Gestures Reveal Mental Models of Discrete and Continuous Change

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Abstract

In studies of analogical transfer, subjects sometimes fail to recognize that problems are structurally isomorphic because of differences in the problems' content. One potential explanation for this finding is that differences in content lead subjects to infer that the problems have different structures. This interpretation would be supported by evidence that subjects construct differing mental models for structurally isomorphic problems. In this study, we show that subjects' gestures reveal their mental models of problems that involve discrete and continuous change. Four subjects talked out loud as they solved a set of four problems that involved constant change. All subjects produced gestures as they spoke, and their gestures revealed both continuous and discrete mental models of the manner of constant change. On problems constructed to evoke mental models of continuous change, subjects tended to produce gestures that incorporated smooth, continuous motions. On problems constructed to evoke mental models of discrete, incremental change, subjects tended to produce gestures that incorporated repeated, sequential, discrete motions. Subjects' gestures sometimes provided more explicit cues to their mental models than did their speech. The results indicate that subjects sometimes constructed differing mental models for structurally analogous problems.

Introduction

People often fail to recognize that a previously learned solution procedure can be used to solve a problem at hand. For example, in studies of analogical transfer, subjects sometimes fail to recognize that problems are structurally isomorphic because of differences in the content or "cover stories" of the problems (Gick & Holyoak, 1980; Holyoak & Koh, 1987). One potential explanation for this finding is that differences in content lead subjects to infer that the problems have different structures (Bassok & Olseth, 1995; Bassok, Wu, & Olseth, 1995). This interpretation would be supported by evidence that subjects construct distinct mental models for structurally isomorphic problems. In this paper, we describe a new method for gaining access to subjects' mental models of problems. This method involves examining the gestures that subjects spontaneously produce when they describe their reasoning about the problems.

It is widely accepted that verbal protocols can provide an accurate and informative account of ongoing cognitive processes (Ericsson & Simon, 1993). As we report in this paper, when subjects provide such protocols, they often gesture spontaneously as they speak. In recent years, several investigators have claimed that, like speech, spontaneous gestures are a window through which speakers' mental processes can be viewed (Goldin-Meadow, Alibali, & Church, 1993; Kendon, 1980; McNeill, 1985; McNeill, 1992). Such studies have shown that, much of the time, the information that speakers express in gestures is identical to the information they express in speech. When gestures match speech in this way, they can provide additional validation for claims made on the basis of subjects' speech. At other times, however, the information that speakers express in gestures differs from the information they express in speech (Church & Goldin-Meadow, 1986; McNeill, 1992; Perry, Church, & Goldin-Meadow, 1988). When gestures mismatch or supplement speech, they can provide information about cognitive processes that speakers do not verbalize.

If gestures do in fact yield evidence about subjects' mental models of problems, this evidence may actually be more compelling than evidence gleaned from verbal protocols. This is because, when problems are presented in words, the words of the text can influence the words subjects use when describing and solving the problems in speech. Spontaneous gestures, in contrast, are not "modeled" for subjects when a problem is presented as text. Thus, if subjects' gestures reveal mental models, they will provide compelling evidence that subjects have actively constructed such models.

There is reason to believe that a study of the spontaneous gestures subjects produce during verbal protocols will indeed be fruitful. Previous studies have shown that most people spontaneously gesture when they explain their reasoning about a problem. Moreover, gestures produced during problem explanations typically convey specific aspects of the problems being explained, or specific strategies for solving those problems. Such substantive gestures have been observed in a variety of age groups and on a variety of tasks. These include preschoolers and elementary school children reasoning about counting tasks (Graham, 1994), Piagetian conservation tasks (Church & Goldin-Meadow, 1986), mathematical equivalence problems (Alibali & Goldin-Meadow, 1993; Perry, et al., 1988), and seasonal change problems (Crowder & Newman, 1993); adolescents reasoning about Piagetian bending rods tasks (Stone, Webb, & Mahootian, 1991); adults reasoning about problems involving gears (Perry & Elder, 1995); and subjects of many age groups reasoning about moral dilemmas (Church, Schonert-Reichl, Goodman, Kelly, & Ayman-Nolley, 1995; Goodman, Church, & Schonert, 1991). Thus, gesture has proven to be a valuable tool for studying cognitive processes at a variety of developmental stages, and in a variety of domains. In the present study, we examined the gestures that adult subjects spontaneously produced when solving word problems that involve constant change (e.g., changes in speed or rate of population growth).

Bassok and Olseth (1995) have argued that, when solving constant change problems, subjects construct mental models of the *manner* in which the entity in the problem changes (i.e., continuously or discretely). They taught subjects a procedure for solving a problem in which an entity changed either continuously (e.g., speed) or discretely (e.g., monthly investments in a savings account). Subjects then attempted to solve analogous transfer problems in which the manner of change differed. Subjects frequently and spontaneously transferred solution procedures learned for discrete change problems to continuous change problems; however, they rarely transferred solution procedures learned for continuous change problems to discrete change problems. Based on these patterns of transfer, Bassok and Olseth argued that subjects constructed differing mental models of the process of change for discrete and continuous problems.

The present study seeks to establish a new and more direct method for assessing the mental models that subjects construct when solving problems that involve constant change. The goal of the study was to investigate whether subjects' spontaneous gestures provide direct evidence that they construct different mental models for problems involving discrete and continuous change.

Method

Participants

Participants were four University of Chicago students, 2 males and 2 females. All were native English speakers. Each was paid $6 for participating.

Procedure

Each subject solved a set of four structurally isomorphic word problems that involved constant change (see Table 1). The set consisted of two problems that were constructed to evoke models of continuous change (Continuous problems), and two that were constructed to evoke models of discrete change (Discrete problems). Each subject was presented with the word problems on paper, one at a time. For each problem, the subject was asked to read the problem aloud, and then to talk out loud as he or she solved the problem. The session was videotaped so that subjects' verbal protocols and gestures could later be analyzed.

Table 1
Problems Used in the Study

Continuous Problems

It takes 35 minutes to inflate a hot air balloon. The rate at which the hot air is pressed into the balloon increases steadily from 10 liters/minute at the beginning of the first minute to 80 liters/minute at the end of the 35th minute. How many liters of hot air are pressed into the balloon over the 35 minute period?

The speed of an airplane increases at a constant rate during a period of 12 minutes from 10 miles/minute to 34 miles/minute. What distance, in miles, will the plane travel during the 12 minute period?

Discrete Problems

A bookcase has 6 shelves. The number of books on each successive shelf from top to bottom increases by a constant from the number of books on the shelf above it. If there are 15 books on the top shelf and 45 books on the bottom shelf, how many books total are in the bookcase?

For a lecture, 10 rows of chairs have been arranged in a lecture hall. The chairs have been set up such that the number of chairs in each row increases by a constant from the number of chairs in the previous row. If there are 25 chairs in the first row and 115 chairs in the 10th row, how many chairs total are there in the lecture hall?

Coding

Each subject's verbal protocol was broken into clauses. Each clause of the verbal protocol and each spontaneous gesture were coded as described in the following sections.

For clauses that were accompanied by gestures, the relationship between gesture and speech was evaluated.

Coding speech. Each verbal clause was coded in terms of whether it indicated a specific manner of change, and if so, which type. Thus, each clause was classified as providing cues for a Continuous model, a Discrete model, or Neither model. The following verbal cues were taken as evidence of a Continuous mental model: (1) mention of the values in the problem using rate-like units; (2) reference to the entire period of time involved in the problem; (3) references to averaging and/or multiplying; (4) explicit references to rates. The following verbal cues were taken as evidence of a Discrete mental model: (1) mention of the values in the problem using amount-like units; (2) reference to the individual units of time involved in the problem; (3) references to dividing and/or repeated addition; (4) explicit references to "the constant". Note that many of these verbal cues are also present in the problem texts. Coding criteria and examples are presented in Table 2.

Table 2
Coding Categories and Examples

Continuous	Discrete
Speech Codes	
"It started at 10 liters per minute"	"At first there were 10 liters"
"Over the 12-minute period..."	"In each of the 12 minutes..."
"The average is 70, times 10 rows"	"90 divided by 9 is 10, so you have to keep adding 10"
"The number of books was going up at a constant rate"	"The constant was 6"
Gesture Codes	
Right hand palm arcs smoothly upward in neutral space in front of subject.	Right hand point, taps table four times, moving left to right.
Right hand point sweeps diagonally across table top away from self.	Right hand palm makes three short hops extending from self into neutral space.

Coding gesture. The stream of manual movement was broken down into individual gestures using criteria developed in previous work (Church & Goldin-Meadow, 1986). The handshape, motion, hand placement, and hand orientation used in each gesture were transcribed without access to the audio portion of the tape (i.e., with the sound turned off). Each gesture was then classified as conveying a Continuous model, a Discrete model, or Neither model. Gestures that incorporated a smooth, continuous motion (e.g., sweeping, arcing, dragging, sliding or lifting) were coded as Continuous. Gestures that incorporated a sequence of discrete movements (e.g., a sequence of at least 3 hops, taps, points, traces, beats, wrist rotations, or finger extensions) were coded as Discrete. Gestures that incorporated a zig-zagging or spiraling motion that moved out from the body were also coded as Discrete. All other gestures were coded as indicating Neither model. These included simple beat (accent or emphasis) gestures, points, flicks, gestures that traced numbers on the table or in the air, and iconic (representational) gestures that did not convey the manner of change (e.g., gestures that depicted the range of values in a problem by indicating two locations in space). Examples of gestures that convey Continuous and Discrete models are presented in Table 2.

Coding the relationship between gesture and speech. For clauses that included both speech and gesture, and that conveyed a model in at least one of the two modalities, the relationship between gesture and speech was identified. This relationship was classified as one of four types: (1) Speech-Gesture Match, in which speech and gesture provide cues for the same model, (2) Speech Explicit, in which speech provided a cue for one of the models, but gesture did not, (3) Gesture Explicit, in which gesture provided a cue for one of the models, but speech did not, and (4) Speech-Gesture Mismatch, in which speech and gesture provided cues for different models.

Results

Did subjects gesture when they described how they solved the problems?

We first examined whether subjects produced gestures along with their verbal protocols. Indeed, all four subjects gestured on every one of the four problems. Subjects varied in how much they spoke, and in how often they produced gestures as well as speech. The total number of clauses subjects produced across the four problems ranged from 88 to 145 (M=124, SD=26.5), and the raw number of gestures produced ranged from 28 to 93 (M=56, SD=27.4). Subjects produced gestures as well as speech in an average of 44% of all clauses (range 32% to 64%, SD=14%).

Did subjects' speech convey continuous and discrete mental models of change?

Next, we examined whether subjects' verbal protocols provided cues for distinct continuous and discrete mental models of the process of change. We isolated the verbal clauses that were "marked" in the sense that speech conveyed a cue for one of the two models. We then examined whether subjects tended to use clauses marked by Discrete cues most often on the Discrete problems, and clauses marked by Continuous cues most often on the Continuous problems. Recall that the problem texts provided many such verbal cues.

As seen in Table 3, Panel A, subjects produced more clauses marked by Discrete cues than by Continuous cues when solving the Discrete problems. All four subjects followed the expected pattern, and the pattern differed significantly from chance performance ($t(3)>10.0$, $p<0.001$). Subjects also produced more clauses marked by Continuous cues than by Discrete cues when solving the Continuous problems, as seen in Panel B. Three of the four subjects showed the predicted pattern. The remaining subject (TM) produced predominantly clauses marked by Discrete cues, suggesting that she had constructed a discrete mental model for the continuous problems. Because TM showed a pattern opposite from the other subjects, the trend toward greater use of clauses marked by Continuous cues on the Continuous problems did not attain significance ($t(3)=2.14$, $0.05<p<0.10$, one-tailed).

Table 3
Proportion of Marked Clauses
that Contained Cues for Each Model

Subject	Continuous	Discrete	N (Marked Clauses)
A: Discrete Problems			
TM	0.03	0.97	35
CF	0.00	1.00	30
SW	0.04	0.96	26
GC	0.00	1.00	16
Mean Proportion	0.02	0.98	
B: Continuous Problems			
TM	0.36	0.64	22
CF	1.00	0.00	19
SW	1.00	0.00	21
GC	0.75	0.25	16
Mean Proportion	0.78	0.22	

Did subjects' gestures convey continuous and discrete mental models of change?

Next, we examined whether, like their speech, subjects' gestures also provided evidence for distinct continuous and discrete mental models of the process of change. Over the set of four problems, every one of the four subjects produced some gestures that conveyed each model. On average, 10% of each subject's total corpus of gestures conveyed a Continuous model (range 4% to 20%) and 19% of each subject's gestures conveyed a Discrete model (range 9% to 32%). The remaining gestures conveyed Neither model.

We isolated the gestures that were "marked" in the sense that they conveyed a cue for one of the two models. We then examined whether subjects tended to use Discrete gestures most often on Discrete problems, and Continuous gestures most often on Continuous problems. As seen in Table 4, Panel A, subjects were indeed more likely to produce Discrete gestures when solving the Discrete problems. All four subjects followed the predicted pattern. Even given the small sample size, the pattern differed significantly from chance ($t(3)=9.76$, $p<0.001$, one-tailed). Subjects were also more likely to produce Continuous gestures when solving the Continuous problems, as seen in Panel B. Indeed, three of the four subjects produced absolutely no Discrete gestures when solving the Continuous problems. The remaining subject (TM) produced predominantly Discrete gestures. Thus, like her speech, TM's gestures indicate that she constructed a discrete mental model for the continuous problems. In fact, her gestures reveal the pattern more strongly than her speech. Because TM's pattern differed from the other subjects, the trend toward greater use of Continuous gestures for the Continuous problems did not attain significance ($t(3)=1.27$, $.05<p<0.15$, one-tailed).

Table 4
Proportion of Marked Gestures
that Conveyed Each Model

Subject	Continuous Gestures	Discrete Gestures	N (Marked Gestures)
A: Discrete Problems			
TM	0.12	0.88	17
CF	0.17	0.83	6
SW	0.27	0.73	11
GC	0.17	0.83	6
Mean Proportion	0.18	0.82	
B: Continuous Problems			
TM	0.12	0.88	17
CF	1.00	0.00	1
SW	1.00	0.00	6
GC	1.00	0.00	3
Mean Proportion	0.78	0.22	

We also examined whether individual subjects differentiated between the two types of problems in their gestures. As noted, subject TM did not systematically differentiate between the two types of problems; she appeared to represent both problems as involving discrete change. Subject SW, in contrast, tended to produce Continuous gestures on Continuous problems, and Discrete gestures on Discrete problems ($\chi 2=8.24$, $p<.001$). The remaining two subjects also showed the predicted pattern, but produced too few gestures to analyze at the individual level.

Did speech and gesture always convey the same mental model?

Next, we examined whether subjects tended to convey the same mental model in speech and in the accompanying gesture. Studies of children's cognitive development have shown that, during transitional periods in the process of acquiring concepts, children often express one strategy for solving a problem in speech, and a second strategy in the accompanying gesture (Alibali & Goldin-Meadow, 1993; Church & Goldin-Meadow, 1986; Perry, et al., 1988). Thus, when children consider multiple (potentially conflicting) strategies for solving problems, their speech and gestures often mismatch. We hypothesized that the adult subjects in our study might also sometimes consider multiple models for a given problem. Like the children in the studies described above, they might also produce gesture-speech mismatches when solving constant change problems. Alternatively, their gestures could provide evidence for a particular model when the accompanying speech did not. Finally, subjects' gestures could provide converging evidence for the model indicated in the accompanying speech.

Table 5
Relationship Between Speech and Gesture

Subject & Problem Type		Match	Speech Explicit	Gesture Explicit	Mis-match	N
TM	C	0.27	0.40	0.23	0.10	30
	D	0.30	0.48	0.21	0.00	33
CF	C	0.50	0.50	0.00	0.00	2
	D	0.19	0.77	0.04	0.00	26
SW	C	0.50	0.00	0.50	0.00	6
	D	0.38	0.48	0.05	0.10	21
GC	C	0.11	0.67	0.22	0.00	9
	D	0.50	0.00	0.50	0.00	6

C=Continuous, D=Discrete

To explore these possibilities, we identified the relationship between speech and gesture in all clauses in which at least one modality (i.e., either speech or gesture or both) provided a cue for one of the models. As seen in Table 5, subjects produced very few clauses in which speech conveyed one model and gesture conveyed the other (Mismatch responses). However, clauses in which gesture explicitly conveyed one of the models while speech conveyed no model were relatively frequent. Individuals differed in how frequently they produced such Gesture Explicit clauses: one subject produced almost none, while others produced them quite regularly. Two of the subjects (SW and GC) frequently produced such clauses for one problem type and not the other. Most interesting, on problems where these subjects frequently produced Gesture Explicit clauses, they never produced Speech Explicit clauses. This opens the possibility that for some individuals, mental models of some problem types may be more readily accessible to gesture than to speech. Alternatively, speakers may choose to express certain aspects of their mental models in speech, and other aspects in spontaneous gestures.

It is also worth noting that gesture often provided converging evidence for the mental model cued in the accompanying speech (i.e., in Match responses). In these responses, gesture provides independent verification that the subject is not simply mimicking the verbal cues provided in the problem text, but has actively constructed the model expressed in speech.

Discussion

This study has shown that subjects' spontaneous gestures can reveal their mental models of discrete and continuous change. All four of the subjects studied gestured as they solved constant change problems, and their gestures revealed both continuous and discrete mental models of the manner of constant change. On problems constructed to evoke mental models of continuous change, subjects tended to produce gestures that incorporated smooth, continuous motions, while on problems constructed to evoke mental models of discrete, incremental change, subjects tended to produce gesture that incorporated repeated, sequential, discrete motions. Most important, subjects' gestures sometimes provided more explicit cues to their mental models than did their speech. These results demonstrate that, in this domain as well as others, gesture can be a compelling source of data about ongoing cognitive processes.

These results also have implications for theories of analogical transfer. Such theories offer two different explanations for the finding that subjects sometimes fail to recognize that a previously learned solution procedure can be used to solve a problem at hand. The traditional account, sometimes called the *interference* account, holds that problem content interferes with subjects' ability to recognize that two problems share the *same* structure (Gentner, 1983, 1989; Gick & Holyoak, 1980). According to this view, subjects fail to transfer a solution procedure because they fail to recognize that the base and target problems are structurally the same. The *interpretative* account, in contrast, holds that differences in content lead subjects to infer that the problems have *different* structures (Bassok & Olseth, 1995). According to this view, subjects fail to transfer because they actively construct different mental models for the base and target problems. The two accounts differ crucially in terms of the processes they impute to problem solvers: The interference account holds that subjects simply *do not recognize* structural similarities, while the interpretative account holds that subjects *actively construct* differing mental models for isomorphic problems.

In this study, subjects' spontaneous gestures revealed that they had constructed different continuous and discrete models for structurally isomorphic constant change problems. Thus, these results support the claim that subjects actively construct different mental models for different types of problems. These findings suggest that the mental representations that subjects actively construct may indeed inhibit them from transferring solution procedures to analogous problems. In this way, evidence from gesture supports the interpretative explanation for the failure of analogical transfer.

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