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Brief article

# Numerical ordering ability mediates the relation between number-sense and arithmetic competence

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ABSTRACT

What predicts human mathematical competence? While detailed models of number representation in the brain have been developed, it remains to be seen exactly how basic number representations link to higher math abilities. We propose that representation of ordinal associations between numerical symbols is one important factor that underpins this link. We show that individual variability in symbolic number-ordering ability strongly predicts performance on complex mental-arithmetic tasks even when controlling for several competing factors, including approximate number acuity. Crucially, symbolic number-ordering ability fully mediates the previously reported relation between approximate number acuity and more complex mathematical skills, suggesting that symbolic number-ordering may be a stepping stone from approximate number representation to mathematical competence. These results are important for understanding how evolution has interacted with culture to generate complex representations of abstract numerical relationships. Moreover, the finding that symbolic number-ordering ability links approximate number acuity and complex math skills carries implications for designing math-education curricula and identifying reliable markers of math performance during schooling.



## 1. Introduction

The intriguing hypothesis that the human symbolic number system is a direct extension of an evolutionarily ancient and developmentally fundamental sense of quantity has gained considerable momentum in recent years (Dehaene, 1997; Nieder & Dehaene, 2009). Work with children and young teenagers has shown a positive relationship between individual differences in approximate number-acuity (i.e., the ability to discriminate non-symbolic quantities) and math achievement (Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008). This result has lent support to the view that complex mathematical concepts are rooted in an approximate number system (ANS) shared across many species ('number sense': Nieder & Dehaene, 2009). However, even assuming human mathematics was kick-started by a universal and evolutionarily ancient system of representing (approximate) non-symbolic magnitudes, considerable work is needed to characterize the intermediary steps that allow this basic system to be co-opted for a functioning grasp of complex mental-arithmetic (Dehaene & Cohen, 2007). We propose that efficient representation of the ordinal associations between numerical symbols is one such step.

Both monkeys (Cantlon & Brannon, 2006) and children as young as 11 months can discriminate approximate quantities in terms of relative order (Brannon, 2002), suggesting that ordinality is a basic property of the ANS. Moreover, we have shown that one's ability to use this ordinal information facilitates the transition from approximate to symbolic numerical representation (Lyons & Beilock, 2009). When trained to relate approximate quantities (that could only be represented initially by the ANS) to a novel

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set of symbols, participants who explicitly focused on the ordinal relations between these symbols learned to use them more accurately in a numerical context. In addition, those individuals most likely to rely on ordinal strategies during novel numerical symbol acquisition also showed greater sensitivity to ordinal information in overlearned numerical symbols (Arabic numerals) in a manner that went beyond the simple count list (e.g., {2 4 6}, {1 4 7}). The above results suggest that a key aspect of transitioning from ANS to symbolic representations of number involves extraction of ordinal information from the ANS and codification of these ordinal relations in terms of direct associations between symbolically represented quantities.

Why might this be the case? The ANS is asymmetrically imprecise: as quantity increases, ANS representational precision decreases (Merten & Nieder, 2009; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). A more ideal number system would be one in which quantity and representational precision are independent. One way to achieve this precision is via a system that determines quantity primarily in relation to other quantities (e.g., the next number is always exactly one more than the current number, whether the current number is 1, 100, or 1000; for similar suggestions, see Le Corre & Carey, 2007; Izard, Pica, Spelke, & Dehaene, 2008). Relative ordinal position ('81' is equally '81 items' and 'the 81st item') is exactly the kind of association one would expect to play a fundamental role in such a system.

Indeed, Nieder (2009) has proposed that symbolic numbers form a system in which the associations between symbols are a fundamental aspect of their meaning, and the strength of these associations may even come to overshadow the relations between the symbols and their actual referents (i.e., the sense of quantity in the ANS that a given symbol represents). Recent evidence supports Neider's view. Several groups (Franklin & Jonides, 2009; Franklin, Jonides, & Smith, 2009; Lyons & Beilock, 2009; Turconi, Campbell, & Seron, 2006) have shown that sets of numerical symbols arranged in increasing order activate a representation of relative order whose influence surpasses the actual magnitudes the numerals represent. That is, the well-known and typically dominant effect of numerical distance seen as a hallmark of the ANS (e.g., Moyer & Landauer, 1967; Buckley & Gillman, 1974) can be completely reversed in numerical symbols when the relative ordinal associations between symbols are accessed.

Nieder's proposal is based on a more general view of symbolic processing in which increasingly abstract symbolic representations derive their respective meanings primarily via their relations with other symbolic representations – and less so by a relation to what they actually represent (Crutch & Warrington, 2010; Deacon, 1997; Peirce, 1955). One implication of this view is that we need numerical symbols not only because they represent large quantities precisely, but also because they facilitate acquisition and storage of complex relations *between* numbers (in a manner that may be more efficient and precise than can be achieved via the ANS alone).

What might be the function of these myriad associations between numerical symbols? Much of mental arithmetic concerns the efficient processing of associations between numbers (Clark & Campbell, 1991; García-Orza, Damas-López, Matas, & Rodríguez, 2009; Verguts & Fias, 2005). Without knowing the correct global ordering of the number symbols (*a la* Lyons & Ansari, 2009; Lyons & Beilock, 2009), it is difficult to see how one can develop a deeper understanding – beyond rigid, rote memorization – of the numerical system that underlies arithmetic more generally. That is, ordinal relations form the building blocks of symbolic numbers as a larger associative system that allows for rapid inference beyond what has been directly learned via repeated stimulus and response. In this way, the degree to which one has mastered ordinality of symbolic numbers should predict a reliable component of complex mental arithmetic. For instance, understanding the correct ordinal relation between successive symbolic numbers may allow for a more general understanding of the functions $n + 1$ and $n - 1$. If one extends the principle to multiples (e.g., {2 4 6}), the ordinality of which has been shown to be better mastered by those more likely to acquire numerical symbols in terms of their relative order in the first place (Lyons & Beilock, 2009), one can form an understanding of multiplication and division that again extends beyond mere memorization of individual arithmetic facts.

To summarize, we propose that representing relative order in numerical symbols may serve as an ideal stepping stone between the ANS and higher math abilities (e.g., complex arithmetic skills). If the ANS is linked to acquisition of symbolic numerical order, then greater ANS acuity should be related to more efficient assessment of ordinal relations in symbolic numbers. Moreover, if ordinal understanding in numerical symbols serves as a foundation for the associations accessed during mental arithmetic, then assessment of ordinal relations in symbolic numbers should predict complex mental-arithmetic ability. Finally, we also hypothesized that the level of one's numerical symbol-ordering ability is a key mechanism that mediates (explains)[1] the previously observed relation between ANS ability and more complex math abilities (Gilmore et al., 2010; Halberda et al., 2008). In this way, we directly test our proposal that relative order in numerical symbols is a stepping stone between the ANS and higher math abilities.

## 2. Methods and results

### 2.1. Participants

University students ($N = 54$; 26 female; $M = 20.5$ years) participated for course credit or monetary compensation.

### 2.2. Procedure

All subjects completed all tasks, organized into three modules. Module 1 included numeral-ordering, letter-ordering, dot-comparison and numeral-comparison tasks

[1] Note that a key difference between a traditional multiple regression analysis and a formal mediation analysis is that with the latter one seeks to explain the mechanism underlying *an observed relation between two variables* (and not the variance of a single outcome measure *per se*).

(task-order was randomized across subjects). Module 2 comprised numeral-recognition (presented first) and 4 mental-arithmetic tasks (presentation order randomized across subjects). Order of Modules 1 and 2 was counterbalanced. Module 3 contained the working-memory task and was completed last. On all tasks, subjects were instructed to respond as quickly as possible without sacrificing accuracy. No feedback was given.

### 2.2.1. Dependent measure: mental-arithmetic

Our dependent measure was performance on four difficult mental-arithmetic tasks: *addition* (three, two-digit addends), *division* (two or three-digit dividend, a one-digit divisor, and a two or three-digit quotient), *multiplication* (a two-digit and a one-digit factor), and *subtraction* (two-digit minuend and subtrahend) (Fig. 1a). Arithmetic problems were adapted and computerized from the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Dermen, 1976). Problems were open-ended (participants typed their responses using the number-pad). Participants were not allowed to work out problems using paper or pencil (though not strictly forbidden, no participants spoke aloud during this task).

Mental-arithmetic performance was calculated as the standardized ($z = \frac{x_i - \bar{x}}{\sigma}$) average (over the four problem types) of the net number of correctly solved problems in a twelve-minute time span (one three-minute block per operation type). This presentation (and scoring) method was chosen to reduce the impact of doing math under time-pressure; as such, participants were not aware of the time-limit (from the participant's perspective, they had simply completed the allotted number of trials for that block). Behavioral results indicated that the mental-arithmetic task was quite difficult [response-time: *M* = 7.75 s, *se* = .36 s; error-rate: *M* = .13, *se* = .02).

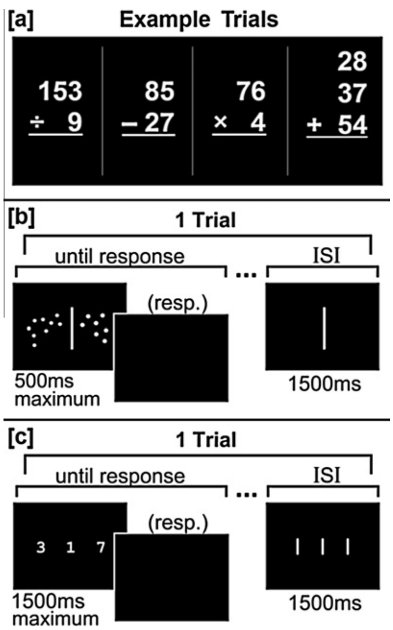


**Fig. 1.** (a) Examples of the complex mental-arithmetic tasks used to estimate mental-arithmetic ability (dependent-measure). (b) Error rates from the dot-comparison task were used to estimate individual differences in participants' ANS acuity (predictive-measure). (c) The numeral-ordering task was used to estimate symbolic number-ordering ability (predictive-measure).

### 2.2.2. Predictive measure (1): ANS acuity

ANS acuity was measured using a canonical dot-quantity comparison task (Fig. 1b). Participants estimated which of two dot-arrays contained more dots (range: 1–9, $|n_1 - n_2| \leqslant 2$). Eight total instances of each numerical pair were seen by each subject. Half of the pairs were equated in individual dot area; half were equated in overall dot area. Orthogonally, half of the pairs were equated in inter-item spacing (local density); half were equated in overall contour-length (outer array perimeter). Hence, participants could not rely on any one of these continuous parameters to estimate array numerosity and perform above chance.

An accepted measure of individual differences in ANS acuity is an estimate of a participant's Weber coefficient *w*, where a smaller *w* corresponds to higher ANS acuity (Halberda et al., 2008; Pica, Lemer, Izard, & Dehaene, 2004; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). Each participant's *w* was estimated by identifying the value of *w* that yielded the best approximation of their actual pattern of errors, as a function of the ratio between quantities being compared. For a given pair of quantities ($n_1$, $n_2$) that we treat as Gaussian random variables, expected error-rate (*ER*) when comparing $n_1$ and $n_2$ can be estimated from one tail of the complementary error-response function *erfc*. This function takes as its input a value from a normalized Gaussian distribution, which, here, is the *w*-weighted, normalized difference between $n_1$ and $n_2$: $ER = \frac{1}{2} erfc\left(\frac{|n_1 - n_2|}{w\sqrt{2(n_1^2 + n_2^2)}}\right)$. Using this method, we obtained a mean *w* of .117, which is close to the previously reported value of .11 in adults (Pica et al., 2004). Furthermore, assuming *w* = .117, agreement between values for *ER* and observed error-rates (as a function of comparison ratio) was high ($R^2$ = .95).

### 2.2.3. Predictive measure (2): symbolic number-ordering ability

In the numeral-ordering task (Fig. 1c), participants decided whether triads of Arabic numerals (range: 1–9) were *all* in increasing order (left to right), irrespective of numerical distance between numbers. Participants completed 2 blocks of 64 trials. The overall pattern of results in all regression analyses was similar for response-times and error-rates; thus, we simplified our measure of performance on this task by averaging standardized error-rates and reaction-times: lower number indicates better performance.

### 2.2.4. Covariates

Covariate tasks included letter-order judgment (identical to the numeral-ordering task, with the exception that the digits were replaced with the letters A to I), numeral-comparison (identical to the dot comparison task, with the exception that quantities were presented as Arabic numerals), numeral-recognition, and a working-memory task (automated reading-span: Conway et al., 2005; Unsworth, Heitz, Schrock, & Engle, 2005). Additional task details and raw behavioral results can be found in Supplementary Information.

### 2.3. Results

First, we replicated the finding that better ANS acuity predicts math ability. Higher acuity (smaller Weber coefficient) was related to better mental-arithmetic performance (higher mental-arithmetic scores) [$r(52) = -.339$, $p = .012$]. To our knowledge, this is the first demonstration that individual differences in ANS acuity predict performance for a complex, symbolic math task in college-aged adults – extending into adulthood the original Halberda et al. (2008) findings examining 14-year-olds.

Second, we tested the hypothesis that symbolic number-ordering ability predicts mental-arithmetic performance. Better performance on the numeral-ordering task was significantly correlated with better mental-arithmetic performance [$r(52) = -.703$, $p < .001$; Fig. 2]. Crucially, this relation remained significant even when controlling for ANS acuity, and performance on the numeral-comparison, letter-ordering, working-memory and numeral-recognition tasks (Table 1). Thus, the relationship between numeral-ordering and mental-arithmetic cannot be merely due to general cognitive factors (working memory), presentation format (Arabic numerals), or even accessing quantity information from symbolic numbers (numeral comparison). Rather, it is the processing of *numerical ordinal information* – over and above these related factors – that is the key property linking the numeral-ordering and mental-arithmetic tasks. We also controlled for ordering ability in a non-numerical (letter ordering) context, which narrows the result still further to understanding ordinal relations among *symbolic numbers* and not ordinal processing in general.

Third, we showed that ANS acuity was positively related to numeral-ordering ability [$r(52) = .408$, $p = .002$], a

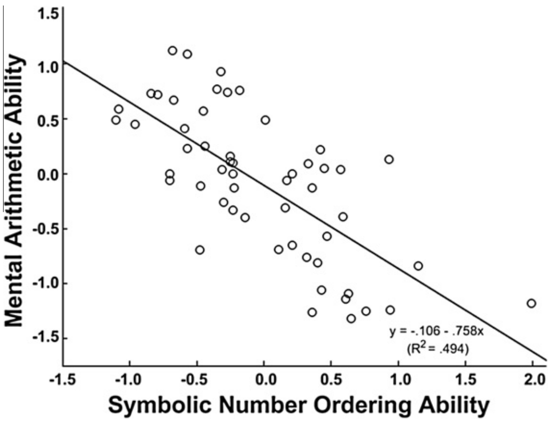


**Fig. 2.** Scatterplot of the relation between symbolic number-ordering ability and complex mental-arithmetic ability. Note that higher numbers indicate *worse* performance for numeral-ordering, but *better* performance for complex mental-arithmetic.

**Table 1**
Mental-arithmetic ability (higher number =: better performance) regressed on several individual-difference variables: *NumOrd*: numeral-ordering (lower number: better performance), *ANS acuity* (lower number: better acuity), *NumComp*: numeral-comparison (lower number: better performance), *LettOrd*: letter-ordering (lower number: better performance), *WorkMem*: working-memory capacity (higher number: higher capacity), *NumRecog*: numeral-recognition (lower number: better performance). Overall model fit: adj. $R^2 = .514$. The rightmost column indicates simple Pearson correlation coefficients (and associated *p*-values) between each predictor and mental-arithmetic ability in the absence of any of the other predictors; $r_{partial}$, by contrast, is the *r*-value between a given predictor and mental-arithmetic ability while controlling for shared variance between all other predictors and both mental-arithmetic ability and the predictor in question.

| Predictor | β (*SE*) | *t* (*p*) | $r_{partial}$ | *r* (*p*) |
|---|---|---|---|---|
| NumOrd | −.7124 (.1568) | −4.543 (.000) | −.552 | −.703 (.000) |
| ANS acuity | −.2308 (1.6014) | −.144 (.886) | −.021 | −.339 (.012) |
| NumComp | .0217 (.1350) | .161 (.873) | .023 | −.305 (.025) |
| LettOrd | −.0190 (.1490) | −.128 (.899) | −.019 | −.382 (.004) |
| WorkMem | .0085 (.0044) | 1.945 (.058) | .273 | .305 (.025) |
| NumRecog | −.0004 (.0004) | −.998 (.324) | −.144 | −.299 (.028) |
| Constant | −.0069 (.6144) | | | |

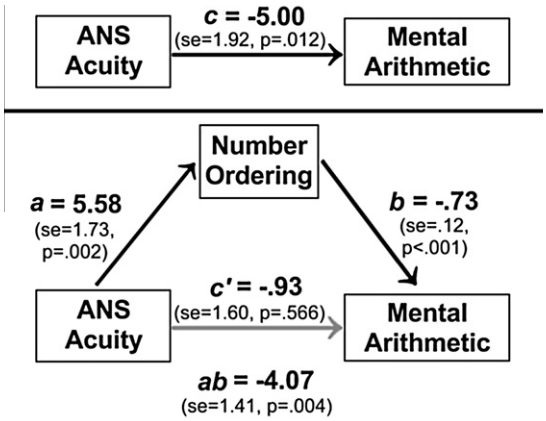


**Fig. 3.** In the mediation framework, one asks whether there is a significant indirect effect (quantified as the product of the unstandardized path coefficients, *a* and *b*) of the mediator (numeral-ordering) that accounts for some portion of the direct effect *c* observed between the original predictor (ANS-acuity) and the outcome (mental-arithmetic) variables. The remaining (unmediated) direct effect is denoted *c*′. Note that, in this framework, the model is constrained by the assumption that $c = ab + c'$. Unlike in a standard multiple regression analysis, we are explicitly asking what portion of the *relation between* ANS-acuity and mental-arithmetic that can be accounted for by the mediating variable (numeral-ordering). Results indicate full (*ab* is significant but *c*′ is not) as opposed to partial (when both *ab* and *c*′ remain significant) mediation.

finding in keeping with the view that relative numerical order in symbols is derived at least in part from quantity representation in the ANS. As seen in Fig. 3, the mediation analysis (Preacher & Hayes, 2008)[2] was consistent with our

[2] We employed the bootstrapping method (Preacher & Hayes, 2008). In this method, one generates percentile-based confidence-intervals (CIs) for indirect effects (*ab*) by a simulated resampling (1000 iterations in the current study) of one's original dataset. Estimating CIs in this manner allows for asymmetric intervals (above and below the mean estimate) and thus relaxes the assumption of multivariate normality. In addition, this method tends to be more robust for small samples.

**Table 2**
Symbolic number-ordering ability regressed on several individual difference variables (see Table 1 for abbreviations/variable details). Overall model fit: adj. $R^2$ = .505.

| Predictor | β (SE) | t (p) | $r_{partial}$ | r (p) |
|---|---|---|---|---|
| LettOrd | .5024 (.1153) | 4.358 (.000) | .532 | .601 (.000) |
| NumComp | .3522 (.1110) | 3.174 (.003) | .417 | .492 (.000) |
| ANS acuity | 3.3420 (1.4233) | 2.348 (.023) | .321 | .408 (.002) |
| WorkMem | −.0001 (.0039) | −.032 (.975) | −.005 | .007 (.610) |
| NumRecog | .0003 (.0004) | .815 (.419) | .117 | .185 (.180) |
| Constant | −.6911 (.5467) | | | |

main hypothesis: the relation between ANS acuity and complex mental-arithmetic performance was fully mediated by symbolic number-ordering ability. This was true even when controlling for all covariate factors shown in Table 1. Note also that ANS acuity did not mediate the relation between numeral-ordering and mental-arithmetic performance ($p$ = .562).

The numeral-ordering task devised here captures a large portion of complex mental-arithmetic variance (Table 1). Why? One view is that number-ordering combines three important components of arithmetic: ANS acuity, symbolic number representation (numeral-comparison task), and ordering ability in general (letter-ordering task). Accordingly, in a multiple regression analysis in which numeral-ordering is treated as the dependent variable, results showed unique variance captured by ANS acuity, symbolic number representation, and general ordering ability (Table 2).

## 3. Discussion

We provide the first evidence that representing relative order in numerical symbols serves as an important intermediary step that links the ANS and more complex arithmetic skills. In addition to predicting complex mental-arithmetic ability, ANS acuity also predicted more efficient symbolic number-ordering ability (*a* in Fig. 3). Furthermore, individuals who were best at processing this ordinal information showed the best mental-arithmetic performance (*b* in Fig. 3; also Fig. 2, Table 1). Crucially, as seen in Table 1, this result obtained even when controlling for general cognitive (working-memory, letter-ordering ability) and several related numerical factors (e.g., ANS acuity and numeral recognition and comparison). Thus, while numeral-ordering is comprised of several related numerical and ordinal factors (Table 2), the relation between symbolic number-ordering ability and complex mental-arithmetic goes beyond the sum of these constituent parts. Finally, symbolic number-ordering ability fully mediated (or accounted for) the relation between ANS acuity and mental-arithmetic performance (*ab* in Fig. 3). The mediation analysis results are of particular interest, given recent work demonstrating a link between ANS acuity and math-ability in kindergarteners (Gilmore et al., 2010) and young adolescents (Halberda et al., 2008). Our data replicate and extend these results to college-age young adults in a demanding mental-arithmetic task (*c* in Fig. 3).

However, our data provide an important revision to the straightforward notion that the ANS serves as a precursor to more complex numerical abilities: a more complete understanding of this link requires an accounting of how symbolic quantities are understood in terms of their relative order. We see this not as a rejection but as a refinement of the hypothesis that the ANS is co-opted for more complex math. For instance, consider the link between ANS acuity and numerical ordering ability (*a* in Fig. 3): those with higher ANS acuity also showed better numeral-ordering ability. One of the ways in which the ANS serves as a launching pad for number processing may be by giving people a sense not only of 'how much' (cardinality) but also of 'which position' (ordinality) (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010). This latter sense of order may then be codified and brought to the fore in the acquisition of numerical symbols (Lyons & Ansari, 2009; Lyons & Beilock, 2009). Consistent with this view, the multiple regression results shown in Table 2 indicate that ANS acuity, symbolic quantity representation, and general symbol-ordering ability each contribute separate variance to symbolic number-ordering ability. In other words, it is precisely by combining these sources that symbolic number-ordering serves as an excellent candidate for linking models of ANS representation with higher math abilities. We therefore propose that an emphasis on the relations between symbols provides the associative building blocks for the system of symbol-symbol relations that underlie complex math in general (Nieder, 2009).

A note of caution: Our model does not by itself allow for inference of how all relevant developmental factors unfold. For instance, while considerable evidence has accrued suggesting an early link between the ANS and higher numerical abilities (e.g., Berteletti et al., 2010; Gilmore et al., 2010; Halberda et al., 2008; Piazza et al., 2010), children's ability to encode and understand ordinal information in a numerical context may also depend on their grasp of the relation between counting and cardinal principles. One view is that this relation may develop first in the subitizing system (via knowledge of object-file indexical pointers), and transfer only later to larger numbers that are typically represented primarily in the ANS (Le Corre & Carey, 2007). While the current dataset cannot distinguish between these two hypotheses, it is worth noting that the regression analysis reported in Table 1 reflects a robust and unique relation between numeral-ordering and complex mental-arithmetic abilities. Thus, regardless of its origin, one's mastery of symbol-symbol ordinal associations between numbers robustly and uniquely predicts one's complex mental-arithmetic ability. Further work aimed at clarifying exactly how and why this is so may provide a firm foothold for understanding the likely complex relation between basic number representation and higher math abilities more generally. In sum, the current data provide strong grounds for hypothesizing that the path to higher math-abilities in children – whether it begins in the ANS or the subitizing system – proceeds in part via acquisition and mastery of ordinal relations among numerical symbols.

To summarize, we propose that representation of order in numerical symbols serves as an ideal stepping stone between the ANS and higher math abilities because (1) ordinality is a property that appears to link the ANS with symbolic numbers, and (2) ordinal associations between symbolic numbers likely serve as an important precursor to the more complex symbol–symbol associations that underlie much of mental arithmetic. Given strong evidence that the ANS is deeply rooted in our evolutionary history (Nieder & Dehaene, 2009) and the fact that symbolic mathematics is a rather recent human cultural invention (Ifrah, 1999; Zhang & Norman, 1995), our results may prove useful in understanding how evolution has interacted with culture to generate complex representations of abstract numerical relationships. Moreover, the strong link between symbolic number-ordering ability and complex mental-arithmetic carries important implications for designing math-education curricula that propel students from the ANS to everyday arithmetic competence. Symbolic number-ordering ability could also serve as a simple and easily administered 'marker' (Goswami, 2006) that captures important information about an individual's current arithmetic skills.

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## Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.cognition.2011.07.009.

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