Expectations in Finance and Macroeconomics

Yueran Ma

Chicago Booth

MFR Summer Session for Young Scholars
Historical Background

- Expectations are central to economic analyses

- 1940s—60s: Extensive effort to understand actual expectations
  - NBER volume led by Franco Modigliani: *The Quality and Economic Significance of Anticipations Data* (1960)

- Rational Expectations Revolution (1970s—)
  - Models dictate expectations agents should hold. Data are redundant.
  - Useful theoretical construct. Not necessarily empirical statement.

- Frictionless Benchmark:
  - Full Information Rational Expectations (FIRE)
Recent Research

Growing *empirical analyses* of expectations data:

- Expectations are *observable*
- Expectations are *important to economic decisions*
- Expectations can be *imperfectly rational*
Recent Research

Growing empirical analyses of expectations data:

- Expectations are observable
- Expectations are important to economic decisions
- Expectations can be imperfectly rational

Domains of analyses:

- Financial markets: stock returns, bond yields, credit spreads
- Macroeconomic outcomes: inflation, GDP
- Corporate decisions: earnings & investment
- Households: income, house prices
Recent Research

Growing empirical analyses of expectations data:

- Expectations are observable
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Two-way interactions between data and theory:

- Data informs theory. Theory organizes & unifies empirical evidence.
Research Program on Expectations

1. Measure and analyze expectations
2. Develop empirically founded, portable models of beliefs
3. Incorporate them in macro/finance analyses
Plan

Part 1. Empirical Evidence on Expectations

1. Informativeness of Expectations Data
2. Empirical Structure of Expectations

Part 2. Models of Expectations Formation

3. Deviations from FI in FIRE
4. Deviations from RE in FIRE

Open Questions, Data Sources, and Additional References
Outline

1. Empirical Evidence on Expectations
   - Informativeness of Expectations Data
   - Structure of Expectations

2. Models of Expectations
   - Deviations from FI in FIRE
   - Deviations from RE in FIRE

3. Summary and Additional Resources
Expectations and Decisions

- Expectations in data have significant explanatory power for decisions
  - Stock returns: Greenwood-Shleifer 14, Andonov-Rauh 18, Giglio-Maggiori-Stroebel-Utkus 20
  - Firm investment: Gennaioli-Ma-Shleifer 16, Richter-Zimmermann 19

- Both in the aggregate and at the firm/individual level

- Survey expectations are informative about decisions
  - Beyond traditional predictors
  - Can help differentiate models of decisions
Investor Expectations and Stock Market Investments

Greenwood-Shleifer 14: Aggregate equity mutual fund flows
Investor Expectations and Stock Market Investments

Figure IV: Expected 1-Year Stock Returns and Equity Share

Giglio-Maggiori-Stroebel-Utkus 20: Individual Vanguard account holders
CFO Expectations and Firm Investment

Geinnaioli-Ma-Shleifer 15: Aggregate CFO expectations and investment
CFO Expectations and Firm Investment

\[ \Delta \text{CAPX}_{qt} = \alpha + \beta E^*_{qt} [\Delta \text{Earnings}] + \lambda X_{qt} + \epsilon_{qt} \]

<table>
<thead>
<tr>
<th>CFO Expectations of Next 12m Earnings Growth</th>
<th>Realized</th>
<th>Next 12m Investment Growth</th>
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<tr>
<td></td>
<td>0.5903</td>
<td>0.5853</td>
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<tr>
<td></td>
<td>(8.14)</td>
<td>(8.41)</td>
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<tr>
<td>Q</td>
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<tr>
<td></td>
<td>(0.37)</td>
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<td>Past 12m Agg. Stock Returns</td>
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<td>0.1975</td>
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<td>(4.20)</td>
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<tr>
<td>Past 12m Credit Spread Change</td>
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<tr>
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<td>(-3.82)</td>
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<tr>
<td>Past 12m Asset Growth</td>
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<tr>
<td></td>
<td>(6.48)</td>
<td>(3.53)</td>
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<tr>
<td>Observations</td>
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<td>57</td>
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<tr>
<td>R-squared</td>
<td>0.610</td>
<td>0.611</td>
</tr>
</tbody>
</table>

Geinnaioi-Ma-Shleifer 15: Firm-level CFO expectations and investment
Interpreting evidence from prices and returns.

Three basic objects:

1. Discount rates
2. (Objective/statistical) expected returns
3. (Subjective) expectations of returns

Can be different with biased expectations.
Relevance of Expectations Data
A Simple Illustration

Two periods: asset pays random dividend \( D \) at \( t = 1 \), discount rate \( R \)

- Expectation of \( D \) is \( \bar{D} + S \) (rational + possible bias) for half; \( \bar{D} \) (rational) for half.
- Price is \( P \) at \( t = 0 \)

1. If \( S = 0 \), \( R \) fluctuates: when \( P \) high
   - Low discount rate
   - Low objective/statistical expected returns
   - Low subjective expectations of asset returns

2. If \( S \) fluctuates, \( R \) stays the same: when \( P \) high
   - Same discount rate
   - Low objective/statistical expected returns (true for rational investors)
   - High subjective expectations of asset returns for biased investors
Relevance of Expectations Data

Why consider beliefs not just preferences?

- Data
  - Predictable negative excess returns & crises (large negative returns)
    - “Instability from beliefs”
  - Example: credit cycles
    - If driven by low risk aversion & risks anticipated
      Loan loss provisions should not be too low during credit booms
    - No “neglected risks”

- Other relevant issues:
  - Firms’ investment/employment decisions. Business cycle fluctuations.
Relevance of Expectations Data

Why consider beliefs not just preferences?

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Expectations an essential part of dealing with an uncertain world
Outline

1. **Empirical Evidence on Expectations**
   - Informativeness of Expectations Data
   - Structure of Expectations

2. **Models of Expectations**
   - Deviations from FI in FIRE
   - Deviations from RE in FIRE

3. **Summary and Additional Resources**
Structure of Expectations

- Tendency to over-extrapolate recent shocks or trends
  - Project them too much into the future

- Holds in many settings
  - **Stock returns**: Greenwood-Shleifer 14
  - **Bond yields**: Piazzesi-Salomao-Schneider 15, Brooks-Lustig-Katz 19
  - **Credit spreads**: Bordalo-Gennaioli-Shleifer 18
  - **Firm earnings**: Gennaioli-Ma-Shleifer 16, Bordalo-Gennaioli-La Porta-Shleifer 19, Richter-Zimmerman 19
  - **Macroeconomic outcomes**: Bordalo-Gennaioli-Ma-Shleifer 20
  - **House prices**: Kuchler-Zafar 18, De Stefani 19
  - **Controlled experiment**: Landier-Ma-Thesmar 20
Extrapolative Tendencies in Expectations

A. Investor Expectations of Stock Returns

Greenwood-Shleifer 14: Extrapolative expectations of stock returns
# Extrapolative Tendencies in Expectations

## A. Investor Expectations of Stock Returns

### Table 5

<table>
<thead>
<tr>
<th></th>
<th>Survey expectation</th>
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<tbody>
<tr>
<td></td>
<td>Gallup N = 135</td>
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<tr>
<td>Log(D/P)</td>
<td>-0.328</td>
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<tr>
<td>[p-val]</td>
<td>[0.000]</td>
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<tr>
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<tr>
<td>[p-val]</td>
<td>[0.000]</td>
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<tr>
<td>cay</td>
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<tr>
<td>[p-val]</td>
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<tr>
<td>Composite ER</td>
<td>-0.572</td>
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<tr>
<td>[p-val]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

Greenwood-Shleifer 14: Extrapolative vs. model return expectations
Extrapolative Tendencies in Expectations

B. Analyst Forecasts of Credit Spreads

Bordalo-Gennaioli-Shleifer 18: Predictable errors in credit spread forecasts
Extrapolative Tendencies in Expectations

C. CFO Forecasts of Firm Earnings

Gennaioli-Ma-Shleifer 16: Predictable errors in CFO earnings forecasts
Extrapolative Tendencies in Expectations
D. Professional Forecasters on Macroeconomic Outcomes

\[ x_{t+h} - F_t^i x_{t+h} = \alpha + \beta \left[ F_t^i x_{t+h} - F_{t-1}^i x_{t+h} \right] + e_{t+h} \]

- Forecast Error
- Forecast Revision

Bordali-Gennaioli-Ma-Shleifer 20: Predictable errors in macro forecasts

Yueran Ma (Chicago Booth)
Extrapolative Tendencies in Expectations

D. Professional Forecasters on Macroeconomic Outcomes

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Forecast Error

Forecast Revision

Bordali-Gennaioli-Ma-Shleifer 20: Predictable errors in macro forecasts
Extrapolative Tendencies in Expectations

E. Controlled Experiments

- Forecast realizations of AR1 process
  - $x_t = \rho x_{t-1} + \epsilon_t$. Randomly assign to $\rho \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$.
  - 40 obs at beginning. Forecast 40 rounds. MTurk & MIT students.
Extrapolative Tendencies in Expectations

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\[
x_{t+h} - F_t x_{t+h} = \alpha + \beta [F_t x_{t+h} - F_{t-1} x_{t+h}] + e_{t+h}
\]

Landier-Ma-Thesmar 20: Predictable forecast errors in simple experiments
Extrapolative Tendencies in Expectations

E. Controlled Experiments

- Forecast realizations of AR1 process
  - $x_t = \rho x_{t-1} + \epsilon_t$. Randomly assign to $\rho \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$.
  - 40 obs at beginning. Forecast 40 rounds. MTurk & MIT students.

\[ F_t^i x_{t+h} = \alpha + \hat{\rho} x_t + e_{t+h}^i \]

Landier-Ma-Thesmar 20: Predictable forecast errors in simple experiments
Are There Exceptions?

A. Some Short-Term Forecasts

1. Short-term corporate earnings (equity analysts)
   - $\beta \approx 0.16$ for near-term analyst earnings forecasts (Bouchaud-Krueger-Landier-Thesmar 19)
   - $\beta \approx -0.3$ for long-term analyst earnings forecasts (Bordalo-Gennaioli-La Porta-Shleifer 19)

2. Short-term corporate sales (Italian firm managers)
   \[
   x_{t+1}^i - F_t^i x_{t+1} = \lambda + \kappa \left[ x_t^i - F_{t-1}^i x_t \right] + e_{t+h}^i
   \]
   - $\kappa \approx 0.1$ (Ma-Ropele-Thesmar-Sraer 20)

3. Short-term interest rate forecasts (professional forecasters)
   - Error-revision coefficient $> 0$ for 3M interest rates.
   - Always $< 0$ for 10Y interest rates
   - Bordalo-Gennaioli-Ma-Shleifer 20, Wang 20, D’Arienzo 20
Are There Exceptions?

B. Unattended Shocks

Figure 3. Response of Forecast Errors of Consumers, Firms, and FOMC Members to Shocks.

Coibion-Gorodnichenko 12: Response of forecast errors to deflationary shocks
Are There Exceptions?

C. Consensus Macro Forecasts

\[
\begin{align*}
\frac{x_{t+h} - \bar{F}_{t+h}}{\alpha + \beta} & = \bar{F}_{t+h} - x_{t+h} \\
\text{Average Forecast Error} & = \bar{F}_{t+h} - x_{t+h}
\end{align*}
\]

\[
\text{Average Forecast Revision} + e_{t+h}
\]

- \( \beta > 0 \) (Coibion-Gorodnichenko 15)
  - Possibly due to informational frictions

- In contrast with individual-level results above: \( \beta < 0 \) for most series (Bordalo-Gennaioli-Ma-Shleifer 20)
Are There Exceptions?

C. Consensus Macro Forecasts

\[
\underbrace{x_{t+h} - \bar{F}_{t}x_{t+h}}_{\text{Average Forecast Error}} = \alpha + \beta \underbrace{\bar{F}_{t}x_{t+h} - \bar{F}_{t-1}x_{t+h}}_{\text{Average Forecast Revision}} + e_{t+h}
\]

- \( \beta > 0 \) (Coibion-Gorodichenko 15)
  - Possibly due to informational frictions

- In contrast with individual-level results above: \( \beta < 0 \) for most series (Bordalo-Gennaioli-Ma-Shleifer 20)

- Difference arises from heterogeneous information sets across forecasts
  - People don’t necessarily react to the same information
  - More on this later
Taking Stock: Empirical Methods

Research analyzing survey data often use **error-revision regressions**

- **Individual level:** \( x_{t+h} - F_t^i x_{t+h} = \alpha + \beta \left[ F_t^i x_{t+h} - F_{t-1}^i x_{t+h} \right] + e_{t+h} \)
- **Consensus level:** \( x_{t+h} - \bar{F}_t x_{t+h} = \alpha + \beta \left[ \bar{F}_t x_{t+h} - \bar{F}_{t-1} x_{t+h} \right] + e_{t+h} \)

**Individual level: Test RE in FIRE**

- **Key advantage:** Do not need to know forecaster information set
  - Forecast revision as a “summary statistic” for information processed
- **Limitations:** Not necessarily behaved for transitory process
  - RE: FR = 0 for i.i.d. process \( \Rightarrow \beta \) not reliably estimated
- Magnitude of \( \beta \) may not be easy to interpret without a given model
  - Also some path dependence given FR includes \( F_{t-1} \)

**Consensus level: Affected by information frictions & FI in FIRE**
Taking Stock: Empirical Findings

Individual-level forecasts:

- Tend to over-adjust to recent observations
  - “Over-extrapolation”, “over-reaction”
- More pronounced when true process more transitory
- More pronounced for longer-horizon forecasts

Consensus (average) forecasts:

- Also affected by informational frictions
  - Infrequent update/not all information is processed
  - Heterogeneous information
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3. Summary and Additional Resources
Deviations from FI: Sticky Information

Mankiw-Reis 02: Infrequent updating (stickiness)

Basic ideas:

- Each period: updating w/ prob $(1 - \lambda)$; no updating with prob $\lambda$
- Use RE conditional on updating

Predictions:

- Individual level: if revise forecasts, forecast errors not predictable
  - Cannot be predicted by forecast revisions
  - Could be predicted by other things (not in info set)

- Consensus level: forecast revisions are insufficient
  - $\bar{F}_{tX_{t+h}} = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j E_{t-jX_{t+h}}$
  - (some people have stale expectations)
  - Positive correlation between average forecast errors & forecast revisions
Deviations from FI: Noisy Information

Woodford 03: Observe noisy signals

Basic ideas:
- Observe noisy representation of current state: $y_{it} = x_t + \omega_{it}$
  - Can interpret as noisy perception, or heterogeneous info.
- Forecasts formed by Kalman filtering: $F_t^i x_t = G y_{it} + (1 - G) F_{t-1}^i x_t$
  - $G \leq 1$ is Kalman gain

Predictions:
- Individual level: forecast errors not predictable
- Consensus level: forecast revisions are insufficient (given $G < 1$)
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Overview

Models of information frictions above:
- “Under-reaction” to information neglected or imperfectly perceived
- Unbiased with respect to information processed

Models of imperfect rationality below:
- “Over-reaction” to information being processed
- Because it’s more representative (diagnostic expectations) or more available in mind (memory-based models)
Some Earlier/Simpler Approaches

Direct extrapolation:

\[ F_t x_{t+1} = x_t + \phi (x_t - x_{t-1}) \]

- Does not have “kernel of truth”
  - Forecasting rule does not adapt to property of true process.
  - \( \phi \) needs to vary in different settings to fit data.

Over-estimation of persistence:

\[ F_t x_{t+1} = \tilde{\rho} x_t, \quad \tilde{\rho} > \rho \]

- Need a way to specify relationship between \( \hat{\rho} \) and \( \rho \).

Lack of adjustment to the setting, subject to Lucas critique
Diagnostic Expectations
Motivation: Representativeness

- Kahneman-Tversky 83: “An attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in a relevant reference class.”

- Assess distribution of attribute $T$ in class $G$

  \[ h(T = t|G) \]

- Following KT, define representativeness of $T = t$ for $G$ as:

  \[ R = \frac{h(T = t|G)}{h(T = t|-G)} \]

- Subjective perception distorts $h(T = t|G)$ by function of $R$
Example of Representativeness: Stereotypes

- Hair color: $T \equiv \{\text{red}, \text{light}, \text{dark}\}, G = \text{Irish}, -G = \text{World}$

<table>
<thead>
<tr>
<th>hair colour</th>
<th>red</th>
<th>light</th>
<th>dark</th>
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<tbody>
<tr>
<td>Irish</td>
<td>10%</td>
<td>40%</td>
<td>50%</td>
</tr>
<tr>
<td>World</td>
<td>1%</td>
<td>14%</td>
<td>85%</td>
</tr>
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</table>

- Given data (Irish), stereotype inflates prevalence of red hair:

$$\frac{h(\text{red hair}|\text{Irish})}{h(\text{red hair}|\text{World})} = 10$$
Example of Representativeness: Stereotypes

Hair color: \( T \equiv \{ \text{red, light, dark} \} \), \( G = \text{Irish} \), \(-G = \text{World}\)

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Given data (Irish), stereotype inflates prevalence of red hair:

\[
\frac{h(\text{red hair} | \text{Irish})}{h(\text{red hair} | \text{World})} = 10
\]

In a dynamic environment, given news:

- Inflate future states whose objective probability goes up the most
- The context is lagged information
Setup

- State of the economy $\Omega_t$ at $t$ follows AR1

  $$\omega_t = \rho \cdot \omega_{t-1} + \epsilon_t$$

- After seeing the state $\omega_t$, decision-maker needs to represent:

  $$h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)$$

- A future state is more representative at $t$ if it has become more likely in light of recent data:

  $$R_t(\omega_{t+1}) = \frac{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \omega_t)}{h(\Omega_{t+1} = \omega_{t+1} | \Omega_t = \rho \cdot \omega_{t-1})}$$

- Reference is information at $t - 1$: $-G = \{ \Omega_t = \rho \cdot \omega_{t-1} \}$
Diagnostic Expectations

- Distorted subjective distribution \( h^\theta_t(\omega_{t+1}) \) is:
  \[
  h(\Omega_{t+1} = \omega_{t+1}|\Omega_t = \omega_t) \cdot \left[ \frac{h(\Omega_{t+1} = \omega_{t+1}|\Omega_t = \omega_t)}{h(\Omega_{t+1} = \omega_{t+1}|\Omega_t = \rho \cdot \omega_{t-1})} \right]^\theta \frac{1}{Z_t}
  \]

- \( \theta \) measures the degree of representativeness bias
  - Typically estimated to be between 0.5 and 1.

- Diagnostic expectations given by:
  \[
  E^\theta_t(\omega_{t+1}) = \int_{\mathbb{R}} \omega \cdot h^\theta_t(\omega) \, d\omega
  \]

- Rational expectations: special case for \( \theta = 0 \) or no news \( \epsilon_t = 0 \)
Diagnostic Expectations

When $\omega_t$ is AR1 with normal $(0, \sigma^2)$ shocks, the distribution $h^\theta(\omega_{t+1})$ is also normal, with variance $\sigma^2$ and mean:

$$E_t^\theta(\omega_{t+1}) = E_t(\omega_{t+1}) + \theta [E_t(\omega_{t+1}) - E_{t-1}(\omega_{t+1})]$$

Can further express $E_t^\theta(\omega_{t+1})$ as:

$$E_t^\theta(\omega_{t+1}) = E_t(\omega_{t+1}) + \rho \theta [\omega_t - E_{t-1}(\omega_t)]$$

Rational Expectations + Over-reaction to Recent Shock $\epsilon_t$

Predictable forecast errors:

$$E_t[\omega_{t+1} - E_t^\theta(\omega_{t+1})] = -\rho \theta \epsilon_t$$
Subjective Probability Distribution

Subjective distribution $h^\theta_t(\omega_{t+1})$ shifts too much to news.
Properties

\[ E^\theta_t (\omega_{t+1}) = E_t (\omega_{t+1}) + \theta [E_t (\omega_{t+1}) - E_{t-1}(\omega_{t+1})] \]

- Rational expectations as special case, when:
  - \( \theta = 0 \), persistence \( \rho = 0 \), or no news

- **“Kernel of truth”**
  - Subjective belief incorporates features of rational beliefs and adapt to the setting of true process: “forward-looking”
  - But exaggerate impact of recent shocks
  - Not subject to Lucas critique (not mechanical dependence on past)

- Refreshes every period
  - Can add more lags beyond \([E_t (\omega_{t+1}) - E_{t-1}(\omega_{t+1})]: -G = \{\Omega_{t-h}\}\)
Non-fundamental reversals:

- Fundamental productivity is AR1: $\omega_t = \rho \omega_{t-1} + \epsilon_t$.
- Predictable reversals in expectations:

$$
\mathbb{E}_t \left[ \mathbb{E}_{t+1}^\theta (\omega_{t+2}) - \mathbb{E}_t^\theta (\omega_{t+2}) \right] \\
= \mathbb{E}_t \left[ (\rho \omega_{t+1} + \theta \rho \epsilon_{t+1}) - (\rho^2 \omega_t + \theta \rho^2 \epsilon_t) \right] \\
= -\theta \rho^2 \epsilon_t
$$

- Expectations at $t$ over-react to $\epsilon_t$. Expectations at $t+1$ do not.
Reduced form relationships, based on Bordalo-Gennaioli-Shleifer 18:

- Long-lived risk-neutral households, lend to firms. Firms invest.
- Everyone subject to diagnostic expectations.
- Credit spread, $\downarrow$ in subjective expected productivity:
  $$S_t \approx b_0 - b_1 E_t(\omega_{t+1})$$
- Investment (w/ time to build), $\uparrow$ in subjective expected productivity:
  $$K_{t+1} \approx a_0 - a_1 E_t(\omega_{t+1})$$
Application: Credit Cycles
Bordao-Gennaioli-Shleifer 18

- Apply diagnostic expectations. Credit spreads and investment follow:

\[ S_t = b_0(1 - \rho) + \rho S_{t-1} - \rho b_1(1 + \theta)\varepsilon_t + b_1\rho^2\theta\varepsilon_{t-1} \]

\[ K_t = a_0(1 - \rho) + \rho S_{t-1} + \rho a_1(1 + \theta)\varepsilon_t - a_1\rho^2\theta\varepsilon_{t-1} \]

- Over-reaction to current news, reversal of past news

- Predictable cycles in prices and quantities
  - Excess optimism after good shocks, on average wanes next period
Biased Beliefs + Information Frictions
Diagnostic Kalman Filter: Bordao-Gennaioli-Ma-Shleifer 20

- Data generating process: \( x_t = \rho x_{t-1} + u_t \)
  - \( \rho \in [0, 1] \) and \( u_t \sim \mathcal{N}(0, \sigma_u^2) \)

- Each forecaster \( i \in [0, 1] \) receives noisy signal
  \[
  s_t^i = x_t + \epsilon_t^i
  \]
  - \( \epsilon_t^i \sim \mathcal{N}(0, \sigma_{\epsilon}^2) \)
  - \( m\epsilon_t^i \) may capture inattention or heterogeneous information/interpretation (Woodford 2003)

- Distorted beliefs characterized by:
  \[
  x_{t+h|t}^i, \theta = x_{t+h|t-1}^i + (1 + \theta) \frac{\sum \rho^h}{\sum + \sigma_{\epsilon}^2} \cdot \left( s_t^i - x_{t+h|t-1}^i \right)
  \]
Predicting Forecast Errors with Forecast Revisions

Forecast Error on Forecast Revision Regression Coefficient

- **Consensus** level:

\[ \beta = \frac{\text{cov} \left( x_{t+h} - x_{t+h|t}, x_{t+h|t} - x_{t+h|t-1}^\theta \right)}{\text{var} \left( x_{t+h|t}^\theta - x_{t+h|t-1}^\theta \right)} = (\sigma_\epsilon^2 - \theta \Sigma) h^* \]

  ▶ Sign \( \propto \sigma_\epsilon^2 - \theta \Sigma \). Forecasters do not react to others’ private info.

- **Individual** level:

\[ \beta^p = \frac{\text{cov} \left( x_{t+h} - x_{t+h|t}^i, x_{t+h|t}^i - x_{t+h|t-1}^i, \theta \right)}{\text{var} \left( x_{t+h|t}^i, \theta - x_{t+h|t-1}^i, \theta \right)} = -\frac{\theta(1 + \theta)}{(1 + \theta)^2 + \theta^2 \rho^2} \]

  ▶ Negative if \( \theta > 0 \).

- Aggregation can change the interpretation of aggregate relationships.
Memory-Based Models
Background

Recent interests in the role of memory in belief formation

- Beliefs shaped by information available in the brain
- Many facts about memory from psychology & neuroscience
  - Kahana: *Foundations of Human Memory*
- Imperfect memory naturally leads to overweighting recent obs

Recent approaches of modeling

- (1) Recall/retrieval: cued recall, costly recall. (2) Noisy memory.
- For AR1 process $x_t = \mu + \rho(x_{t-1} - \mu) + \epsilon_t$,
  generally apply to biases of mean $\mu$ (simple & effective)
Imperfect Retrieval
Afrouzi-Kwon-Ma 20

Model in a nutshell

\[ \min S_t \mathbb{E} \left[ \min E_t x_{t+h} \mathbb{E} \left[ (F_t x_{t+h} - x_{t+h})^2 \mid S_t \right] + C_t(S_t) \right] \]

- **\( S_t \):** Retrieved info set. **\( C_t(S_t) \)** retrieval cost.
  \[ C_t(S_t) \equiv \omega \left[ \exp \left( 2 \ln(2) \cdot \gamma \cdot I(S_t, x_{t+h}\mid x_t) \right) - 1 \right] / \gamma \]

- **\( F_t x_{t+h} = \)**
  \[ \left( \frac{\omega T}{(1 - \rho h)^2} \right) \frac{1}{1+\gamma} \]

- Degree of bias larger when \( \rho^h \) smaller.
Imperfect Retrieval
Afrouzi-Kwon-Ma 20

Model fit using experimental forecasts of AR1 processes
Imperfect Retrieval

Afrouzi-Kwon-Ma 20

Model fit using experimental forecasts of AR1 processes

Implied Persistence

Actual Persistence

Data forecasts
Adaptive
Extrapolative
Diagnostic
Constant gain learning

Actual Persistence

0 0.2 0.4 0.6 0.8 1

0 0.2 0.4 0.6 0.8 1

0

Yueran Ma (Chicago Booth)
Noisy Memory
da Silveira-Sung-Woodford 20

Model in a nutshell

\[
\hat{\mu}_t = E[\mu|m_t] + \gamma_t (x_t - (1 - \rho)E[\mu|m_t] - \rho x_{t-1})
\]

\begin{itemize}
  \item $\lambda_t$ affects memory cost. $x_t$ observed after memory state $m_t$ is formed.
  \item $\gamma_t$ is function of memory precision; depend on $\lambda_t$ (optimally chosen).
  \item High cost/low precision $\Rightarrow$ more weight on $x_t$.
\end{itemize}
Model fit using experimental forecasts of AR1 processes
1. Incorrect persistence $\hat{\rho} > \rho$
   - $F_t x_{t+h} = \mu + \hat{\rho}^h(x_t - \mu)$
   - Unclear how $\hat{\rho}$ varies with $\rho$. Bias ↓ when $h \uparrow$.

2. Over-react to recent shocks
   - Diagnostic expectations:
     $F_t x_{t+h} = E_t x_{t+h} + \theta(E_t x_{t+h} - E_{t-1} x_{t+h}) = \mu + \rho^h(x_t - \mu) + \theta \rho^h \epsilon_t$
   - Bias ↓ when $\rho \downarrow$ (same as RE when $\rho = 0$). Bias ↓ when $h \uparrow$.

3. Biases in beliefs about long-run mean (memory models)
   - $F_t x_{t+h} = (1 - \rho^h) \hat{\mu}_t + \rho^h x_t$
   - Bias likely ↑ when $\rho \downarrow$. Bias ↑ when $h \uparrow$.
   - Parsimonious for greater bias when process transitory, horizon long.
Outline

1. Empirical Evidence on Expectations
   - Informativeness of Expectations Data
   - Structure of Expectations

2. Models of Expectations
   - Deviations from FI in FIRE
   - Deviations from RE in FIRE

3. Summary and Additional Resources
Summary

- Substantial information from data on expectations
  - Impact on decisions.
  - Structure of beliefs.

- Progress in unification of empirical evidence & models
  - Empirical structure increasingly clear
  - Modeling approaches taking shape

- Potential unification with biases in judgment in general
  - Stereotypes. Representativeness & availability heuristics.
  - Role of perception and memory.

- New venues for understanding economic activities
  - Credit cycles. Financial crises. Over- and under-investment...
Open Questions

1. Expectations of fundamentals vs. output vs. prices/returns

2. Heterogeneity and disagreement

3. Beliefs about central tendencies vs. tails
   - Most expectations data so far about central tendencies
   - But beliefs about tails can be important, e.g., credit cycles, tech boom

4. Interaction with financial structure
   - Biased expectations can apply to firms, stock markets, credit cycles...
   - Dependence on settings and financial structure?
Testing deviations from rational expectations
- Requires large $T$. Large $N$ and small $T$ is tricky.

Kendall/Stambaugh bias (time series) & Nickell bias (panel with individual/firm fixed effects)
- Suppose with AR1 process $x_{t+1} = \rho x_t + u_{t+1}$
  Bias in estimating $\rho$: $E[\hat{\rho} - \rho] = -\left(\frac{1+3\rho}{T}\right) + o\left(\frac{1}{T^2}\right)$
- Suppose use $z_t$ to predict $x_{t+1}$, where $z_{t+1} = \phi z_t + v_{t+1}$
  Want to estimate $x_{t+1} = \beta z_t + e_{t+1}$
  Bias in estimating $\beta$: $\gamma E[\hat{\rho} - \rho]$, where $\gamma = \sigma_{uv}/\sigma_u^2$
- Forecast errors likely auto-correlated with overlapping horizons

Be careful interpreting tests using average beliefs
Expectations Data

- **Duke CFO Survey**: [www.cfosurvey.org](http://www.cfosurvey.org)
  - Forecasts of next 12 months
  - Earnings, sales, CAPX, R&D, employment, wage, productivity, price; S&P 500 returns.
  - Aggregate and sectoral data. Quarterly since 1998.

- **IBES Analyst (on WRDS)**
  - Quarterly, annual, and long-term growth forecasts
  - Mainly EPS and sales. Firm-level data, since 1980s.

- **IBES Firm Guidance (on WRDS)**
  - Quarterly and annual forecasts
  - Mainly EPS and sales. Firm-level data, since 1980s.
Expectations Data

- **American Association of Individual Investors**
  - Next 6 months stock market sentiment (qualitative)
  - Aggregate data. Weekly since late 1980s.

- **Gallup**
  - Next 12 months stock market sentiment
  - Aggregate data. Monthly since 1990s (with gaps).
  - See Robin Greenwood’s website

- **Shiller**
  - Individual stock market confidence index
  - Aggregate data. Monthly since early 2000s.

- **RAND American Life Panel**
  - Stock returns within range. Since 2008.
Expectations Data

- **Michigan Survey of Consumers**
  - Inflation, consumer sentiment, stock returns (occasional)
  - Aggregate and household level data. Monthly (not panel)

- **New York Fed Survey of Consumer Expectations**
  - Inflation, spending, unemployment rate, house prices, etc.
  - Proprietary. Monthly since late 2012.

- **Survey of Professional Forecasters**
  - GDP, inflation, interest rates
  - Aggregate and forecaster-level. Quarterly since late 1960s.

- **BlueChip Survey**
  - GDP, inflation, Treasury rates, AAA/BAA yield
References and Additional Readings

Some Earlier Work


References and Additional Readings

Expectations and Decisions


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References and Additional Readings

Structure of Expectations


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