

# CHAPTER 3

## *GESTURE'S ROLE IN LEARNING ARITHMETIC*

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Mathematical concepts are traditionally viewed as abstract and formal. However, recent research suggests that there is an embodied component to these concepts—that notions like infinity, continuity, and number are metaphorical and grounded in our experience of the world (Lakoff & Núñez, 2000). The evidence for this view has largely come from analyses of mathematical language. But the spontaneous gestures that speakers produce when they talk about math have also been used to argue that mathematical concepts are metaphorical and embodied (Núñez, 2008). For example, gesture can reveal that a speaker construes a problem dynamically even when the speaker's words focus exclusively on the problem's static notation (e.g., Marghetis & Núñez, 2010).

Perhaps, then, it is not surprising to find that children often gesture when talking about arithmetic, the foundational branch of mathematics. For example, toddlers routinely point when counting (e.g., Fuson, 1988; Gelman & Gallistel, 1978; Graham, 1999; Saxe, 1977), and school-age children often move their hands when explaining their solutions to mathematical equivalence problems (Perry, Church, & Goldin-Meadow, 1988). Gesture enables young speakers to use their bodies to instantiate number and simple operations on those numbers; for example, producing a sweeping gesture under a string of numbers to indicate that they should be added together.

But learners are not the only ones who gesture when talking about arithmetic. Parents frequently move their hands when counting sets with their young children (Suriyakham, 2007), and instructors frequently move their hands when teaching arithmetic to individual students (Goldin-Meadow, Kim, & Singer, 1999) and in their classrooms (Flevaris & Perry, 2001). Some gestures are simple deictic points (e.g., grounding a spoken number with a point to a numeral written on the board); others are more complex and representational (e.g., demonstrating division by two by holding up two hands in front of the body and then moving the hands away from each other).

Here we describe the pedagogical role that gesture can play in teaching and learning arithmetic. We focus on two arithmetic skills—counting and simple arithmetic operations. We first show that gesture can serve as a window into children's early arithmetic understanding; gesture provides an additional and often overlooked modality for discovering what children know about arithmetic. We then show that gesture can serve as an instructional tool in arithmetic lessons, both the gestures that learners produce themselves and the gestures they see their teachers produce. Gesture can thus not only be used to discover what children know about arithmetic but may also change what they know.

### **GESTURE REFLECTS WHAT CHILDREN KNOW ABOUT ARITHMETIC**

## Counting

Toddlers routinely use their fingers when counting, and their hand movements may not only enable children to count more accurately (e.g., Gelman & Gallistel, 1978) but may also reveal aspects of their early counting skills and understanding that are not apparent in their talk (e.g., Gelman & Gallistel, 1978; Saxe, 1977). Gelman and Gallistel (1978) found that 2-year-olds pointed on at least some trials when asked to count. Saxe (1977) examined the counting behaviors of 3- and 4-year-old children who were asked to compare and reproduce sets containing from 4 to 9 objects. At age 3, children either did not gesture or failed to coordinate the points they did produce with their number words. By age 4, children almost always produced pointing gestures while counting, and they did so successfully, in a one-to-one manner, on the majority of trials (63%). Broadly consistent with Saxe's findings, in a longitudinal study of parent-child naturalistic interactions, Suriyakham (2007) found that at 30 months, children produced more counting utterances without pointing gestures than with them; by 38 months, they had increased their pointing gestures, producing roughly equivalent numbers of counting utterances with and without gesture. Together these studies indicate that pointing while counting is a developmental advance. Although Suriyakham did not compare the accuracy of counts with and without gesture, other research findings suggest that counts accompanied by gesture are more accurate than counts without gesture (e.g., Alibali & DiRusso, 1999; Potter & Levy, 1968; Saxe & Kaplan, 1981).

Other studies have examined the specificity of children's pointing gestures, asking whether children point more when counting larger versus smaller sets and whether they point more when counting the elements in a set versus labeling the cardinal value of the set. Graham (1999) asked 2-, 3-, and 4-year-old children to count sets of two, four, and six object arrays. Not surprisingly, children use small numbers and count small sets before learning to use large numbers and count large sets. If children gesture primarily when the counting problem is hard, we might expect them to gesture more on large arrays (four and six objects) than on small arrays (two objects), which can be enumerated without counting (i.e., on sets than can be subitized; Gelman & Gallistel, 1978; Saxe & Kaplan, 1981; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2011). The 4-year-olds did just that (the 2- and 3-year-olds seemed to be challenged by all three arrays and gestured equally often on all of them). Thus, by age 4, children seem to be recruiting gesture when the counting task becomes difficult, indicating that they are not applying this behavior indiscriminately but rather to help them obtain the correct answer.

Suriyakham (2007) asked whether 30- and 38-month-olds use gesture differently when counting items in a set (e.g., "one, two, three") versus when labeling the cardinal value of items in the set (e.g., "I have three trucks"). At 30 months, children produced gesture equally often in counting versus cardinal contexts. However, at 38 months, they produced gesture more often in counting than in cardinal contexts, thus clearly differentiating between the two, an important step in the acquisition of the cardinal meaning of number words (e.g., Le Corre, Van de Walle, Brannon, & Carey, 2006; Wynn, 1990). Given prior research indicating that gesture often predicts and promotes verbal understanding (Goldin-Meadow, 2003), these findings raise the possibility that the children who use gesture differentially when counting, compared with when labeling set size, might be just those children who are ready to advance in their understanding of cardinal number.

Even when judging others' explanations of counting, children seem to rely on gesture. When children notice a speaker's counting words, they also notice that speaker's counting gestures but not vice versa. Graham (1999) asked very young children to "help" a puppet learn to count. Half the time the puppet counted correctly, but the other half the time the puppet added an extra number (e.g., the puppet would say "one, two three" while counting two objects). In addition, when the puppet made these counting errors, he either produced the same number of pointing gestures as number words (three in this example), a larger or smaller number of pointing gestures (four or two pointing gestures), or no pointing gestures at all. The child's job was to tell the puppet whether his counting was correct and, if incorrect, to explain why the puppet was wrong. The interesting result from the point of view of this discussion is whether children made reference to the puppet's number words (speech only), the puppet's points (gesture only), or both (gesture+speech) in their explanations. Two-year-olds did not refer to either gesture or speech, 3-year-olds referred to gesture but not speech (gesture only), and 4-year-olds referred to both gesture and speech (gesture+speech). Few children across all three ages referred to the puppet's counting words without also referring to the puppet's counting

gestures. In other words, when they noticed the puppet's words, they also noticed his gestures but not necessarily vice versa.

In summary, pointing while counting emerges early and, at first, is not perfectly coordinated with the number of objects in a set or with the number words the child produces. By age 3, children produce more pointing gestures while counting than while labeling the cardinality of sets (Suriyakam, 2007). By age 4, children selectively produce pointing gestures while counting—they point more when counting larger sets than smaller sets perhaps because they realize that pointing increases the accuracy of their count. Moreover, children notice the accuracy of others' points while counting even when they do not notice the accuracy of their own counting words. These findings suggest that pointing gestures that accompany counting reflect what the child knows about counting. The findings also indicate the fundamental and persistent embodied nature of our mathematical representations.

Going one step further, researchers have suggested that pointing gestures can also play a role in the development of counting. For example, pointing while counting may help children partition items into those counted versus not counted (Potter & Levy, 1968). Moreover, pointing while counting has been hypothesized to play a supportive role in the development of the fundamental counting principles, including one-to-one correspondence (i.e., that each number in the count sequence is paired with an item in the set; Gelman & Gallistel, 1978), the stable order principle (i.e., that the count words need to be uttered in a fixed order; Wiese, 2003), and the cardinal principle (i.e., that the last number in the count sequence indicates set size; Fayol & Seron, 2005).

Butterworth (1999) uses the fact that impaired finger representation (finger agnosia) is associated with imprecise numerical representations as support for the claim that fingers play a role in the development of counting. Finger gnosia (i.e., the ability to recognize fingers) is typically assessed by asking children or adults to indicate which finger has been touched or by asking them to determine whether two fingers were touched simultaneously or successively without visual access to this information (e.g., Fayol, Barrouillet, & Marinthe, 1998; Noël, 2005). Children's performance on a finger gnosis test at age 5 predicted their math skills (but not their reading skills) at age 8 and did a better job of predicting math skills than performance on intellectual tasks (Fayol et al., 1998; see also Marinthe, Fayol, & Barrouillet, 2001). The tight relation between finger gnosis and performance on math tasks could be due to the proximity of the neural areas that subserve these functions (both the parietal cortex and the precentral gyrus are involved in finger and number representations; Noël, 2005). Alternatively, fingers could be serving a functional role in numerical representations—under this view, poor finger gnosis impairs children's ability to use their fingers to count, which in turn impairs their ability to develop strong numerical skills (Gracia-Bafalluy & Noël, 2008). Finger-differentiation training has, in fact, been found to improve numerical skills in first-grade students who scored low on a finger gnosis test (Gracia-Bafalluy & Noël, 2008).<sup>1</sup>

A related but somewhat different research approach has asked whether preschool children are better at using and understanding finger representations for set size than using number word representations for set size (Crollen, Seron, & Noël, 2011; Nicoladis, Pika, & Marantette, 2010) perhaps because finger representations provide a more iconic representation (Wiese, 2003), which could facilitate arriving at the correct count. Nicoladis et al. (2010) tested this possibility in two studies: (a) they asked children to put a number of toys into a box, which was identified by a number word or a hand shape; and (b) they presented children with a set of toys and asked the children to tell them how many toys were in the set, using either a number word or a hand shape. In neither study was gesture found to have an advantage over number words. Although these studies do not show a gesture advantage, it is important to note that the tasks used are different from those used in the studies reported earlier, which examine children's spontaneous reliance on fingers during counting tasks. This difference suggests that children's spontaneous use of gesture while counting may reflect implicit knowledge about the connection between counting and cardinality rather than explicit knowledge that can be used to answer a "how many" question. As suggested by both Nicoladis et al. (2010) and Crollen et al. (2011), it is also possible that spontaneous use of gesture while counting goes beyond reflecting children's implicit numerical knowledge to help them develop explicit numerical knowledge.

Taken together, these findings raise the possibility that early pointing gestures not only reflect what

children know about early number but also play a role in creating that knowledge. To test this hypothesis, we need to experimentally manipulate gesture, a point to which we return in a later section.

## Arithmetic Operations

### *Early Calculation*

Levine, Jordan, and Huttenlocher (1992) examined how 4½- to 7-year-old children used their fingers while solving addition and subtraction problems presented in three different formats: (a) number fact problems—for example, “How much is  $m$  and  $n$ ?” “How much is  $m$  take away  $n$ ?”; (b) story problems—for example, “Mike had  $m$  balls; he got  $n$  more; how many balls did he have all together?” “Kim had  $m$  crayons; she lost  $n$ ; how many crayons did she have left?”; and (c) nonverbal number problems—for example, a set of disks was placed on a mat and then covered with a box; another set of disks was placed on the mat and the disks were slid under the box one by one through a small opening on the side of the box; the child was asked to lay out the number of disks that were under the box on the mat. A comparable procedure was used for the subtraction problems. For each of the calculation problem types, finger strategies were coded if children were observed counting on their fingers or if they held up their fingers for any term in a problem without overtly counting them.

Levine and colleagues (1992) hypothesized that using fingers when solving calculation problems could serve the same function as manipulatives; that is, it could help children carry out a counting strategy when they cannot retrieve the correct answer and also reduce the working memory load of mentally imagining the to-be-counted objects. They thus predicted that children would use fingers most often on the number fact problems that made no reference to objects, next most often on the story problems that referred to objects that were not physically present, and least often on the nonverbal number problems that referred to physically present objects. They found this ordering for finger strategies in the two oldest age groups, children ages 5 years 6 months to 5 years 11 months and children ages 6 years 0 months to 6 years 5 months (children below 5½ years of age rarely used their fingers when calculating). Moreover, for each problem type, the older age groups were more accurate when they used their fingers than when they did not (90% versus 78% for nonverbal problems, 83% versus 66% for story problems, 75% versus 54% for number fact problems). In fact, accuracy on the two harder types of problems (story problems and number fact problems) when fingers were used (83%, 75%) was as good as accuracy on the easiest problems (nonverbal problems) without gesture (78%). These findings suggest that using fingers on story problems and number fact problems can substitute for the presence of concrete objects and thus raise performance levels.

Subsequent studies showed a difference in how often middle- versus low-income kindergarten and first-grade children used their fingers when asked to solve calculation problems in the same three formats. Middle-income children used fingers more often than low-income children in kindergarten, and low-income children used fingers more often than middle-income children in first grade (e.g., Jordan, Huttenlocher, & Levine, 1992; Jordan, Levine, & Huttenlocher, 1994). Following up on these studies, Jordan, Kaplan, Ramineni, and Locuniak (2008) studied middle- and low-socioeconomic status (SES) children from the fall of kindergarten through the spring of second grade and examined how these children used their fingers to solve number fact problems and whether finger use correlated with problem-solving success over developmental time. Finger use decreased between kindergarten and second grade for middle-SES children but increased over this period for low-SES children. Moreover, consistent with the Levine et al. (1992) findings, using finger strategies in kindergarten was a strong, positive predictor of calculation success in second grade. By the end of second grade, finger use was a negative predictor of children’s calculation success.

Siegler and colleagues (e.g., Siegler & Jenkins, 1989; see also Beller & Bender, 2011) have shown that how children use their fingers when solving calculation problems at the least reflects, and may even scaffold, their calculation knowledge and fluency. Initially, children use the “count all” strategy, holding up the number of fingers represented by each addend of the problem (e.g., holding up two fingers, then another four, for the problem  $2 + 4$ ) and then counting all of the raised fingers (i.e., “one, two, three, four, five, six”). They then shift to the “count on” strategy, either counting on from the first addend (i.e., counting on from the 2,

“three, four, five, six”) or using the more efficient min strategy and counting on from the largest addend (i.e., counting on from the 4, “five, six”).

In summary, young children use their fingers to represent the cardinal value of sets and to count when solving calculation problems, particularly when other manipulatives are not available. The way in which they use their fingers changes over the course of development depending on their knowledge and the ease with which they can retrieve answers from memory. At early ages, children who use their fingers perform better on calculation problems than children who do not. In contrast, at later ages, children who use their fingers perform worse on calculation problems than children who do not most likely because the non-gesturers at this age are able to mentally represent the cardinal value of sets and retrieve the answers to calculation problems without counting (although the non-gesturers could also be recognizing the quantity without mentally representing the size of the sets; that is, they may simply have memorized the math facts and be using retrieval strategies to answer the problems).

### *Mathematical Equivalence*

One basic concept that children need to master, one that is surprisingly difficult, is the meaning of the equal sign. Children as old as 9 or 10 years often think that the equal sign is merely an instruction to add up all of the numbers in a problem and fill in the blank. This interpretation works well if the problem is a simple one laid out in a traditional format (as most problems in U.S. textbooks are; McNeil et al., 2006) (e.g.,  $7 + 6 + 4 = \underline{\quad}$ ). But the same children who can solve this problem correctly fail if the blank is situated on the left side of the equation (e.g.,  $7 + 6 + \underline{\quad} = 17$ ) or if an additional number is placed on the right side of the equation (e.g.,  $7 + 6 + 4 = 7 + \underline{\quad}$ ), thus failing to appreciate the meaning of the equal sign (that the quantities on the two sides of the equation must add up to equivalent numbers). When children fail to solve a problem like  $7 + 6 + 4 = 7 + \underline{\quad}$  correctly, they typically either add up all of the numbers in the problem and put, in this case, 24 in the blank, or they add up all of the numbers on the left side of the equal side and put 17 in the blank. When asked to explain their incorrect solutions to these problems, children typically say what they did (e.g., “I added the 7, the 6, the 4, and the 7 and got 24,” the add-all-numbers strategy, or “I added the 7, the 6, and 4 and got 17,” the add-to-equal-sign strategy).

However, most children also gesture as they explain their solutions in speech. Some convey the same information in their gestures as they convey in speech (e.g., they say, “I added 7, 6, and 4 and got 17” while pointing at the 7, 6, and 4, and then the blank, the add-to-equal-sign strategy in both speech and gesture). These children produce what have been called “gesture-speech matches” (Church & Goldin-Meadow, 1986; Goldin-Meadow, 2003). However, other children convey different information in their gestures than they convey in their speech—they produce “gesture-speech mismatches” (Church & Goldin-Meadow, 1986; Goldin-Meadow, 2003). For example, a child might say, “I added 7, 6, and 4 and got 17” (an add-to-equal-sign strategy) while pointing at all of the numbers in the problem, at the 7, 6, and 4 on the left side of the equation and the 7 on the right side (an add-all-numbers strategy), thus making it clear that she did, at some level, know that the 7 on the right side of the equation was there and might be important. Note that this second child seems to have an understanding (however implicit) of two pieces of information: (a) there are two distinct sides to the equation (reflected in the add-to-equal-sign strategy the child conveyed in speech), and (b) there is an additional addend on the right side of the equation (reflected in the add-all-numbers strategy conveyed in gesture). These two pieces of information are not yet integrated into a single framework but will have to be if the child is to solve the problem correctly.

Children who produce mismatches in their explanations of a task have information relevant to solving the task at their fingertips and could, as a result, be on the cusp of learning the task. If so, they may be particularly susceptible to instruction. Perry, Church, and Goldin-Meadow (1988) gave 9- and 10-year-old children instruction on problems of the  $4 + 5 + 3 = \underline{\quad} + 3$  variety. Prior to instruction, all of the children solved the problems incorrectly, and all of their spoken explanations were incorrect. However, the children differed with respect to their gestures: Some produced gestures that did not match their speech, whereas others produced only matching gestures. After the instruction period, the children were given a second test to see how much they learned. Children who produced mismatches prior to instruction learned more than children who produced no mismatches (Perry et al., 1988).

When children's responses to problems of this type are charted over the course of instruction, we can see a child systematically progress through three periods characterized by the relation between gesture and speech: (a) the child first produces the same strategy in both speech and gesture and that strategy is incorrect; (b) the child then produces two different strategies, one in speech and a different one in gesture; both strategies may be incorrect, or one may be correct and the other incorrect (if there is a correct strategy, it is typically produced in gesture and not speech); and (c) the child again produces the same strategy in both speech and gesture, but now that strategy is correct (Alibali & Goldin-Meadow, 1993). Gesture, when taken in relation to speech, signals that the child is ready to take the next step in learning about mathematical equivalence. Interestingly, when a child fails to pass through step (b) and goes directly from step (a) to step (c), the child's understanding of mathematical equivalence is likely to be fragile—the child is unable to generalize the knowledge gained during instruction and does not retain the knowledge on a follow-up test (Alibali & Goldin-Meadow, 1993).

The gestures that children produce when talking about mathematical equivalence thus provide insight into ideas that have not yet appeared in their speech. Gesture offers a second window into children's knowledge of arithmetic operations, a window that can tell us who is ready to learn that operation. If teachers not only listen to what their students say, but are also sensitive to the information they convey in their gestures (as they often are; Goldin-Meadow & Singer, 2003), they can use this information in planning their lessons and thus promote student learning.

## **GESTURE CAN CHANGE WHAT CHILDREN KNOW ABOUT ARITHMETIC**

We have seen that gesture can reflect what children know about arithmetic and, importantly, can provide insight into knowledge not evident in their speech. But recent research has shown that gesture can go beyond reflecting what children know about arithmetic to play a role in changing what they know. Gesture can play a role in knowledge change in two ways: The gestures children see and the gestures children do can influence whether they will learn a particular arithmetic concept.

### **The Gestures Children See**

#### *The Gestures Parents Produce During Counting*

Many studies have noted the presence of pointing gestures when sets of objects are being counted. As a set of objects is being counted, adults as well as young children frequently point to each object in turn as they recite the count list (e.g., Fuson & Hall, 1983, Gelman & Meck, 1983; Graham, 1999; Saxe & Kaplan, 1981; Schaeffer, Eggleston, & Scott, 1974). However, we do not know whether exposure to more gesture in the input, either during counting or when set sizes are labeled, affects children's ability to count or their understanding of the cardinal principle.

Suriyakham (2007) examined parents' use of gesture when they counted and labeled set sizes for their 30- and 38-month-old children. Parents, like their 38-month-old children (described earlier), use gestures significantly more often when counting than when labeling set sizes. Moreover, the kinds of gestures they produce in these contexts differ. When counting, they most often sequentially point to each of the items in the set. In contrast, when gestures were produced in the context of a cardinality utterance, the most common gesture they produced was a single point to the set, and the second most common gesture in this context was an iconic hand shape (e.g., holding up two fingers to indicate "two").

Suriyakham (2007) next examined whether children whose parents produced more gestures when counting and labeling set sizes had stronger numerical knowledge than children whose parents produced fewer gestures when counting and labeling set sizes. She found that children whose parents were in the high gesture group said more number words than those whose parents were in the low gesture group; they also performed better on the Point-to-X task (in which children are presented with two sets, for example, one with three items and one with four items, and are asked to point to four items). Importantly, the two groups of parents did not differ significantly in terms of the frequency of their number talk, differing only in terms of how much accompanying gestures they produced along with this talk.

These correlational data are consistent with the possibility that the gestures children see in the context of

number talk promote their numerical understandings. However, in order to test this hypothesis, we need to carry out studies that experimentally manipulate children's exposure to gestures in the context of adults' verbal counts and their labeling of set size. This kind of approach is illustrated in Alibali and DiRusso's (1999) study of children's own points during counting and in several studies described in the next section that manipulate gestural input in the context of equivalence lessons (Church, Ayman-Nolley, & Mahootian, 2004; Ping & Goldin-Meadow, 2008; Valenzeno, Alibali, & Klatzky, 2003). Only one exploratory study has tried to use this approach to ask whether early number talk that is accompanied by gesture is more effective in promoting number development than early number talk in speech only. Suriyakham (2007) presented children whose cardinal number knowledge had been assessed on a pretest using the Give-a-Number task and the Point-to-X task with one of three types of training: (a) sets were counted, accompanied by a pointing gesture to each element, and then given a cardinal label; (b) sets were counted, accompanied by a pointing gesture to each element, and then given a cardinal label that was accompanied by a gesture that circled the entire set; and (c) the same training as (b) except that children also were asked to make a circling gesture to indicate the set size. Posttest assessments showed most growth for children in condition (a) possibly because the circling gesture, used to indicate that the cardinal label applied to the entire set rather than one object in the set, was an unfamiliar gesture, not found in naturalistic parent-child interactions; the gesture may not have been interpretable and perhaps was even distracting. The findings suggest that the juxtaposition of a pointing gesture in the context of counting and no hand movement (or at least no circling hand movement) in the context of labeling the cardinal value, a pattern that is seen frequently in naturalistic parent-child interactions, is particularly helpful to children in promoting understanding of cardinal number.

### ***The Gestures That Teachers Produce During Mathematical Equivalence Lessons***

Teachers routinely gesture in classrooms (Crowder & Newman, 1993; Flevares & Perry, 2001; Neill, 1991; Roth & Welzel, 2001; Zukow-Goldring, Romo, & Duncan, 1994), particularly if they are experienced (Neill & Caswell, 1993). This practice raises the question: Does gesture occur in math lessons, and, if so, is it used often enough to make a difference?

Goldin-Meadow, Kim, and Singer (1999) observed teachers conducting one-on-one individual tutorials with 9- and 10-year-old children who had not yet mastered mathematical equivalence with respect to addition. They found that the teachers expressed 40% of the problem-solving strategies they taught in gesture. When in a classroom situation, teachers also use gesture to convey their message. Flevares and Perry (2001) found that mathematics teachers used from five to seven nonspoken representations of mathematical ideas per minute (almost one every 10 seconds), and gesture was by far the most frequent nonspoken form for all of the teachers (the others were pictures, objects, and writing). Moreover, when the teachers combined two or more nonspoken representations, one of those forms was always a mathematically relevant gesture—gesture was the glue that linked the different forms of information to one another and to speech. Interestingly, the teachers often used their nonspoken representations strategically, responding to a student's confusion with a nonspoken representation. The teachers would repeat their own speech while clarifying the meaning of their utterance with gesture. This seemed to work—the children would then frequently come up with the correct answer.

However, these studies do not tell us whether the gestures that children see their teachers produce on a task lead children to improve their performance on that task the next time around. The few experimental studies that have been done suggest that children learn more from a lesson that contains gesture than from a lesson that does not contain gesture (Church, Ayman-Nolley, & Mahootian, 2004; Valenzeno, Alibali & Klatzky, 2003), even when the gestures are not directed at objects in the immediate environment (Ping & Goldin-Meadow, 2008). However, much more work needs to be done before we fully understand the conditions under which gesture promotes learning. Take, for example, the role of gesture-speech mismatches in instruction. We might have guessed that gesture would get in the way of learning when it conveys information that is different from the information conveyed in speech; that is, when it mismatches speech. We know that communication can suffer when speakers produce gestures that convey different information from their speech (Goldin-Meadow & Sandhofer, 1999; Kelly & Church, 1998; McNeil, Alibali, & Evans, 2000). Yet teachers frequently produce gesture-speech mismatches when teaching children who produce their own mismatches, and those children profit from their instruction (Goldin-Meadow & Singer, 2003). These

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findings suggest that mismatching gesture and speech could be good for learning. Indeed, in a study in which the instruction that children received in mathematical equivalence was experimentally manipulated, Singer and Goldin-Meadow (2005) found that presenting two strategies in a gesture-speech mismatch (one correct strategy in speech and a different, also correct strategy in gesture) was more effective than presenting those same two strategies entirely in speech. Perhaps in these instances gesture provides a representation that matches the child's next developmental state and, in this way, facilitates learning.

More generally, recommendations for mathematics curricula encourage teachers to present ideas through a variety of representations—diagrams, physical models, written text (National Council of Teachers of Mathematics, 1989). Shavelson, Webb, Stasz, and McArthur (1988), among others, recommend that teachers translate among alternative symbolic representations of a problem (e.g., math symbols and number line) rather than working within a single symbolic form. Gesture can serve as one of these representational formats, one that has a strong visual component. Gesture is unique, however, in that unlike a map or diagram, it is transitory—disappearing in the air just as quickly as speech. But gesture also has an advantage—it can be, indeed must be, integrated temporally with the speech it accompanies. We know that it is important for visual information to be timed appropriately with spoken information in order for it to be effective (Baggett, 1984; Mayer & Anderson, 1991). Thus, gesture used in conjunction with speech may present a more naturally unified picture to the student than a diagram used in conjunction with speech. If gesture were to become recognized as an integral—and inevitable—part of conversation in a teaching situation, it could perhaps be harnessed, offering teachers an excellent vehicle for presenting to their students a second perspective on the task at hand.

## **The Gestures Children Do**

### ***Counting***

As noted earlier, many studies show that children tend to point to the objects in a set while counting them, and that this pointing behavior is correlated with more accurate counting. However, few studies have experimentally manipulated children's pointing gestures, which is necessary to test whether gesturing promotes better counting performance. In one study, 2-, 4-, and 6-year-olds were asked to count sets with two, three, seven, or eight objects. One group was encouraged to use pointing gestures while counting, and the other group was prevented from using gestures, which was achieved by displaying the objects in a cage that made it impossible for the participants to point while counting. Findings showed that the 4-year-olds' counting accuracy was significantly better in the gesture condition (Saxe & Kaplan, 1981). Alibali and DiRusso (1999) also explored gesture's role in children's counting by comparing three conditions: the child gestured while counting, the child was restricted from gesturing while counting, and the child watched a puppet gesture while counting. They found that children were most accurate in the two conditions in which counting was accompanied by gesture, theirs or the puppet's. But they were least likely to make errors coordinating number words and objects when they themselves produced the gestures. Thus, children's production of gestures while counting leads to earlier one-to-one correspondence between number words and objects than the use of number words alone. An important question is whether efforts to accelerate the accuracy of counting through the gesture accelerate the learning of associated mathematical concepts such as one-to-one correspondence and the cardinal principle.

### ***Early Calculation***

As we have seen, children's use of finger strategies when they are first learning to solve calculation problems is associated with performance accuracy. These correlational findings are consistent with the possibility that the gestures children produce when carrying out calculations scaffold their development, but we cannot conclude from these studies that gestures play a causal role in the development of calculation skill.

Researchers have suggested that children's finger use during calculation promotes their calculation skill. For example, Jordan et al. (2008) hypothesize that finger strategies help children progress from solving calculation problems with concrete objects to solving symbolic calculation problems. Siegler and Shipley (1995) made the more specific suggestion that finger counting when solving calculation problems can help



children accurately make the association between number fact problems and their solutions. However, the critical experiments are yet to be done. For example, to determine whether finger counting in the context of calculation problems is causally related to children's later success in solving these problems through retrieval strategies, we need to experimentally manipulate children's finger strategies during calculation, as in Alibali and DiRusso's (1999) experiment on children's counting.

## *Mathematical Equivalence*

The gestures that children see during a math lesson influence what they will learn from the lesson. It is likely that the information conveyed in a teacher's gestures has an impact on child learning, but the impact of teacher gesture on children may also be through child gesture—children who see their teachers gesture a concept are themselves likely to gesture the concept; in turn, these children are particularly likely to learn the concept (Cook & Goldin-Meadow, 2006). Indeed, evidence suggests that encouraging children to gesture prior to a mathematical equivalence lesson facilitates learning.

Broaders, Cook, Mitchell, and Goldin-Meadow (2007) asked children to explain how they solved six mathematical equivalence problems with no instructions about what to do with their hands. They then asked the children to solve a second set of comparable problems and divided the children into three groups: Some were told to move their hands as they explained their solutions to this second set of problems, some were told not to move their hands, and some were given no instructions about their hands. Children who were told to gesture on the second set of problems added strategies to their repertoires that they had not previously produced; children who were told not to gesture and children given no instructions at all did not. Most of the added strategies were produced in gesture and not in speech, and, surprisingly, most were correct. Telling children to gesture appeared to bring out ideas that they had not previously expressed. Moreover, articulating those ideas in gesture seemed to help the children become ready to learn about mathematical equivalence. When later given instruction in mathematical equivalence, children who were told to gesture, and added strategies to their repertoires, profited from the instruction and learned how to solve the math problems. Being told to gesture thus encouraged children to express ideas that they had previously not expressed, which in turn led to learning.

But can gesture, on its own, create new ideas? To determine whether gesture can create new ideas, we need to teach speakers to move their hands in particular ways. If these speakers can extract meaning from their own hand movements, they should be sensitive to the particular movements they are taught to produce and learn accordingly. Alternatively, all that may matter is that speakers move their hands. If so, they should learn regardless of which movements they produce. To investigate these alternatives, Goldin-Meadow, Cook, and Mitchell (2009) manipulated gesturing during a math lesson. They found that children required to produce correct gestures learned more than children required to produce partially correct gestures, who learned more than children required to produce no gestures. This effect was mediated by whether, after the lesson, the children added information to their spoken repertoire that they had conveyed only in their gestures during the lesson (and that the teacher had not conveyed at all). The findings confirm that gesture is involved not only in processing old ideas but also in creating new ones. This study suggests that we may be able to lay the foundations for new knowledge simply by telling learners how to move their hands (see Cook, Mitchell, & Goldin-Meadow, 2008, for related findings) or by moving our hands ourselves when teaching them (Cook & Goldin-Meadow, 2006).

Why might the act of gesturing facilitate learning? One possibility is that gesturing reduces demand on speakers' working memory, which in turn frees cognitive effort that can be applied to learning. If asked to remember an unrelated list of items while explaining how they solved a math problem, speakers are able to maintain more items in verbal working memory (and thus recall more items) when they gesture during the explanation than when they do not gesture. This effect has been found in both children and adults (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001). The effect has also been found when speakers are asked to recall unrelated items in visual (rather than verbal) working memory (Wagner, Nusbaum & Goldin-Meadow, 2004); speakers maintain more items in visual working memory (and thus recall more items) when they gesture during their explanations than when they do not gesture, suggesting that gesturing does not lighten the load on working memory by transferring items from a verbal store to a visual store. In

addition, gesturing reduces demand on working memory even when the gestures are not directed at visually present objects, that is, even when the objects are removed and the gestures are produced in space. If asked to remember an unrelated list of items while explaining how they solved conservation of quantity problems that elicit iconic gestures, speakers remember more items when they gesture during the explanation than when they do not gesture (Ping & Goldin-Meadow, 2010). Gesturing thus confers its benefits by more than simply tying abstract speech to objects directly visible in the environment.

Importantly, it is not just moving the hands that reduces demand on working memory—it is the fact that the moving hands convey meaning. Producing gestures that convey different information from speech (i.e., mismatches) reduces demand on working memory less than producing gestures that convey the same information in speakers who are experts on the task (Wagner et al., 2004). Somewhat paradoxically, we find the opposite effect in speakers who are novices—producing gestures that convey different information from speech (mismatches) reduces demand on working memory more than producing gestures that convey the same information as speech (Ping & Goldin-Meadow, 2010). This result could explain why mismatch is particularly good for learning in novices because a reduction in the demand on working memory would free up cognitive resources that can then be applied to the to-be-learned task. However, in both the expert and the novice, the meaning relation that the gesture holds to speech determines, at least in part, the extent to which the load on working memory is reduced. This saved effort can then be applied to grappling with new information.

## THE ROLE OF THE BODY IN TEACHING ARITHMETIC

Hostetter and Alibali (2008) have proposed that gestures emerge from perceptual and motor simulations underlying the speaker's thoughts (see also Rimé & Schiaratura, 1991). This proposal is based on recent theories claiming that linguistic meaning is grounded in perceptual and action experiences (Barsalou, 1999; Glenberg & Kaschak, 2002; Richardson, Spivey, Barsalou, & McRae, 2003; Zwaan, Stanfield, & Yaxley, 2002). If so, gesture could be a natural outgrowth of the perceptual-motor experiences that underlie language. Under this view, the richer the simulations of action experiences, the more speech will be accompanied by gesture. For example, speakers tend to gesture more when describing dot patterns that they had constructed with wooden pieces than dot patterns that they had viewed on a computer screen (Hostetter & Alibali, 2010; see also Cook & Tanenhaus, 2009).

Focusing on acquisition and learning, Karmiloff-Smith (1992) described a process of redescription in which action representations are continuously redescribed into different representational formats. The process culminates in a verbal format that brings with it explicit awareness. Because gesture has its roots in action, it can exploit the effects that action has on thinking (cf. Beilock, Lyons, Mattarella-Micke, Nusbaum, & Small, 2008; see also Beilock & Goldin-Meadow, 2010; Goldin-Meadow & Beilock, 2010). But gesture is also a step removed from action and can thus selectively highlight components of action that are relevant to the to-be-learned problem and leave out components that are not relevant. As a result, gesture's contribution to learning may be used as a stepping-stone in the transition from concrete action to abstract thought.

One final point deserves mention with respect to action, gesture, and learning. Because gestures are external and can closely resemble the actions they represent (i.e., they can be relatively concrete), they have the potential to serve a scaffolding function comparable to the one that manipulatives serve in teaching math (Mix, 2010). Manipulatives are concrete objects designed to instantiate mathematical notions in external form and thus to off-load some of the mental effort involved in learning those notions (e.g., Cuisinaire rods—blocks that illustrate the decomposition of numbers,  $1 + 6 = 2 + 5 = 3 + 4$ ). They are typically used in the earliest stages of learning and are gradually replaced with more abstract symbols. Future work is needed to explore whether gesture's closeness to action does, in fact, contribute to its effect on learning and, if so, to determine how we can best manipulate gesture to enhance learning.

In summary, although mathematics is typically considered an abstract and a formal system, there is evidence of a dynamic component underlying this apparently static system in both learners and experts. Because of its close relation to action, gesture has the potential to capture this dynamic component and thus play a role in how arithmetic is taught and learned. The learner's own gestures may reveal an understanding of arithmetic not found in speech. Gesture can thus serve as a tool for recognizing when a child is ready to profit from instruction in a particular arithmetic notion. But gesture has the potential to do more than just

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reflect a child's knowledge of arithmetic. It can also play a role in changing that knowledge—the gestures that teachers produce can lead children to make inferences they would not have made if the teacher used speech alone, perhaps by bringing action (perceptual-motor) information into the mix. Gesture may thus serve as a pedagogical tool uniquely qualified to instill in children an understanding of arithmetic.

## NOTE

1 Finger representations continue to play a role in numerical representations throughout the lifespan, even though adults rarely rely on finger counting to solve mathematical problems. Adults' finger counting habits (i.e., starting to count with the left hand versus the right hand) modulate the Spatial-Numerical Association of Response Codes (SNARC) effect—the association of spatial location and number, characterized by faster responses to lower numbers on the left and higher numbers on the right. In particular, the SNARC effect is reduced for those who start counting with the right hand rather than the left (Fischer & Zwaan, 2008). Supporting the role of finger counting in neural representations of number, findings from an event-related fMRI study showed that adults' motor areas contralateral to the hand used for counting small numbers are activated when small numbers or number words are presented, even when no overt hand movements are made (Hauk & Tshentscher, 2013).

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