Building Intuition about the DFT

5

Note Title 9/10/2008

A

George

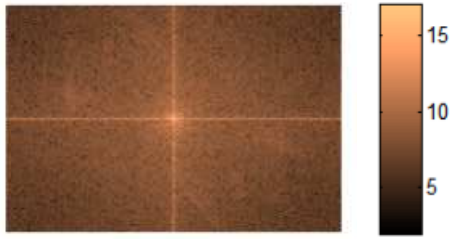
$f[m_1, m_2]$

B

DFT of

$F[k_1, k_2]$

C



Rescaled DF T

$\log(F[k_1, k_2] + 1)$

```
f = double(rgb2gray(imread('George.jpg')));
f = f(1:420,1:630);

F = fftshift(fft2(f));
figure(1);
A - subplot(131);imagesc(f);axis image;colormap copper;axis off
B - subplot(132);imagesc(abs(F));axis image;colormap copper;colorbar;axis off
C - subplot(133);imagesc(log(1+abs(F)));axis image;colormap copper;colorbar;axis off
```

What happens if we set the DC component to zero?

```
k1_dc = 211;
k2_dc = 316;
```

k1 and k2 corresponding to DC (zero frequency) component.

```
Fa = F;
Fa(k1_dc,k2_dc) = 0;
fa = (ifft2(fftshift(Fa)));
```

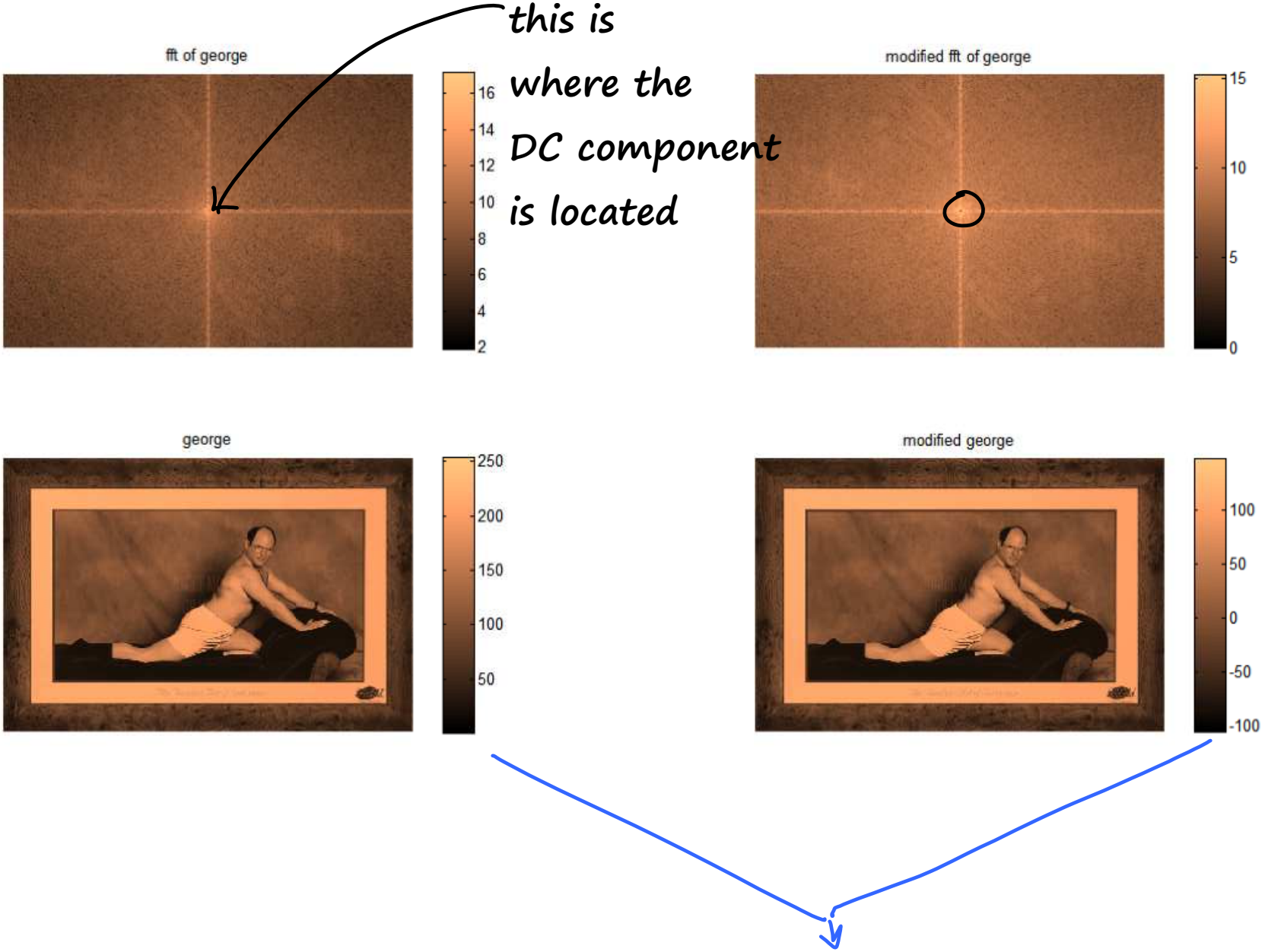
First, let's think about what the zero frequency component is mathematically.

$$F[k_1, k_2] = \sum_{\underline{m} \in \{0,1,\ldots,M-1\}^2} f[\underline{m}]\, e^{-j2\pi \underline{m}^T \underline{k}/M}$$

$$= \sum_{\underline{m}} f[\underline{m}]\, e^{-j2\pi (m_1 k_1 + m_2 k_2)/M}$$

$$F[0,0] = \sum_{\underline{m}} f[\underline{m}]\, e^{0} = \sum_{\underline{m}} f[\underline{m}]$$

So if we set F[0,0]=0 and perform the inverse DFT, then our


this is
where the
DC component
is located
fft of george
16
14
12
10
8
6
4
2
modified fft of george
15
10
5
0
george
250
200
150
100
50
modified george
100
50
0
-50
-100
note different color scales

what happens if we set F[k1,k2]=0 whenever k1=0 ?

```
Fa = F;
Fa(k1_dc,:) = 0;
fa = (ifft2(fftshift(Fa)));
```

Consider some arbitrary, fixed k2.

$$F[k_1,k_2] = \sum_{\underline{m}} f[\underline{m}]\, e^{-j2\pi(m_1k_1+m_2k_2)/M}$$

$$F[0,k_2] = \sum_{\underline{m}} f[\underline{m}]\, e^{-j2\pi m_2 k_2/M}$$

$$= \sum_{m_2=0}^{M-1} \left(\sum_{m_1=0}^{M-1} f[m_1,m_2] \right) e^{-j2\pi m_2 k_2/M}$$

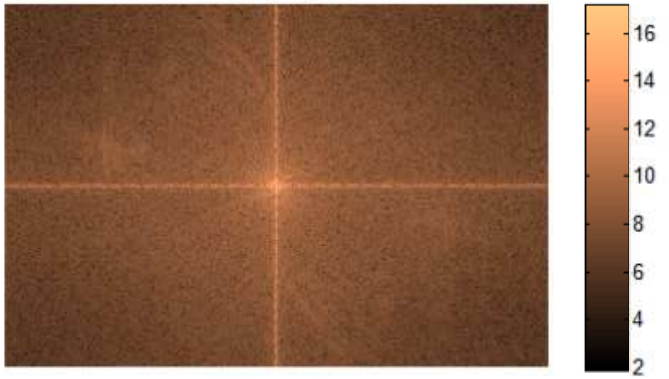
sum of column m_2

So let $g[m_2] = \sum_{m_1=0}^{M-1} f[m_1,m_2]$

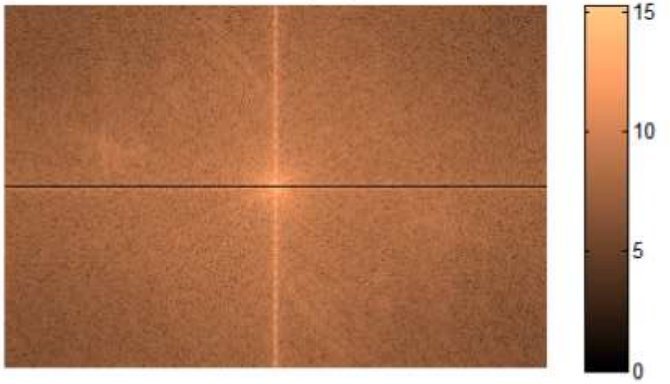
= the sum of the m_2^{th} column.

Then F[0,k2] is just the DFT of g.
So setting F[0,k2]=0 for all k2
and taking the inverse DFT
gives an image where each
column sums to zero.

fft of george



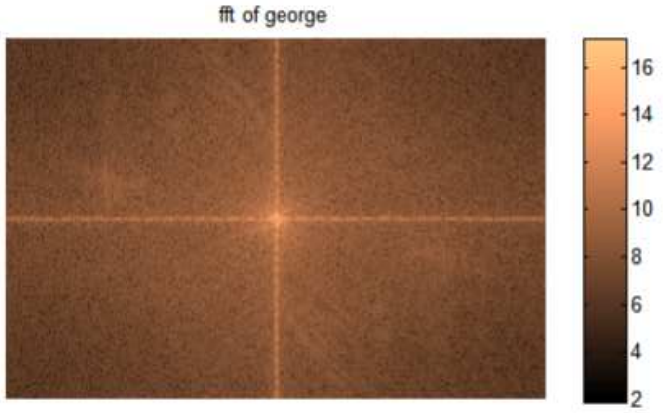
modified fft of george

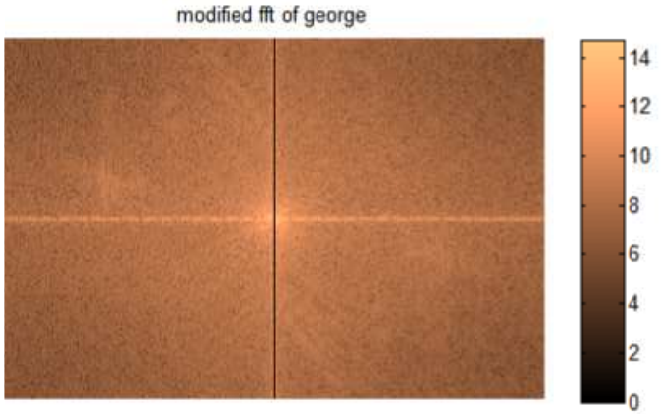


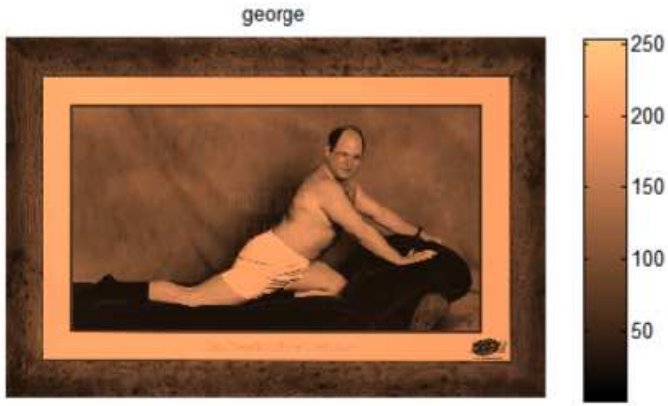
george

modified george

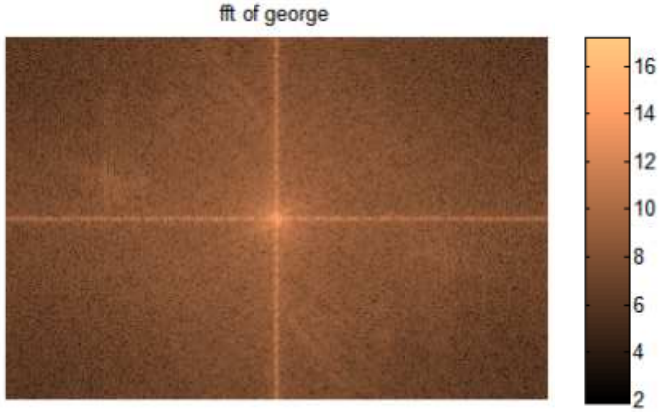
What happens if we set F[k1,k2]=0 whenever k2=0?

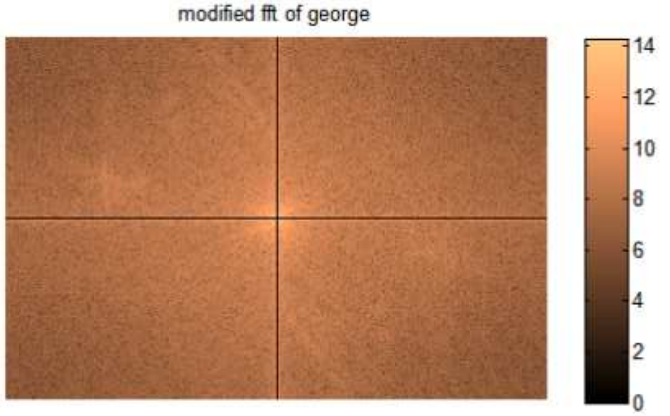


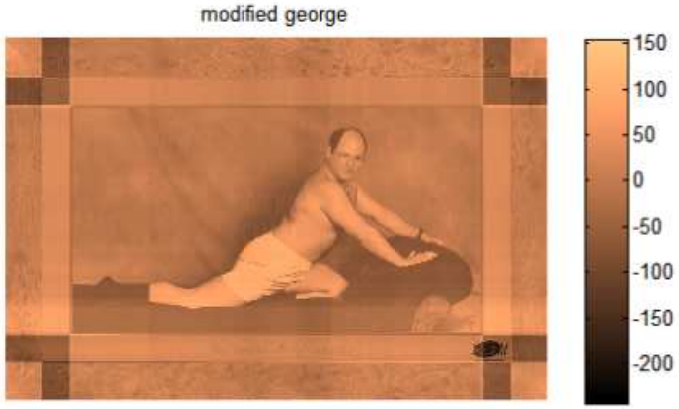




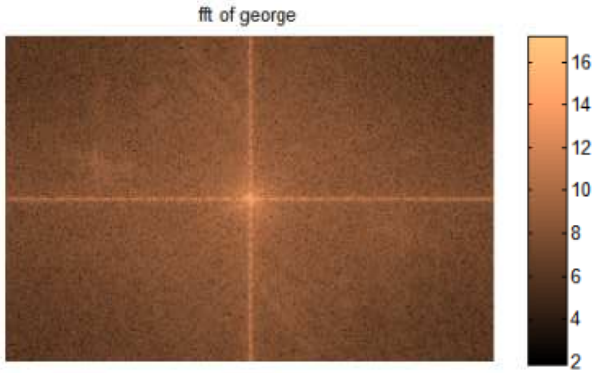
What if we zero out both the vertical and horizontal DC components ?

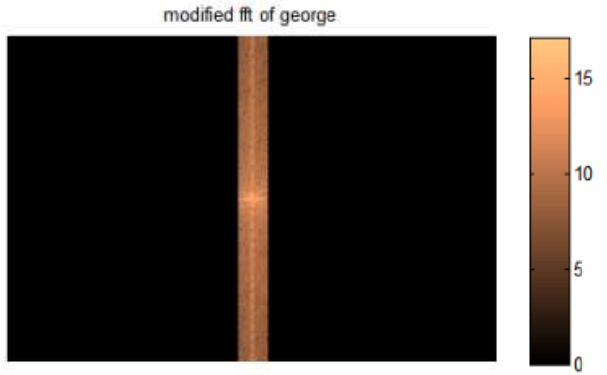


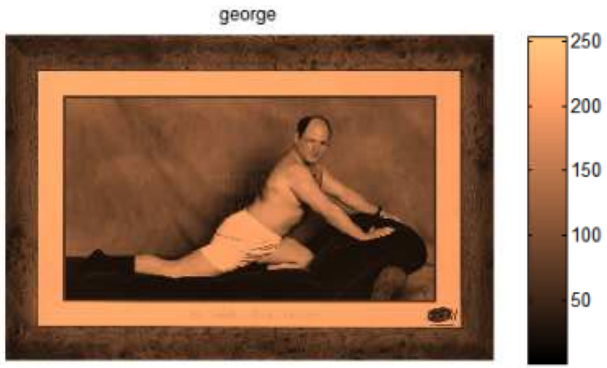




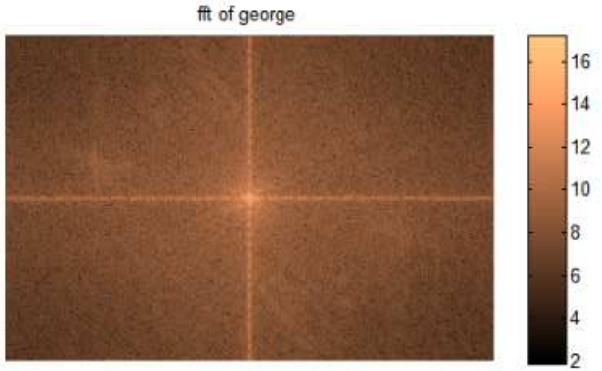
What happens if we set F[k1,k2]=0 whenever k2 > c or k2 < -c
for some c? (This corresponds to zeroing out high horizontal frequencies.)

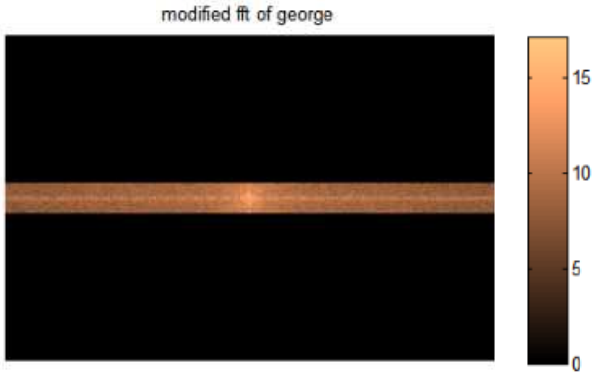


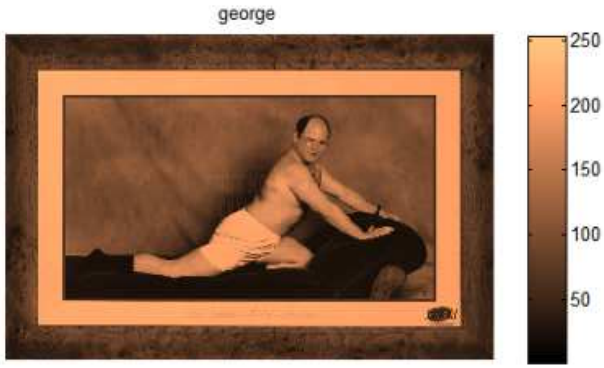


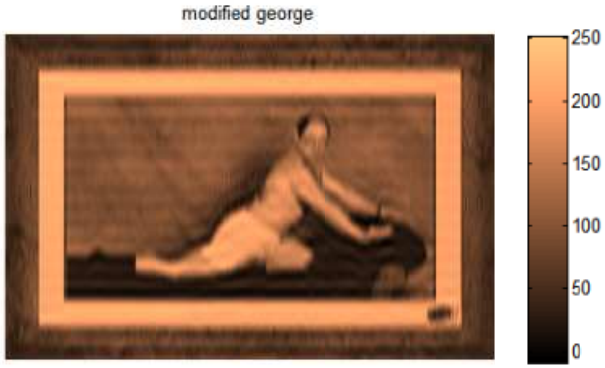


What happens if we set F[k1, k2]=0 whenever k1 > c or k1 < -c for some c? (This

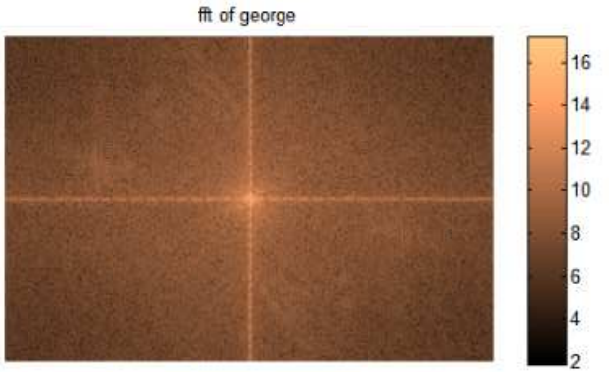


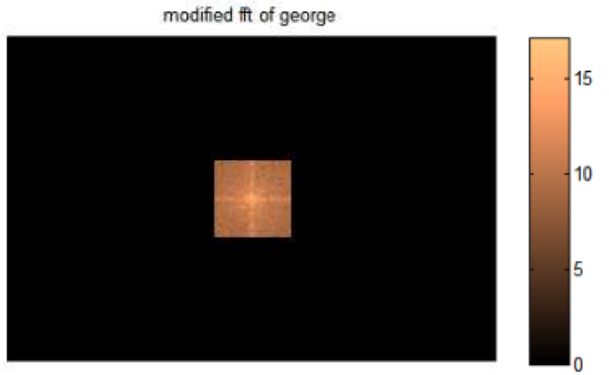


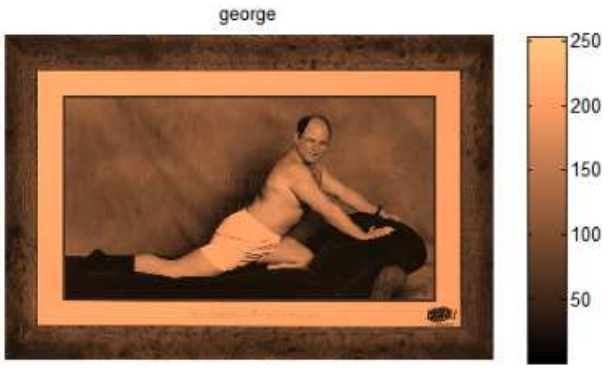




What happens if we zero out high vertical and horizontal







What happens if we zero out low vertical and horizontal