d = \int f^i \rho^i_d \, \text{d}i$ and $\lambda = M / \rho^d$. Note that Assumption (18) is reasonable, as the balance sheet cost eventually ties to binding leverage constraints. In a richer model, a more risk-averse dealer (so with a smaller $\rho^i_d$) tends to have a lower leverage and hence is less likely to hit the constraint; this gives rise to a lower effective balance sheet cost.

We provide proofs in the Internet Appendix; in both cases, the equilibrium holdings for each individual dealer $i$ are given by:

$$x_i^d (\tau) = X_i (\tau) \frac{\rho^i_d}{\rho_d} \quad \text{and} \quad q_i^d (\tau) = Q_i (\tau) \frac{\rho^i_d}{\rho_d}.$$  

The greater the risk-bearing capacity $\rho^i_d$ (or, the lower the holding cost $\lambda = M / \rho^d$), the larger the equilibrium balance sheet expansion (both direct holdings and repo positions). We leave empirical tests of these predictions to future research.

4.1.4. Shadow price for OIS curves

One of our key empirical objectives is to track the changes in the (in)convenience yield of Treasuries during the COVID-19 crisis episode. To this end, we seek a maturity-matched benchmark for comparison that isolates the (in)convenience yield. In our model, the benchmark asset can be defined as a derivative asset that has the same cash flows as physical Treasury bonds but is free from balance sheet costs. Empirically, we proxy for the yield of this derivative asset with OIS rates. In practice, the weight imposed by the SLR constraint on interest rate derivative contracts is not exactly zero, but it is about two orders of magnitude smaller than the weight on Treasury securities.

Suppose that dealers in our model are quoting prices for OIS contracts, which are zero-net supply in equilibrium. Denote by $\rho_{OIS, \tau}$ the price of an OIS contract with tenor $\tau$, which takes the following functional form (one can show that $A(\tau) = \frac{1-e^{-\tau^r}}{\kappa}$ as in Section 4.1.3):

$$\rho_{OIS, \tau} = \frac{\rho_{OIS, \tau}}{\rho_t} \cdot \exp \left[ - \left( A(\tau) r_t + C_{\rho_t}^{\tau} (\tau) \right) \right], \quad \text{with } \Delta T \in [0, \beta].$$  

In contrast to Eq. (16), the drift of $dP_{\rho_t}^{OIS, \tau} / dt$ must satisfy the following standard Euler equation but without the balance sheet cost $A_{\rho_t}$:

$$\mu_{\rho_t}^{OIS, \tau} - r_t = A(\tau) \frac{\alpha^2}{\rho_d} \int_0^\tau x_{\rho_t} (u) A(u) \, \text{d}u + \left( e^{d(\tau - C_{\rho} (\tau))} - 1 \right) \frac{\beta^2}{\rho_d} \int_0^\tau x_{\rho_t} (u) \left( e^{d(\tau - C_{\rho} (\tau))} - 1 \right) \, \text{d}u.$$  

One can solve for $\left\{ C_{0, r}^{\rho} (\cdot), C_{\rho}^{\tau} (\cdot) \right\}$ following the same technique as in Section 4.1.3 (for details, see the Internet Appendix).

The Treasury-OIS spread at tenor-$\tau$ roughly captures the Treasury’s extra holding cost $\Lambda_t$ during the remaining time-to-maturity $\tau$, discounted by the equilibrium pricing kernel. Hence through the lens of our model, Treasury-OIS spreads observed in the empirical data capture the Treasury (in)convenience yields.

4.1.5. Summary and model mechanism

The following proposition summarizes what we have shown.

**Proposition 1.** Consider the scenario of a potential supply shock, i.e., $\beta > 0$. Given the habitat agents’ demand $H_t (\tau) = -\theta (\tau) \tilde{\rho}_t$, the equilibrium is characterized by:

1. The dealer’s balance sheet size is $B_t = \tilde{\rho}_t \Theta$, and the repo spread equals to balance sheet cost $\Delta_t = \Lambda_t = \lambda B_t = \lambda \Theta \tilde{\rho}_t$.
2. The dealer sector holds $x_i^d (\tau) = \frac{\rho^i_d}{\rho_d} \theta (\tau) \tilde{\rho}_t$ directly and hedge funds hold $Q_i (\tau) = \frac{\rho^i_d}{\rho_d} \theta (\tau) \tilde{\rho}_t$ via repo financing.
3. The Treasury (OIS) prices $P_t^{\tau} (\rho_{OIS, \tau})$ in Eq. (15) (Eq. (19)) solve the ODE system of $\left\{ C_{0, r}^{\rho} (\cdot), C_{\rho}^{\tau} (\cdot) \right\}$ (or $\left\{ C_{0, r}^{\rho} (\cdot), C_{\rho}^{\tau} (\cdot) \right\}$) in Eq. (IA.10) (Eq. (IA.11)) given in the Internet Appendix.

Fig. 5 illustrates the workings of our model; the mechanism applies to tenor-$\tau$ bonds, as well as the entire maturity spectrum. After a supply shock hits habitat agents ($\tilde{\rho}_t$ jumps from 0 to $\beta > 0$), the dealers’ balance sheet $B = -H$ expands due to market clearing. The greater the (negative) demand shock size $\beta$, the larger the dealers’ balance sheet size, and hence the higher the balance sheet cost $\Lambda = \lambda B$. Fundamentally, this balance sheet cost is tied to the scarcity of intermediary capital (He and Krishnamurthy, 2012; 2013) and/or debt overhang (Andersen et al., 2019).

Hedge funds step in to absorb the supply shock from habitat agents via collateralized repo financing from dealers. Since a reverse repo takes up space on the dealers’ balance sheet, in equilibrium, dealers pass the balance sheet cost $\Lambda$ through to hedge funds via the repo spread $\Delta$ on a one-to-one basis, adversely affecting the repo demand $Q$ from the hedge fund sector. In equilibrium, both dealers and hedge funds achieve optimal risk sharing.

The dealers’ direct holding $X$ pins down their pricing kernel and hence equilibrium bond prices via their Euler equation (16). As noted in Fig. 5, the balance sheet cost $\Lambda$, as the holding cost or inconvenience yield per unit of Treasury bonds, also enters in the Euler equation (16). In contrast, the OIS curve, which does not entail the balance sheet cost, is driven by the dealers’ pricing kernel only.
4.2. Treasury market breakdown in the COVID-19 crisis in 2020

We now consider a supply shock to long-term Treasuries, i.e., \( \theta(t) = 1_{[T-t_f]} \) and \( \beta > 0 \), as motivated by the quantity evidence in Section 2.1. We then present further supporting evidence on asset pricing during the COVID-19 crisis.

4.2.1. Model implications: Treasury inconvenience yield Parameterization

We set \( \bar{t} = 1 \) as the supply of Treasury bills were mostly unchanged through 2020-Q1 (He and Krishnamurthy, 2020); \( T \) is set at 30 years as the longest bond issued by the US Treasury Department. We set \( r_f = 0 \) as the Fed cut the federal funds rate during the COVID-19 crisis to 0 to 25 bps on March 18. We explain the key model parameters, i.e., the size of supply shock \( \beta \) and the balance sheet cost parameter \( \lambda \), shortly; the remaining parameters (e.g., \( \rho, \sigma \)) follow the parameterization in Greenwood and Vayanos (2014) whenever possible.

In our model, the balance sheet cost matters only through \( \lambda \theta \beta = \theta \beta (T - \bar{t}) \beta \) in the distress state. We first normalize the primary dealers’ aggregate balance sheet size to be one; during the COVID-19 crisis, the Treasury supply shock was about 15.625% of the primary dealers’ balance sheet.28 We therefore normalize \( \theta = 1 \) (by choosing \( \theta = 1/(T - \bar{t}) \)), and set \( \beta = 15.6\% \). The balance sheet cost \( \Lambda \) is taken from Fleckenstein and Longstaff (2020), who estimate the average balance sheet cost to be 81 bps.29 As we assume \( \Lambda = \lambda B \) and \( B \) has been normalized to 1 before the crisis, \( \lambda \) is pinned down at 0.0081.

Model predictions

In Fig. 6 Panel A, we plot the equilibrium yield curves at the normal (stress) state \( \beta = 0 \) (\( \beta = \beta > 0 \)). The yield at the end-long rises when hedge funds and dealers are absorbing the aggregate supply shock.30 Importantly, the yield curve steepens in the stressed state, consistent with yield curve movements in March 2020 as shown in Fig. 1.

In Fig. 6 Panel B, we plot the equilibrium GCF–tri-party repo spread \( \Delta \) (left axis) and 10-year Treasury-OIS spread (right axis) in the stress state, both as a function of the supply shock size \( \beta \). The equilibrium repo spread \( \Delta \) equals the balance sheet cost \( \Lambda = \lambda \theta \beta \), which is linear in the supply shock size \( \beta \) in the stress state. This represents the (state-dependent) inconvenience yield of Treasuries, driving a positive implied 10-year Treasury-OIS spread.

4.2.2. Empirical evidence of the Treasury inconvenience yield

In Fig. 7, we plot the daily series of the 10-year and 3-month Treasury-OIS spreads (in the left panel) during the COVID-19 crisis. Consistent with the model prediction (Fig. 6 Panel B), the 10-year Treasury-OIS spread is indeed positive. Moreover, it fell before March 9, 2020, consistent with a standard flight-to-safety to long-term Treasuries, but shot up afterwards amid a selling pressure by investors who scrambled for cash. On March 15, right before the Fed announced direct purchases of Treasuries, the 10-year Treasury-OIS spread jumped up by about 30 bps, consistent with an increase of \( \Lambda \) in our model. The 10-year Treasury-OIS spread began to ease afterwards, likely because the Fed’s direct purchases weakened the supply shock.

The right panel of Fig. 7 shows the daily series of the GCF–tri-party repo spread during the COVID-19 crisis. Consistent with the model prediction (Fig. 6 Panel B), the repo spread is mostly positive. Indicating a surging balance sheet cost \( \Lambda \) associated with the supply shock, the repo spread spiked up as high as 60 bps during the two-week period from March 9 to March 23. Holders of long-term Treasury securities found it hard to finance these positions in the repo market.

Together with the results in Section 2.1, we show that large sales of long-term Treasuries sharply drove up both the long-term Treasury yield and the repo spread during the COVID-19 crisis. Primary dealers only absorbed a small amount of this supply through direct purchases. Dealers’ provisions of repo funding to levered investors were limited too, likely because of the SLR constraint. Direct purchases by the Fed eventually absorbed the supply, relieving the strain on dealers’ balance sheets.

4.2.3. Term structure of Treasury-OIS spread

Fig. 7 also shows that in contrast to the spike in the 10-year Treasury-OIS spread, the 3-month Treasury-OIS spread dropped on March 15, during the COVID-19 crisis, implying a steepening of the Treasury-OIS curve along the maturity dimension. Although our model mechanism relies on the supply shock from habitat agents, and it is also widely documented that the supply shock during March 2020 was concentrated at the long end (He and Krishnamurthy, 2020), we acknowledge that our model, building in balance sheet constraints like the SLR, is inconsistent with a steepening of the Treasury-OIS curve.

In our stylized model, the Treasury-OIS spread is driven by the balance sheet cost, which is dictated by a simple accounting rule. One dollar of 3-month T-bills and one dollar of 10-year Treasuries require the same amount of compensation per unit of time, as both are counted as the same for the balance sheet size. Importantly, the time-varying balance sheet cost in Eq. (12), which is modeled by a binary Markov chain, is mean reverting. Therefore, conditional on

---

28 As shown in Fig. 3, approximately $500 billion net of Treasuries were sold by the preferred habitat agents (investors other than hedge funds and primary dealers). The total assets held by primary dealers are approximately $2.2 trillion. Data on primary dealers’ balance sheets are manually collected from their financial statements filed with the SEC via EDGAR (https://www.sec.gov/edgar/search) or their own website. Note that these are the balance sheets at the level of broker-dealers rather than their holding companies.

29 Fleckenstein and Longstaff (2020) compare the repo rate implied in the 5-year Treasury futures and the actual repo rate for the same Treasury note, and argue that the difference captures the cost of holding the Treasury note onto the balance sheet (which corresponds to \( \Lambda \) in our model).

30 As in Greenwood and Vayanos (2014), the equilibrium yield at the short-end also rises because all Treasury bonds are priced by the same marginal investors.
the supply shock, for longer-maturity Treasury securities, dealers (or levered investors) expect smaller balance sheet costs per unit of holding time until maturity. This logic also provides an explanation for the pattern in Fig. 6 Panel B: the model-implied repo spread, which can be viewed essentially as the inconvenience yield of a one-day Treasury security, is more sensitive to the underlying shock than the 10-year Treasury-OIS spread.

A steepening of the Treasury-OIS curve can be delivered with a simple model extension. During the COVID-19 crisis, dealers who had absorbed a significant amount of long-term Treasuries would have likely to aggressively cut their positions in T-bills (as their prices were largely unaffected). Dealers would keep doing so until they sold all of their T-bill holdings, and hence likely were no longer marginal in determining the equilibrium pricing of T-bills (here, we impose no-naked-short-selling constraints). Interestingly, this narrative is indeed consistent with the data as shown in Fig. 3: from late February to March 23, primary dealers’ T-bill holdings decreased from $30 billion to almost zero.\[31\]

Recall the Treasury prices \( p_{\bar{R}t} \) derived in Eq. (15) in the baseline model. With the simple extension, we denote by \( \bar{p}^*_{\bar{R}t} \) the equilibrium Treasury price. For Treasuries with \( \tau < \bar{t} \approx 1 \), dealers—due to the naked-short-selling constraint—are off from their Euler equation in the distressed state \( \bar{R}_t = \beta \), and for these securities we have \( \bar{p}^*_{\bar{R}t} > p_{\bar{R}t}^* \),\[32\] but \( \bar{p}^*_{\bar{R}t} = p_{\bar{R}t}^* \) still holds for Treasuries with \( \tau > 1 \). For illustration, suppose that there exists a constant \( \delta > 0 \), so that for 3-month T-bills (i.e., \( \tau = 1/4 \)), we have:

\[
\bar{p}^*_{\bar{R}t} = e^{-\delta \tau} p_{\bar{R}t}^* = \exp \left[ -\left( \bar{A}(\tau) r_t + \bar{C}_{\bar{R}}(\tau) - \delta \tau \right) \right] > p_{\bar{R}t}^*.
\]

Finally, because derivative contracts are free from the naked-short-selling constraint, the OIS pricing \( \bar{p}^{OIS}_{\bar{R}t} \) in Eq. (19) still holds in the extended model. Therefore, the extended model essentially lowers the Treasury-OIS spread by \( \delta \) for 3-month T-bills without touching the 10-year Treasury-OIS spread, which helps generate a steepening in the Treasury-OIS spread curve.\[33\]


While our focus is on the COVID-19 crisis, the essential framework of our model can also account for the movements of yields and yield spreads during the 2007–2009 financial crisis. This serves as a useful additional validation.

We posit that in the early stages of the 2007–2009 financial crisis, there was a positive demand shock for Treasuries (in contrast to a supply shock in March 2020), and the intermediary sector short-sold some bonds to meet this excess demand. Since this is an episode prior to the SLR regulation, the analysis in this section ignores the balance sheet cost (it is trivial to add back the balance sheet cost). Also, as there is no clear term structure pattern in the 2007–2009 financial crisis, we eliminate the tenor-dependence of demand shocks by setting \( \theta(\tau) = 1 \) (so \( \Theta = T \)) and drop \( \tau \) whenever appropriate.

4.3.1. Excess Treasury demand and naked short-selling

When Treasury demand surges, \( \bar{R}_t = \beta < 0 \) and \( H_t > 0 \). Following the same logic as in Section 4.1.2, it is reasonable to conjecture that, in equilibrium, dealers and hedge
funds would short sell bonds to absorb this excess demand. To accommodate this case with empirically realistic limits on short-selling, we assume that naked short-selling is costly. The short-selling costs can include potential repudiation loss or regulatory penalty when dealers fail to deliver the short-sold bonds.\footnote{Fleming and Garbade (2005) discuss the costs of failing to deliver, including foregone interest, counterparty credit risk, labor costs, and worsened customer relations. We could equivalently introduce a securities lending market in which dealers engage in a search for Treasury bonds to borrow from some habitat agents, and then sell to other habitat agents who demand these bonds, as long as these security lending activities entail some cost (for instance, it is difficult to locate the securities).}

For each tenor \( t \), denote by \( q^d_t (t) = \max (\gamma (t) - q(t), 0) \geq 0 \) the (absolute) amount of naked short-selling that the dealer is engaged in, with a marginal cost \( \Gamma_t \geq 0 \) (to be specified shortly). Any representative dealer’s wealth dynamics \( dw^d_t = -r_t w^d_t dt \) follow:

\[
\int_0^T x_t (\tau) \left( \frac{dP_t}{P_t} - r_t d\tau \right) d\tau + \int_0^T [q^d_t (\tau) \Delta_t - n^d_t (\tau) \Gamma_t] d\tau dt.
\]

(21)

The dealer faces the exact same problem as before in (6), except for one difference: now the dealer can short sell \( n^d_t \) dollars of bonds at a cost of \( n^d_t \Gamma_t \). The dealer’s total naked short-selling equals \( n^d_t = -x_t - q^d_t \) if it is positive; otherwise, there is no naked short-selling with \( n^d_t = 0 \).\footnote{For instance, \( x_t (t) = -1.5, q^d (t) = 1, \) and \( n^d (t) = 0.5 \) imply that the dealer short-sells 1 unit of Treasury via repo, and short-sells another 0.5 through either naked positions or security lending. During the 2007–2009 crisis dealers were actively borrowing Treasury securities (see Section 4.3.3 for details).}

To be consistent with the framework in Section 3.3, we continue to assume that the only way for hedge funds to establish Treasury positions \( q^h \)—whether long so \( q^h > 0 \) or short so \( q^h < 0 \)—is through dealers; this implies the same wealth dynamics for hedge funds given by (4). More specifically, if hedge funds decide to short Treasuries, then they need to engage in reverse repo with dealers first—essentially, hedge funds borrow Treasury securities from dealers and then sell; and dealers satisfy this “securities borrowing” demand using the naked short-selling technology.

As we show below, in equilibrium the hedge funds are shorting with \( Q^h < 0 \), implying that dealers who supply these securities are shorting \( N^d = -Q^h > 0 \) amount of Treasuries. To close the model, we assume that the naked short-selling cost \( \Gamma_t \) is increasing in the equilibrium aggregate naked short-selling \( N_t \equiv N^d + N^h \). For simplicity, we consider the following form:

\[
\Gamma_t = \Gamma (N_t) = \gamma N_t \text{ for some } \gamma > 0.
\]

4.3.2. Equilibrium characterization and implications

A positive naked short-selling amount, \( N_t > 0 \), must occur in equilibrium because market clearing implies that:

\[
N_t = -Q_t - X_t = H_t - \beta_t > 0.
\]

As in Section 4.1.2, in equilibrium both hedge funds and dealers absorb the habitat agents’ demand in proportion to their risk-bearing capacities, respectively:

\[
X_t = -N_t^d = \frac{\rho_t}{\rho_h + \rho_d} \beta_t, \text{ and } Q_t = -N_t^h = \frac{\rho_d}{\rho_h + \rho_d} \beta_t,
\]

The equilibrium naked short-selling cost \( \Gamma_t = \gamma N_t > 0 \) pins down the shadow price of repo financing \( \Delta_t \). To see this, each dealer solves the following problem:

\[
\max \left\{ n^d_t \cdot \Gamma_t + q^d_t \cdot \Delta_t \right\}
\]

with a linear constraint \( n^d_t = -x_t - q^d_t \). Note that we removed the nonnegativity constraint of \( q^d_t \geq 0 \) here.\footnote{In Section 3.4, for clarity of exposition, we impose \( q^d_t \geq 0 \) in the dealer’s problem in (5) so that \( q^d_t \) admits the narrow interpretation of dealers’ reverse repo positions to finance long positions of levered investors (as in the COVID-19 crisis). Another issue worth discussing is that we have assumed away tenor-dependent demand shocks by setting \( \theta (t) = 1 \) for all \( t \in [0, T] \) in this section. In the equilibrium for a general \( \theta (t) \), \( N_t (t) = H_t (t) \) and the cost of naked short selling \( \gamma N_t (t) \) will be \( t \)-dependent. This would be inconsistent with our general collateral (GC) repo market setting, in which the repo wedge \( \Delta_t \) is uniform across all tenors. The special repo market studied in Duffie (1996) precisely captures this tenor-\( t \) dependent demand, and would be an interesting direction for further research.}

By interpreting \( q^d_t < 0 \) as dealers engaging in repo transactions with, i.e., sending securities to, hedge funds, if they demand reverse repos, then \( q^h < 0 \). Dealers need to find securities somewhere; it is (potentially) costly because \( q^d_t < 0 \) increases the total naked short-selling position \( n^d_t \) to \( \max (\gamma x_t (t) - q^d_t (t), 0) \). Because dealers are free to cut naked short-selling (and save \( \Gamma_t > 0 \)) and hence cut repo transactions \( q^d_t \), the equilibrium indifference condition for dealers with \( n^d_t = N^d_t > 0 \) implies that:

\[
\Delta_t = -\Gamma_t = -\gamma N_t = \gamma T \beta_t < 0.
\]

(22)

Note that analogous to the supply shock case in Section 4.2 where dealers accommodate hedge funds’ cash borrowing at the cost of expanding their balance sheet, in this demand shock case, dealers accommodate hedge funds’ securities borrowing by engaging in more costly naked short-selling \( N_t \). In the equilibrium, dealers pass through the naked short-selling cost by charging hedge funds \( \Gamma_t = \gamma N_t \) per dollar of transaction, just like the balance sheet cost \( \Delta_t = \lambda B_t \) in the supply shock case. Importantly, both activities are implemented via the repo market. Analyzing the consequences of frictions in this market is our main contribution to the literature.

Following a demand shock on Treasury bonds, the equilibrium collateralized borrowing/lending rate falls below the risk-free rate, as \( \Delta_t = R_t - r_t < 0 \). But dealers are desperate to borrow and short sell Treasury bonds to satisfy the habitat agents’ surging demand. This mechanism is similar to Treasury being “special” in Duffie (1996). We then derive the equilibrium bond pricing as in Section 4.1.5, recognizing that now the holding cost of an additional Treasury bond is negative with \( \Delta_t = \gamma T \beta_t < 0 \), which leads to a convenience yield.

For the purposes of illustration, most parameters, including the shock size \( \beta_t \), take the same values as those in the COVID-19 crisis in Fig. 6. The new parameter governing the shorting cost \( \gamma \) is calibrated at 0.0538 so that upon the demand shock, the 10-year Treasury-OIS spread is around
–40 bps to match the corresponding empirical moment in Section 5. With these parameters, Fig. 8 Panel A shows the equilibrium yield curves at the normal (stressed) state $\tilde{\beta} = 0$ ($\tilde{\beta} = \beta < 0$). In contrast to the supply shock studied in Section 4.2, the demand shock $\tilde{\beta} = \beta < 0$ pushes down the entire yield curve. Because dealers (hedge funds) are in a short position in equilibrium, long-term bonds provide a hedging benefit for the marginal investor and hence demand a lower premium. This explains a downward sloping yield curve in Panel A. Fig. 8 Panel B shows the convenience yield $\Delta$ (left axis) and 10-year Treasury-OIS spread (right axis) after the demand shock, both as a function of the shock size $\beta < 0$. As a widely documented empirical regularity in the safe asset literature, a Treasury convenience yield in the context of 2007–2009 financial crisis explains the negative implied 10-year Treasury-OIS spread.

We finally discuss the model implication on repo spreads, highlighting one crucial conceptual difference between supply and demand shocks. In the supply shock case examined in Section 4.2, a good proxy for Treasury (in)convenience yield $\Delta = R_t - r_t$ is the repo spread, i.e., the wedge between the dealers’ GCF lending rate and their tri-party borrowing rate; the latter is close to the risk-free rate $r_t$ given the abundance of collateral. In contrast, when there was a shortage of Treasury bonds in the 2007–2009 financial crisis, the repo rate $R_t$ could still be proxied by the GCF (or bilateral repo) rate, but it would be inappropriate to use the tri-party rate, which is still a collateralized borrowing rate, to proxy for the risk-free rate $r_t$. Our analysis hence clarifies that the GCF–tri-party repo wedge is ultimately driven by (SLR-type) balance sheet constraints on cash borrowing. It should be nonpositive in the 2007–2009 financial crisis when such constraints were absent.

4.3.3. Treasury convenience yield in 2007–2009 financial crisis

Fig. 9 shows the weekly series of the holdings of Treasuries by foreign investors, the Fed, and primary dealers, as well as the repo amounts of primary dealers, from January 1, 2007, to December 31, 2008. The three event dates considered are July 31, 2007, when Bear Stearns liquidated two of its subprime hedge funds, September 15, 2008, when
Lehman Brothers filled for bankruptcy, and November 25, 2008, when the Fed announced the direct purchases of agency MBS. We observe a flight-to-safety to long-term Treasuries in the 2007–2009 crisis, in sharp contrast to their selling pressure during the COVID-19 crisis (Fig. 3). In particular, the top left panel shows a large increase (by about $350 billion) in foreign investors’ holdings of long-term Treasuries since July 31, 2007, while the top right panel shows a decrease in the Fed’s holdings. In stark contrast to March 2020, the Fed reduced its holdings of Treasuries to help accommodate the surging demand for safety. Primary dealers were net short in Treasuries, especially coupon Treasuries, and were on the net lending side in the repo market. That is, they maintained short positions in their market-making portfolios of Treasuries and often borrowed Treasuries in the repo market, consistent with our theory presented above. Their net short cash positions and net repo lending amounts trended lower towards the end of 2008, suggesting an easing of the shortage of Treasuries.

The excess demand of Treasuries would imply a positive shorting cost $\Gamma_1$, affecting the Treasury-OIS spread in our model (Fig. 8 Panel B). The left panel of Fig. 10 shows the daily series of the 10-year Treasury-OIS spread from January 2007 to December 2008, which stayed below zero mostly and reached as low as $-50$ bps. That is, amid a flight-to-safety to long-term Treasuries (with decreasing 10-year Treasury yield shown in the right panel), the large shorting cost pushed the Treasury-OIS spread to negative levels, consistent with our model. The Treasury-OIS spread then moved up notably after the Fed’s TSLF program was introduced in March 2008, which likely eased the shortage of Treasuries and dampened the shorting cost $\Gamma_1$. We also observe that the 3-month Treasury-OIS spread stayed slightly more negative, showing that the excess demand for T-bills is even stronger.

Regarding the repo market, daily series of tri-party repo rates are not readily available. Daily GCF repo rates extend back to 2005, but we find large month-end drops of GCF repo rates in this period, making it difficult to draw reliable inferences. That being said, using the month-end series of tri-party repo rates available through filings of money market funds with the SEC (Krishnamurthy et al., 2014; Hu et al., 2019), we do find a negative GCF-tri-party...
repo spread, consistent with the absence of SLR-like balance sheet constraints around 2008.

5. Regression analysis of dealer constraints and costs

The empirical evidence presented so far follows an event-study approach and reveals that (a) the shock to the demand of long-term Treasuries is negative in the COVID-19 crisis and positive in the 2007–2009 financial crisis, (b) dealers incur a balance sheet cost in holding long-term Treasuries and intermediating repos during the COVID-19 crisis, and (c) dealers incur a cost in short-selling Treasuries in the 2007–2009 crisis. In this section, we conduct a formal regression analysis using the full sample from January 2006 to April 2020.

5.1. Dealer constraints and costs across subperiods

Guided by our theoretical framework in Section 4, we first divide the full sample period according to variations of dealers’ balance sheet costs and shorting costs ($\Lambda_t$ and $\Gamma_t$ in the model, respectively).

The implementation of SLR that drove dealers’ balance sheet costs phased in progressively from 2012 onward (see Section 2.2.2). We define the pre-SLR and post-SLR periods as before and after September 2014, respectively, when the denominator of the SLR was finalized and banks learned how binding the rule would be. We expect the balance sheet cost $\Lambda_t$ to be effectively positive post-SLR, but either negligible or minor pre-SLR.37 Moreover, according to the NBER business cycle classifications, the Great Recession (GR) ended in June 2009, so we define the pre-GR and post-GR periods as before and after June 2009, respectively. Given that primary dealers entered the GR with large net short positions of Treasuries, we expect the shorting cost $\Gamma_t$ to be positive, though likely abated significantly around March 2008–June 2009, a period during which the Fed allowed dealers to obtain Treasuries against non-Treasury collateral in the TSLF program (Fleming et al., 2010). We hence define the pre-TSLF and post-TSLF periods as before and after March 2008, respectively.

In sum, the full sample period consists of four subperiods that fit with the interpretation of our model: (1) pre-TSLF, when $\Gamma_t$ is positive and $\Lambda_t$ is zero, leading to a positive repo spread and positive Treasury-OIS spread; (2) post-TSLF but pre-GR, when $\Gamma_t$ is reduced substantially but still positive and $\Lambda_t$ is zero, (3) post-GR but pre-SLR when both $\Gamma_t$ and $\Lambda_t$ are zero, and (4) post-SLR when $\Gamma_t$ is zero and $\Lambda_t$ is positive, implying a negative Treasury-OIS spread.

5.2. Results

The left panel of Fig. 11 reports the average daily GCF–triparty repo spread, which is only over the last two subperiods due to limitations of daily tri-party repo rates. Consistent with our characterizations of these subperiods, the GCF–triparty repo spread is only slightly positive post-GR, about 4 bps before the SLR formally phased in, and then increased to about 11 bps afterwards. Column (1) of Table 1 reports regressions of the repo spread on the subperiod dummies to formally test the significance of its change:

$$\text{GCF–Triparty}_t = \alpha + \beta D_{\text{Post–SLR}} + \epsilon_t,$$  \hspace{1cm} (23)

where the full sample from August 1, 2012, to April 30, 2020, is used (recall that the daily tri-party repo rate series are available only from August 2012). The significantly positive coefficient on $D_{\text{Post–SLR}}$ confirms the significance of the incremental changes.

Regarding the balance sheet cost $\Gamma_t$, the effect of the SLR rule on dealers’ balance sheet is likely to come in over an extended period of time rather than as an immediate effect. Our post-SLR dummy in the regression reported in column (1) of Table 1 captures this average effect over this extended period, but it could be confounded by other factors. We explore a variation based on quarter-end effects to mitigate this concern and further quantify the potential effect of the SLR on the repo spread. In particular, for-

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37 Though not in effect officially until 2018, banks could have had reputation concerns due to the disclosures and also had begun to prepare for the final adoption long before, which would make the SLR exert influence much earlier.
Table 1

**Repo spread regressions**
Column (1) reports the results of regressing the daily GCF-tri-party repo spread on dummy variables for the subperiods over the whole sample from August 1, 2012, to April 30, 2020. Columns (2) and (3) report the results of regressing the GCF-tri-party repo spread on the dummy for quarter ends (\(D\_\text{QuarterEnd}\)) using the post-SLR and pre-SLR periods, respectively, while column (4) reports those of regressions on \(D\_\text{Post-SLR} \times D\_\text{QuarterEnd}\), and their interaction term using the full sample. Quarter fixed effects are included in all regressions except that in column (1). The t-statistics based on standard errors clustered at calendar quarters are reported in parentheses. The significance levels are represented by \(^* p < 0.1\), \(^{**} p < 0.05\), and \(^{***} p < 0.01\).

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<th>(1) All</th>
<th>(2) Post-SLR</th>
<th>(3) Pre-SLR</th>
<th>(4) All</th>
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<td>(D_\text{Post-SLR})</td>
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Table 2

**Treasury-OIS spread regressions**
The first column reports the results of regressing the daily 10-year Treasury-OIS spread on dummy variables for subperiods over the full sample period from January 2006 to April 2020. The last two columns report results of regressing the daily Treasury-OIS spread on the daily GCF-tri-party repo spread for the pre-SLR and post-SLR periods, respectively, with the full sample period as from August 2012 to April 2020. The t-statistics based on standard errors clustered at calendar quarters are reported in parentheses. The significance levels are represented by \(^* p < 0.1\), \(^{**} p < 0.05\), and \(^{***} p < 0.01\).

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</tr>
<tr>
<td>GCF-Tri-Party</td>
<td>0.163</td>
<td>0.407*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.495)</td>
<td>(1.821)</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>3565</td>
<td>413</td>
<td>1504</td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td>0.835</td>
<td>0.005</td>
<td>0.078</td>
</tr>
</tbody>
</table>

effects is statistically significant with the interaction term \(D\_\text{QuarterEnd} \times D\_\text{Post-SLR}\).

Turning to the Treasury-OIS spread, the right panel of Fig. 11 reports the average daily Treasury-OIS spread over the four subperiods. Again, consistent with our theory, the average Treasury-OIS spread was low, at about –40 bps during the pre-TSLF period. The Fed’s TSLF program reduced this spread to –9 bps, which then turned positive during the post-GR period. It reached about 16 bps before the SLR phased in and climbed to about 34 bps afterwards. The first column of Table 2 reports a regression of the 10-year Treasury-OIS spread similar to Eq. (23) but with dummy variables for all subperiods. The coefficients measure the incremental change of a subperiod relative to the previous subperiod (hence, to obtain the average level of Treasury-OIS spread for the post-SLR period as in Fig. 11, one needs
to take the sum of the intercept and all the dummy coefficients). The significantly positive dummy coefficients confirm that dealers’ balance sheet constraints have become more and more binding since the 2007–2009 crisis.

The last two columns of Table 2 report contemporaneous time-series regressions of the 10-year Treasury–OIS spread on the repo (GCF–tri-party) spread for the pre-SLR and post-SLR periods, respectively. The regression coefficient is low and insignificantly different from zero pre-SLR because the balance sheet costs \( \lambda_1 \) are likely to be negligible. Post-SLR, instead, the regression coefficient is significantly positive because both the GCF–tri-party and Treasury-OIS spreads contain a positive \( \lambda_1 \) (which is potentially time varying).

6. Conclusion

In sharp contrast to most previous crisis episodes, the Treasury market experienced severe stress and illiquidity in March 2020 during the COVID-19 pandemic, raising concerns that the safe-haven status of US Treasuries could be eroding. We document that some large owners of Treasuries substantially reduced their holdings during March 2020 and the intermediation sector struggled to absorb this supply shock.

To understand the inelastic response of the intermediation sector, we build a model in which the balance sheet constraints of dealers and supply shocks from habituat agents affect equilibrium Treasury yields. A novel element of our model is to introduce repo financing as an important part of dealers’ intermediation activities, through which levered investors obtain leverage. Both direct holdings of Treasuries and reverse repo positions of dealers are subject to balance sheet constraints like SLR, which have been implemented as part of regulation reforms since the 2007–2009 crisis. Consistent with model implications, the spread between the Treasury yield and OIS rate and the spread between dealers’ reverse repo and repo rates are both highly positive during the COVID-19 crisis. Over the whole sample of 2006–2020, both the Treasury-OIS and GCF–tri-party spreads increased after 2015 when the regulatory reforms phased in, with their correlation turning significantly positive. Our model framework can also account for price and quantity changes of Treasury securities that received a demand shock in the 2007–2009 crisis.

Appendix

In this Appendix, we provide details of the data and variables used in empirical analysis. We obtain the daily series of constant maturity Treasury (CMT) yields from the H.15 reports of the Federal Reserve. We obtain the daily series of overnight index swap (OIS) rates from Bloomberg. The OIS is a fully collateralized interest rate swap contract that exchanges a constant cash flow against a flow of floating payment indexed to the geometric average of the daily effective federal funds rate. OIS contracts with maturities of up to one year have only one final payment, while cash payments for those with maturities longer than one year are made quarterly. Hence, OIS rates are effectively zero-coupon yields for maturities of up to one year and par yields for maturities longer than one year, both comparable to the CMT yields. We take the difference between the CMT yield and the maturity-matched OIS rate as the Treasury-OIS spread.

We use overnight repo rates of the Treasury securities in both the tri-party and GCF repo markets. Daily series of GCF repo rates are provided by the Depository Trust & Clearing Corporation (DTCC), available starting from 2005, calculated as the average interest rate across repo transactions weighted by volume within a day.\(^\text{38}\) The tri-party repo rates are from multiple sources. First, we obtain a daily series of the tri-party general collateral rate (TGCR) computed by the Federal Reserve Bank of New York, available from August 22, 2014. The TGCR is calculated as the volume-weighted median of transaction-level tri-party repo data, excluding GCF repo transactions and transactions to which the Federal Reserve is a counterparty.\(^\text{39}\) Second, we obtain a daily series of tri-party repo rates from August 1, 2012, to August 21, 2014, calculated by the Bank of New York Mellon, the largest of the two clearing banks. These repo rates are also calculated as volume-weighted medians on each business day for new, overnight repo trades with US Treasuries (excluding STRIPS) as collateral assets.\(^\text{40}\) Third, for November 2010–July 2012, we obtain monthly series of tri-party repo rates using the overnight tri-party repo trades between MMFs and dealers reported in the N-MFP fillings with the SEC, similar to Hu et al. (2019). Specifically, we calculate the volume-weighted medians of all overnight tri-party repo trades on the last business day of each month. Fourth, for October 2006–April 2010, we use the month-end value-weighted average overnight repo rates (weighted by notional amounts) constructed in Krishnamurthy et al. (2014) based on quarterly N-CSR, N-CSRS, and N-Q fillings with the SEC. MMFs file these reports at different month-ends throughout each quarter, so the monthly series of repo rates can be calculated.

The weekly series of primary dealers’ net positions and financing amounts of Treasury securities are obtained from the FR2004 data collected by the Federal Reserve Bank of New York. The data are reported on a weekly basis, as of the close of business each Wednesday.\(^\text{41}\) The reported series are netted and aggregated across all primary dealers, available for four categories, including T-bills, coupon-bearing nominal securities (coupons), Treasury inflation-protected securities (TIPS), and Floating Rate Notes (FRNs).

\(^{38}\) The series can be downloaded at http://www.dtcc.com/charts/dtcc-gcf-repo-index#download.

\(^{39}\) The TGCR is one of the three overnight repo rates provided by the Federal Reserve Bank of New York as important reference rates for financial markets, together with the broad general collateral rate (BGCR) and the secured overnight financing rate (SOFR). The calculation of BGCR includes all trades used in the calculation of TGCR plus the GCF repo transactions. The calculation of SOFR includes all trades used in the calculation of BGCR plus the bilateral Treasury repo transactions cleared through the delivery-versus-payment (DVP) service offered by the FICC but filtered to remove a portion of transactions considered “special.” For further details, see https://www.newyorkfed.org/markets/treasury-repo-reference-rates-information.

\(^{40}\) The data can be found at https://repoindex.bnymellon.com/repoindex/.

\(^{41}\) For details, see https://www.newyorkfed.org/markets/gds/search.
that began issuance in January 2014 and reported from 2015. The net positions include both spot cash positions and Treasury derivatives like futures (Fleming and Rosenberg (2007)).

Regarding financing amounts, the FR2004 data separate repo from other financing activities like security lending contracts from April 3, 2013, but repo and securities lending contracts are blended together earlier. Hence, we use the amounts of repo and reverse repo from April 3, 2013, onward, but total financing amounts of cash in and cash out before, which will be referred to as repo and reverse repo for convenience. Note that the financing contracts are defined from the perspective of dealers, so dealers borrow cash through repo and lend cash out through reverse repo. For both repo and reverse repo, the amounts are available for overnight and term contracts separately. We mainly focus on the total amount by adding the overnight and term financing amounts together, but will briefly discuss the breakdown in the Internet Appendix.

The daily VIX series are obtained from the Chicago Board Options Exchange (CBOE). We also obtain daily series of constant maturity yields of TIPS from the H.15 reports of the Federal Reserve. We compute the break-even inflation rate as the difference between the CMT nominal yield and the TIPS yield of the same maturity, which is a market-based measure of expected inflation. We also obtain daily series of inflation swap rates from Bloomberg as an alternative measure of expected inflation. To measure the uncertainty and tail event probability of inflation, we obtain the weekly series of the standard deviation and the probability of a large increase in inflation (of more than 3%) based on inflation density estimates using 5-year inflation caps and floors.42

We obtain the amounts of Treasury holdings and issuance from various sources. First, the monthly series of net issuance amount (gross minus retirement) of Treasury notes and bonds are obtained from the SIFMA.43 Second, quarterly series of flows into Treasury securities of pension funds, mutual funds, insurance companies, and hedge funds are obtained from the Financial Accounts of the United States – Z.1, provided by the Federal Reserve.44 The amounts for mutual funds exclude T-bills, while those for others include them. We use the quarterly change in the market values of Treasury holdings for hedge funds, but the flow series for insurance companies, pension funds, and mutual funds adjust for capital gains. None are seasonally adjusted. To give a sense of the magnitude of the capital gains adjustment, the level change is around $240 billion while the flow is around $200 billion for mutual funds in 2020:Q1. Third, monthly series of foreign net purchases of US long-term Treasury securities are obtained from the Treasury International Capital (TIC) system.45 We group them into four categories: Europe, Asia, Caribbean (including a lot of popular tax haven countries), and other (including all other countries and international organizations). Nonmarketable Treasuries are excluded. Fourth, weekly series of the total face value of US Treasury securities held by the Federal Reserve are provided in their H.4.1 release.46 We group these series into T-bills, nominal coupons, and TIPS.

We obtain the weekly series of the total net assets (TNA) of prime, government, and tax-exempt MMFs from the ICI. We obtain the weekly seasonally adjusted series of the commercial and industrial loans (C&I), cash assets, Treasury and agency securities, fed funds sold and cash lent out through reverse repo,47 and deposits, of all commercial banks, provided in the H.8 release of the Federal Reserve.48

References


42 These series are available at https://www.minneapolisfed.org/banking/current-and-historical-market--based-probabilities.
43 The series can be found at https://www.sifma.org/resources/research/us-marketable-treasury-issuance-outstanding-and-interest-rates/.
44 The series can be found at https://www.federalreserve.gov/releases/z1/20200921/html/default.htm.
45 The series can be found at https://www.treasury.gov/resource-center/data-chart-center/tic/Pages/ticsec.aspx.
46 The series can be found at https://www.federalreserve.gov/releases/h41/.
47 These include total federal funds sold to, and reverse repos with, commercial banks, brokers and dealers, and others, including the Federal Home Loan Banks.
48 The series can be found at https://www.federalreserve.gov/releases/h8/.


He, Z., Song, Z., 2021. Agency MBS as safe assets. working paper.


