MIXWILD: A new freeware program for multilevel statistical modeling of intensive longitudinal data

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MixWild - mixed models with intensive longitudinal data

Stage 1 models for an occasion-varying continuous outcome

- Mixed-effects Location Scale Model (MIXREGLS)
  - mixed-effects (aka multilevel or hierarchical linear) model with random subject intercept (location effect) and random subject WS variance (scale effect)

- Mixed-effects Multiple Location Scale Model (MIXREGMLS)
  - mixed-effects (aka multilevel or hierarchical linear) model with multiple random subject location effects (intercept and slope) and random subject WS variance (scale effect)

$\Rightarrow$ is there only a random subject intercept (MIXREGLS) or random subject intercepts and slopes (MIXREGMLS)?

Stage 2 models (optional)

$\Rightarrow$ Stage 1 random subject effect estimates (e.g., intercept, slope, scale) and other subject-level variables can be used as regressors and interaction terms to predict a Stage 2 subject-level outcome

- Multiple regression for continuous subject-level outcome
- Logistic multiple regression for binary/ordinal subject-level outcome

Mixed-Effects Location Scale Models for EMA data

- Hedeker, Mermelstein, & Demirtas (2008). An application of a mixed-effects location scale model for analysis of Ecological Momentary Assessment (EMA) data. *Biometrics, 64*, 627-634.
- Hedeker, D., Mermelstein, R.J., & Demirtas, H. (2012). Modeling between- and within-subject variance in EMA data using mixed-effects location scale models. *Statistics in Medicine, 31*, 3328-3336.
- Maher JP, Nordgren R, Huh J, Chou CP, Hedeker D, & Dunton GF. (in press). Do fluctuations in positive affective and physical feeling states predict physical activity and sedentary time? *Psychol Sport Exerc*.
- Maher JP, Intille S, Hedeker D, & Dunton GF. (in press). Greater variability in daily physical activity is associated with poorer mental health profiles among obese adults. *Ment Health Phys Act*.

Multilevel (mixed-effects regression) model for measurement y of subject i $(i = 1, 2, \ldots, N)$ on occasion j $(j = 1, 2, \ldots, n_i)$

$$y_{ij} \;=\; \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \upsilon_i + \epsilon_{ij}$$

$\boldsymbol{x}_{ij} = p \times 1$ vector of regressors (including a column of ones)

$\boldsymbol{\beta} = p \times 1$ vector of regression coefficients

$\upsilon_i \sim N(0, \sigma^2_\upsilon)$ BS variance; how homogeneous/heterogeneous are subjects?

$\epsilon_{ij} \sim N(0, \sigma^2_\epsilon)$ WS variance; how consistent/erratic are the data within subjects?

Model with no covariates: $y_{ij} = \beta_0 + \upsilon_i + \epsilon_{ij}$

- υ_i is subject's mean (deviation from β_0)
  - if subjects are alike, $\upsilon_i \approx 0$ and σ_υ^2 will approach 0
  - if subjects are different, $\upsilon_i \neq 0$ and σ_υ^2 will increase from 0

  $\Rightarrow$ magnitude of σ_υ^2 indicates how different subjects are from each other (heterogeneity)

- ϵ_{ij} is subject i's error at time j (deviations from their mean)
  - if subjects are all well-fit, $\epsilon_{ij} \approx 0$ and σ_ϵ^2 will approach 0
  - if subjects are not well-fit, $\epsilon_{ij} \neq 0$ and σ_ϵ^2 will increase from 0

  $\Rightarrow$ magnitude of σ_ϵ^2 indicates how data vary within subjects (erraticism)

Log-linear models for variances

$$\text{BS variance} \qquad \sigma^2_{\upsilon_{ij}} \;=\; \exp(\boldsymbol{u}'_{ij}\boldsymbol{\alpha}) \qquad \text{or} \quad \log(\sigma^2_{\upsilon_{ij}}) \;=\; \boldsymbol{u}'_{ij}\boldsymbol{\alpha}$$

$$\text{WS variance} \qquad \sigma^2_{\epsilon_{ij}} \;=\; \exp(\boldsymbol{w}'_{ij}\boldsymbol{\tau}) \qquad \text{or} \quad \log(\sigma^2_{\epsilon_{ij}}) \;=\; \boldsymbol{w}'_{ij}\boldsymbol{\tau}$$

- $\boldsymbol{u}_{ij}$ and $\boldsymbol{w}_{ij}$ include covariates (and $\mathbf{1}$)
- subscripts i and j on variances indicate that these change depending on covariates $\boldsymbol{u}_{ij}$ and $\boldsymbol{w}_{ij}$ (and their coefficients)
- exp function ensures a positive multiplicative factor, and so resulting variances are positive

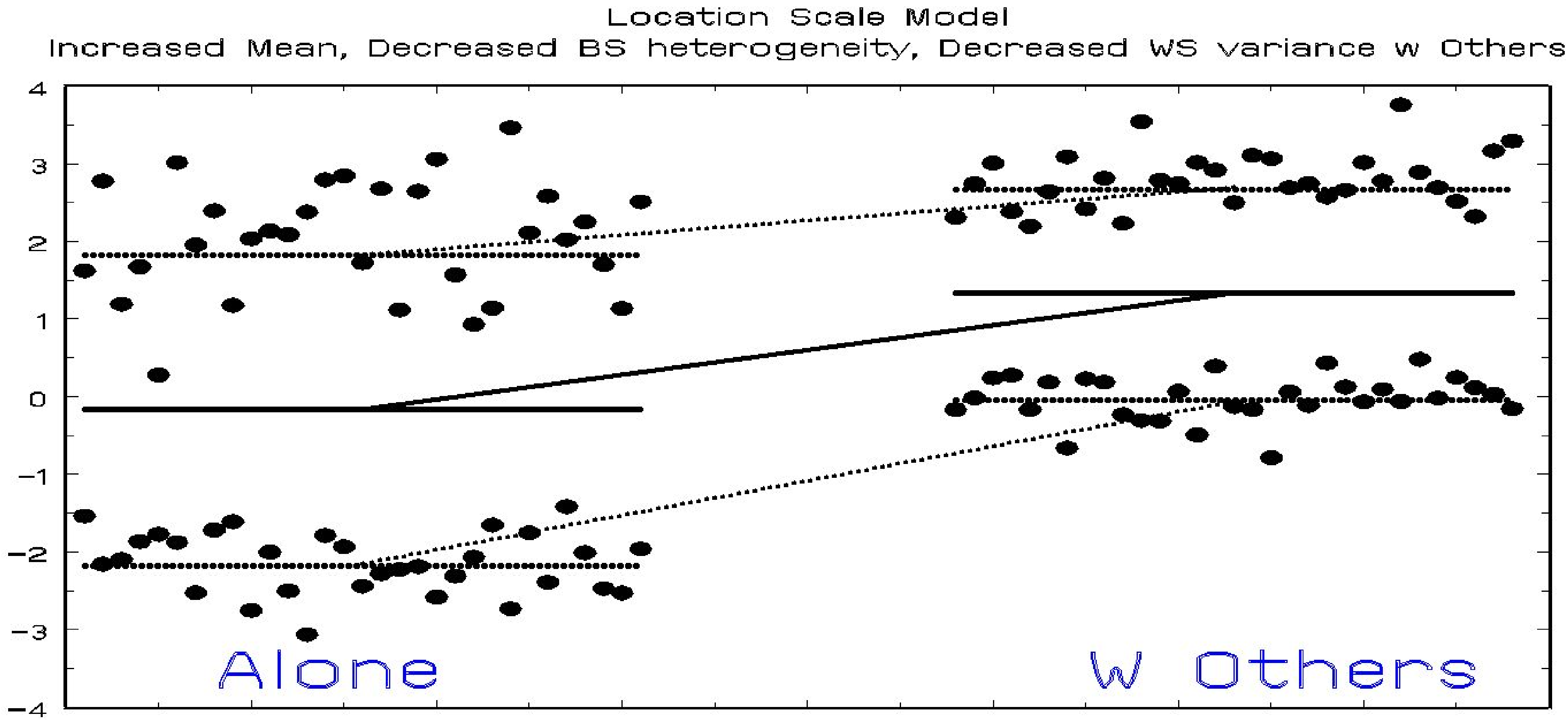
How can WS variables influence BS variance?

$$\sigma^2_{\upsilon_{ij}} = \exp(\boldsymbol{u}'_{ij}\boldsymbol{\alpha})$$

- Do rainy days and Mondays get everyone down?
- Is Tuesday just as bad as Stormy Monday for all?
- Are all kids happy on the last day of school?

Example: strong positive effect of being alone on BS variance of positive and negative mood

$\Rightarrow$ being alone increases subject heterogeneity (or, subjects report more similar mood when with others)



- Means are increased with others
- Subjects are more similar to each other when with others (BS var)
- Within-subject data are more consistent with others (WS var)

WS variance varies across subjects

$$\sigma^2_{\epsilon_{ij}} = \exp(\boldsymbol{w}'_{ij}\boldsymbol{\tau} + \omega_i) \quad \text{where} \quad \omega_i \sim N(0, \sigma^2_\omega)$$

$$\log(\sigma^2_{\epsilon_{ij}}) = \boldsymbol{w}'_{ij}\boldsymbol{\tau} + \omega_i$$

- ω_i are log-normal subject-specific perturbations of WS variance
- ω_i are "scale" random effects - how does a subject differ in terms of the variation in their data
- υ_i are "location" random effects - how does a subject differ in terms of the mean of their data

Multilevel model of WS variance

$$\log(\sigma^2_{\epsilon_{ij}}) \;=\; \boldsymbol{w}'_{ij}\boldsymbol{\tau} + \omega_i$$

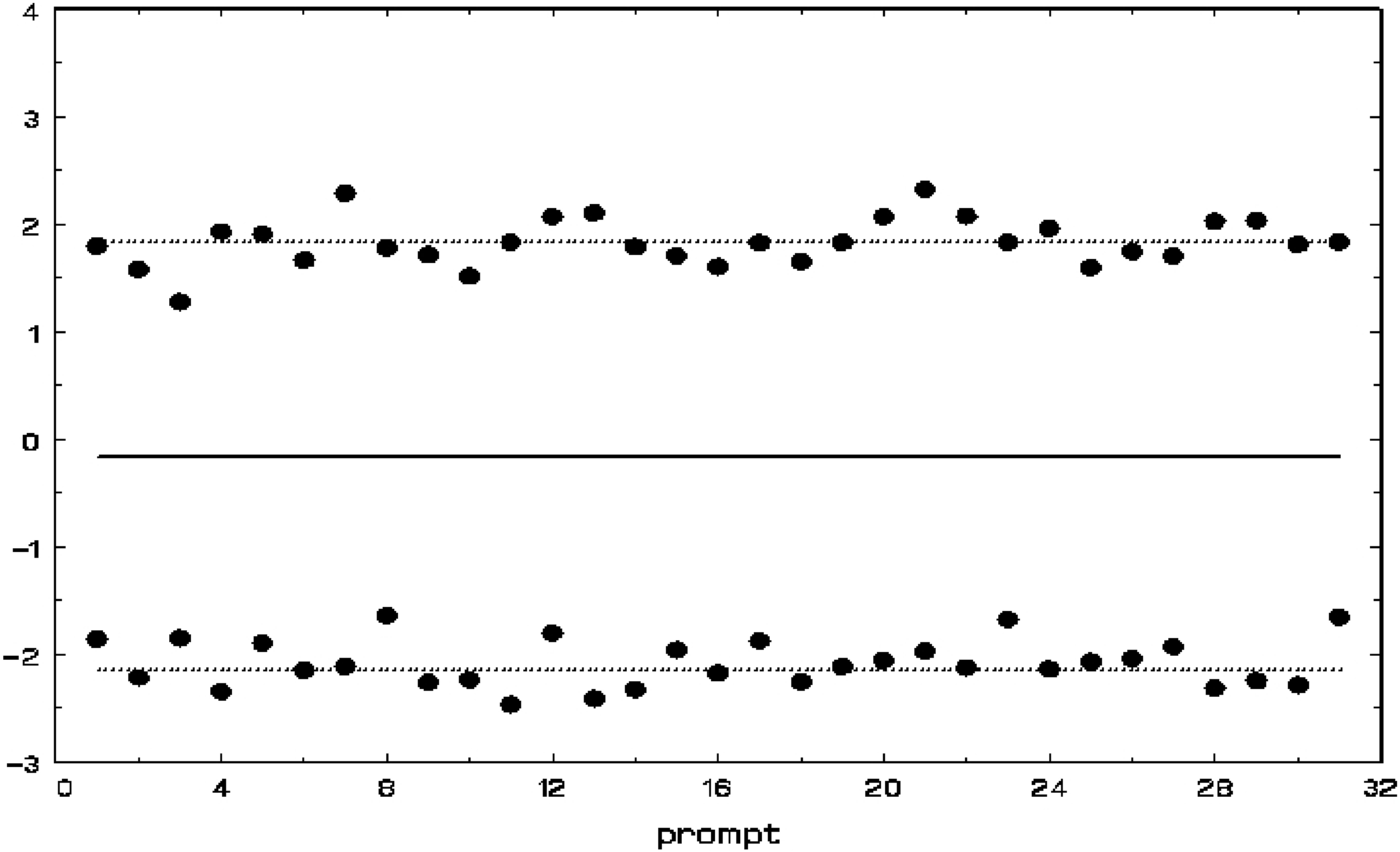
Why not use some summary statistic per subject (say, calculated subject standard deviation S_{y_i}) in a second-stage model?

$$S_{y_i} = \boldsymbol{x}'_i\boldsymbol{\beta} + \epsilon_i$$

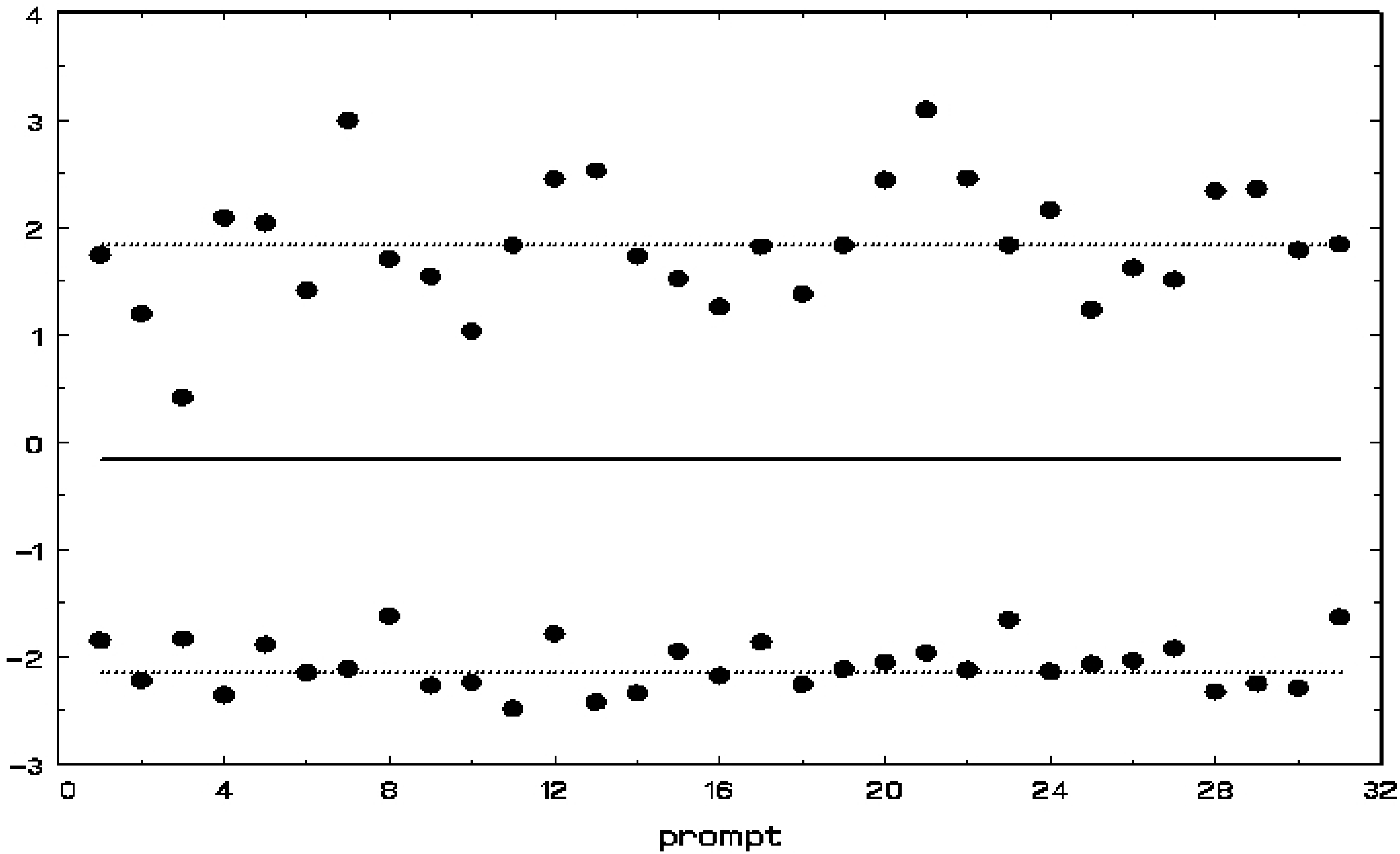
latter approach

- treats all standard deviations as if they are equally precise (but some might be based on 2 prompts or 40 prompts)
- does not recognize that these are estimated quantities (underestimation of sources of variation)
- does not allow occasion-varying predictors

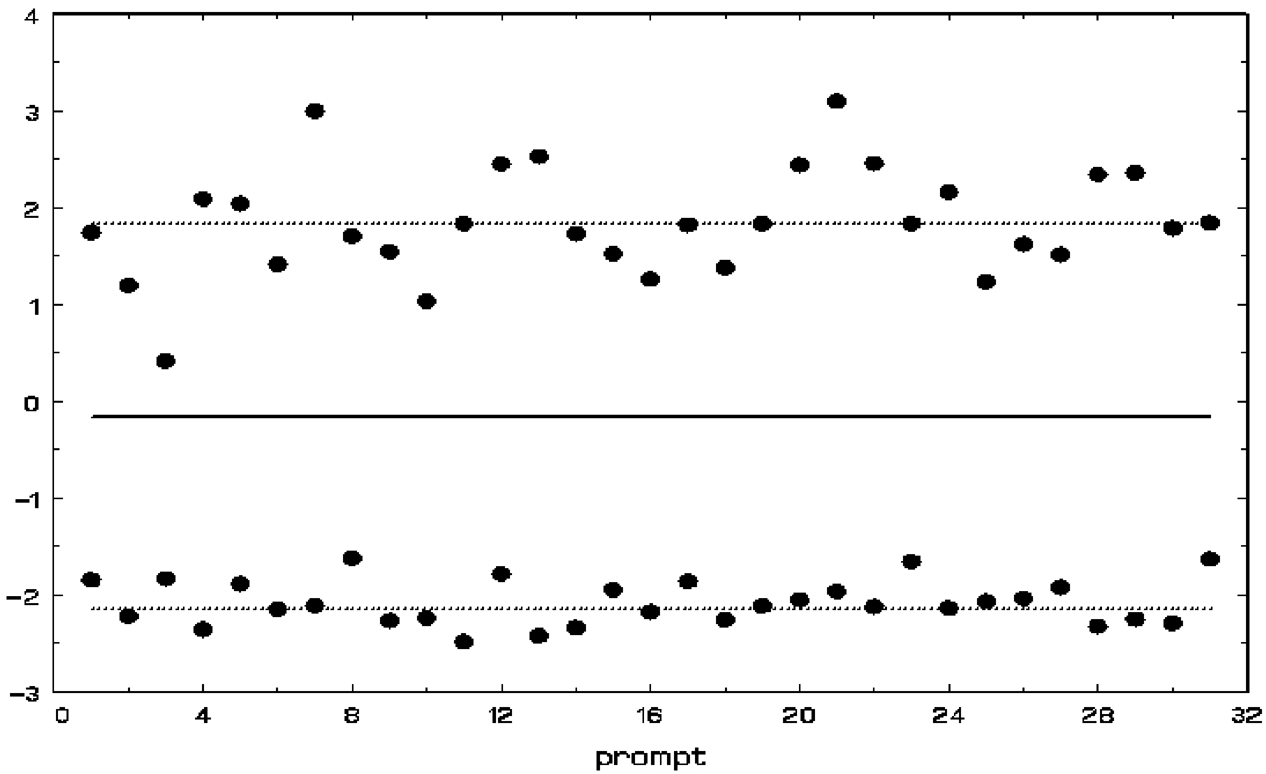
$\Rightarrow$ We use multilevel models for mean response, why not for variance?



Location random effects for two subjects



Location and scale random effects for two subjects



Model allows covariates to influence

- mean: level of solid line
- BS variance: dispersion of dotted lines
- WS variance: dispersion of points

additional random subject effects on: mean and WS variance

MIXREGLS and beyond

- Stand-alone program that implements the mixed-effects location scale model for continuous normal outcomes

  Hedeker, D. & Nordgren, R. (2013). MIXREGLS: A program for mixed-effects location scale analysis. *Journal of Statistical Software, 52(12)*, 1-38.

- Download MIXREGLS zip file via
  https://hedeker-sites.uchicago.edu/
  includes program executable file, manual, examples

- Now within MixWild GUI for both Mac and Windows
  - extended to allow for multiple random location effects (MIXREGMLS; random intercept, slopes, and scale)
  - random effects from Stage 1 analysis can be used to predict a Stage 2 subject-level outcome (either continuous or binary/ordinal)

Mixed-Effects Multiple Location Scale Models (MIXREGMLS)

$$y_{ij} = (\beta_0 + \upsilon_{0i}) + (\beta_1 + \upsilon_{1i})X_{ij} + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \epsilon_{ij}$$

$i = 1, 2, \ldots, N$ subjects $\quad j = 1, 2, \ldots, n_i$ occasions

BS variance

$$\begin{bmatrix} \upsilon_{0i} \\ \upsilon_{1i} \end{bmatrix} \sim N\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} \sigma^2_{\upsilon_0} & \sigma_{\upsilon_0\upsilon_1} \\ \sigma_{\upsilon_0\upsilon_1} & \sigma^2_{\upsilon_1} \end{bmatrix}\right\}$$

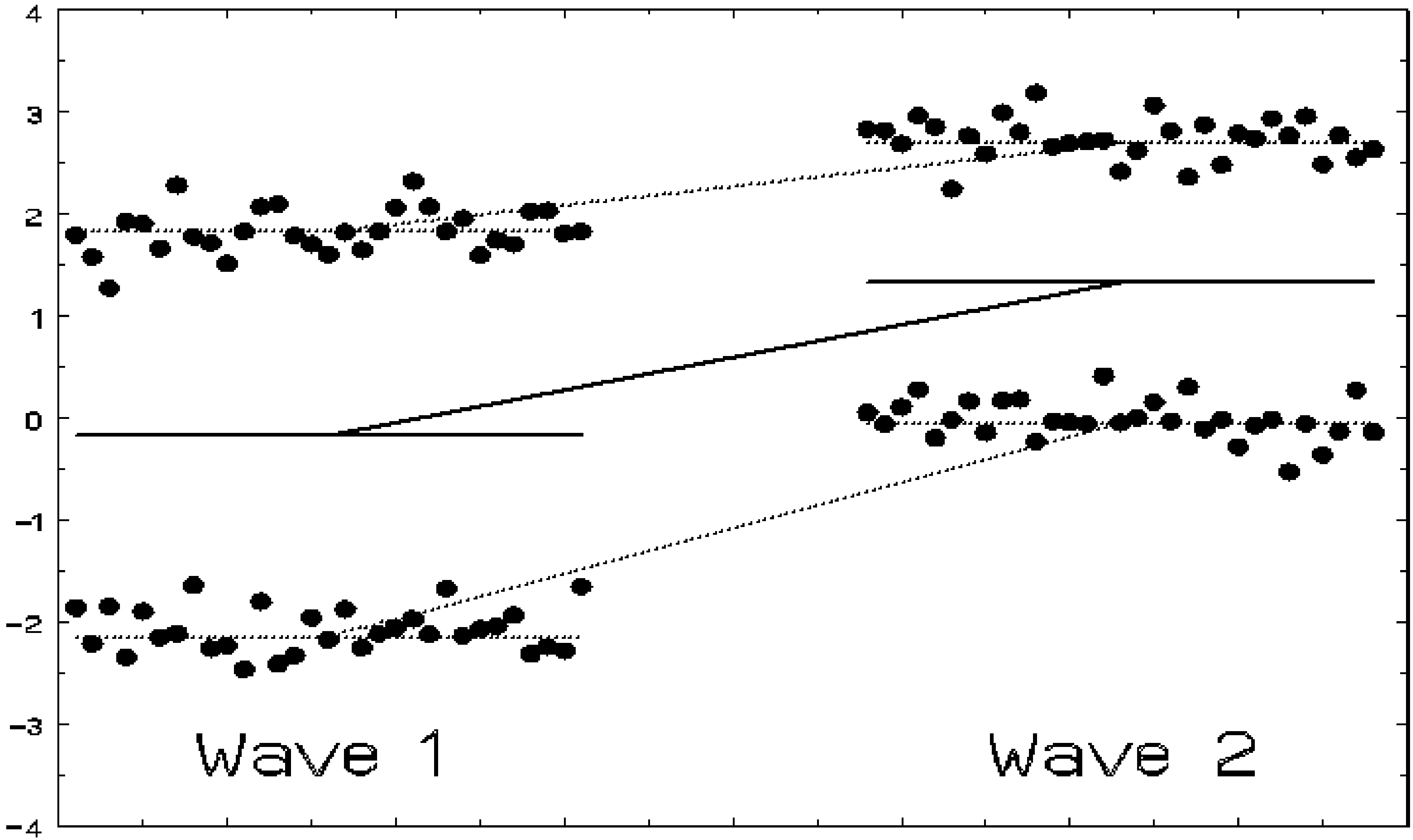
WS variance $\epsilon_{ij} \sim N(0, \sigma^2_{\epsilon_{ij}})$

$$\sigma^2_{\epsilon_{ij}} = \exp(\boldsymbol{w}'_{ij}\boldsymbol{\tau} + \omega_i) \quad \text{where} \quad \omega_i \sim N(0, \sigma^2_{\omega})$$

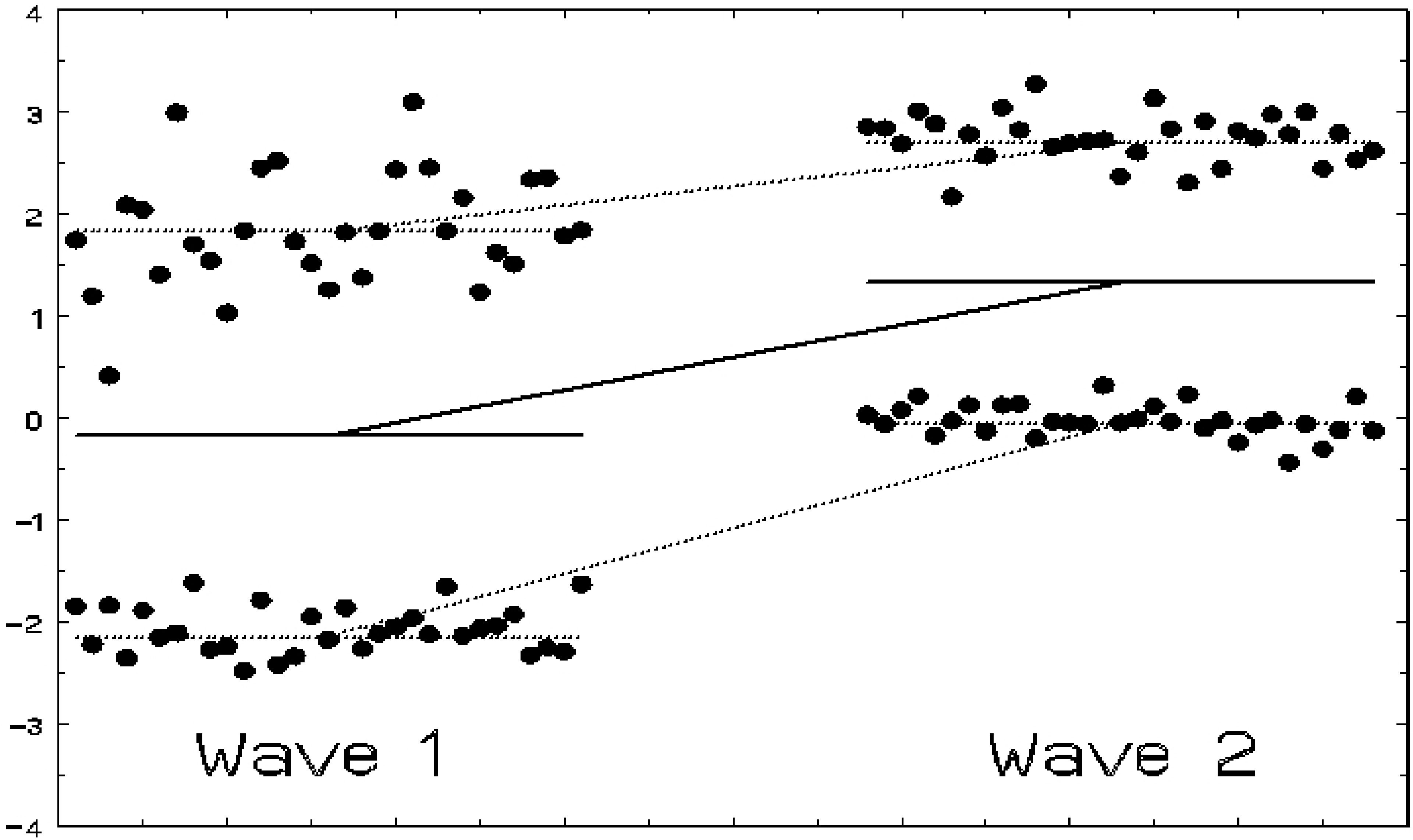
All random effects please rise!

$$\begin{bmatrix} \upsilon_{0i} \\ \upsilon_{1i} \\ \omega_i \end{bmatrix} \sim N \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \sigma^2_{\upsilon_0} & \sigma_{\upsilon_0 \upsilon_1} & \sigma_{\upsilon_0 \omega} \\ \sigma_{\upsilon_0 \upsilon_1} & \sigma^2_{\upsilon_1} & \sigma_{\upsilon_1 \omega} \\ \sigma_{\upsilon_0 \omega} & \sigma_{\upsilon_1 \omega} & \sigma^2_{\omega} \end{bmatrix} \right\}$$

- multiple subject location random effects (i.e., intercept and slope) and random subject scale
- can have more than 2 random location effects (i.e., multiple random slopes), but estimation time is increased with each additional random effect



- population intercept and trend (solid line)
- random intercept and trend for 2 subjects (dotted lines)
- error variance is the same



- population intercept and trend (solid line)
- random intercept and trend for 2 subjects (dotted lines)
- error variance varies across time and subjects (random scale)

Dataset

- CSV file; first line with variable names, next lines with variables separated by one or more blank spaces
- Numeric information only
  - no characters at all (e.g., use 0/1 instead of M/F)
  - no hidden characters or word processing format codes
  - missing values should have a numeric coding of -99, for example
- Data should be sorted by the level-2 ID (subject ID for longitudinal data, or cluster ID for clustered data)
- Data usually contain level-2 ID, dependent variable, covariates (the order of variables does not matter)

Stage 1 analysis

`MIXREGLS` fits and lists results for three submodels

Submodel one : mean effects and BS variance covariate effects

Submodel two : + WS variance covariate effects

Submodel three : + random scale and location/scale association

`MIXREGMLS` fits and lists results for four submodels

Submodel one : mean effects and BS variance-covariance matrix

Submodel two : + WS variance covariate effects

Submodel three : + random scale effects

Submodel four : + associations of random location and scale

Stage 2 analysis (optional, but hopefully useful)

Stage 1 random subject effect estimates (e.g., intercept $\hat{\upsilon}_{0i}$, slope $\hat{\upsilon}_{1i}$, scale $\hat{\omega}_i$) and other subject-level variables $\boldsymbol{x}_i$ can be used as regressors and interaction terms to predict a Stage 2 subject-level outcome y_i

- Multiple regression for continuous subject-level outcome

$$y_i = \beta_0 + \beta_1 \hat{\upsilon}_{0i} + \beta_2 \hat{\upsilon}_{1i} + \beta_3 \hat{\omega}_i + \boldsymbol{x}_i' \boldsymbol{\beta} + \varepsilon_i$$

- Logistic regression for binary/ordinal subject-level outcome

$$\log \left[\frac{P(y_i = 1)}{1 - P(y_i = 1)} \right] = \beta_0 + \beta_1 \hat{\upsilon}_{0i} + \beta_2 \hat{\upsilon}_{1i} + \beta_3 \hat{\omega}_i + \boldsymbol{x}_i' \boldsymbol{\beta} + \varepsilon_i$$

Since the random subject effects are estimates with estimated uncertainty, "plausible value" replications of the the random effects are performed (Mislevy, 1991, *Psychometrika*); akin to multiple imputation for missing values.