Mergers, Employee Hold-Up and the Scope of the Firm: An Intrafirm Bargaining Approach to Integration

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Abstract

We explore the scope of the firm in a setting where employee wage contracts are nonbinding and firms cannot contract with one another on their respective employment decisions. Specifically, we consider two divisions that have scope for beneficial interaction, and examine whether it is best for them, given this incomplete contracting environment, to produce jointly within the same firm or to interact over the market. Employing a multilateral bargaining framework, we analyze how employee wages, firm profits and employment levels are altered by merger when employees have some hold-up power. Among other results, our analysis suggests that merged production is more likely when the optimal contributions by the two firms to joint production are more unequal (leading to productive and bargaining externalities), while nonintegration is more likely the greater the productive gains to joint interaction between the firms (despite such gains being equally realizable under both merger and nonintegration).

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1 Introduction

Accompanying mergers, one frequently observes significant employee renegotiations. Indeed, in many mergers, negotiations to ensure top employees or key collections of employees are "on board" seem to be viewed as a necessary precondition to implementing the merger. Yet at first pass such activity seems rather anomalous. While some haggling might be expected in an environment where new positions are being created and others eliminated, renegotiations with top employees seem to be prevalent even when the operations of the merging firms are left relatively intact. Employing a standard analysis, it is unclear why such employees should command different compensation after a merger, and what determines the extent to which top employees are retained or eliminated.[1]

Media accounts often depict such post-merger haggling as deriving from a "culture clash" of dissimilar environments or procedures in the merging firms.[2] However, such explanations often seem to smack of ex-post rationalizations: When a merger proceeds smoothly observers nod approvingly towards "obvious" synergies, while when there are public disputes or defections, vague differences in corporate cultures are discovered and attributed.

In this paper we pursue an alternative explanation for renegotiations and key-employee turnover surrounding mergers. In particular, we will argue that such phenomena can be understood as a natural consequence of a multilateral bargaining process between the firm and its employees, in a setting where employees have some hold-up power through their threat to quit. This setting seems to be a natural one to analyze. The fact that workers are not contractually bound to their firms (i.e., labor contracts are generally *at-will* in nature), but some key workers are (at least temporarily) irreplaceable, appears to be an essential ingredient to labor renegotiations surrounding mergers. Furthermore, it also seems essential that a model of employee renegotiations captures the multilateral nature of the bargaining process between workers and their firms. Firms must negotiate simultaneously with multiple employees or unions, and the outcome of negotiations with one party often appear to affect how negotiations proceed with the other parties. The central theme of this paper is that a careful depiction of such a bargaining environment can in fact serve to rationalize renegotiation behavior observed in mergers, and in turn, can shed light on the costs and benefits

[1]For a few recent representative examples of renegotiations accompanying mergers, see "Labor Sees Opportunity in Airline Tie-Ups – Unions Weigh Support As a Way to Extract Better Contract Terms," *Wall Street Journal*, 2/21/08; "New Clout – A Labor Union's Power: Blocking Takeover Bids, Steel-Company Buyers Learn They Must Get USW on their side," *Wall Street Journal*, 5/9/07; "Talks to Buy Delphi Hit Snag with Union," *Wall Street Journal*, 4/17/07. Similar accounts can be found for a number of recent mergers of computer firms and investment banks. Such phenomena is all the more curious when one observes that frequently the merging firms were already interacting extensively with one another over the market.

[2]For recent examples, see "Is Tribal Conflict Inevitable in Mergers and Takeovers," *Financial Times*, 7/16/08; "At InBev, a Gung-Ho Culture Rules; American Icon Anheuser, A Potential Target, Faces Prospect of Big Changes," *Wall Street Journal*, 5/28/08; "The Art of Joining Different Cultures," *Financial Times*, 6/29/07; "How to Make Peace Between Two Warring Cultures," *Financial Times*, 5/23/07.

of a merger and the scope and boundaries of a firm.

In previous work (Stole and Zwiebel, (1996a, 1996b)), we have developed such a general model of intrafirm bargaining to analyze the process of wage determination and its subsequent effect on the firm's decisions when employee contracts are nonbinding.[3] Here we adopt this framework to analyze mergers, and in the process derive a set of intuitive, testable results on merger-related renegotiations, wage-related costs and benefits of mergers, and on the hiring and firing decisions made by firms associated with mergers.

Our analysis is based on a Property Rights approach to the firm, whereby ownership affects economic outcomes due to contractual incompleteness, by confering certain rights in states of the world not addressed by contract.[4] In particular, we identify an important manner of contractual incompleteness, and analyze its implications for ownership, and consequently, mergers. We differ from the previous literature, however, in the form of contractual incompleteness which we analyze, and in doing so, we obtain a novel set of implications.

Specifically, the primitive incomplete contracting assumption which we make and analyze is that employment contracts are nonbinding and unverifiable to outsiders. The at-will nature of labor contracts (i.e., that employees can sever labor contracts at little cost) is an empirical regularity, and can be seen as a consequence of involuntary servitude laws.[5] We further presume here that firms interacting on the market cannot contract with one another on how many employees the other firm is to hire, or which employees they choose to hire.[6] We will show that these simple contractual restrictions alone are sufficient to generate interesting consequences for mergers.

We consider two productive entities that may either be joined together as a firm (merged production) or may instead remain separate firms but still interact with one another over the market (nonintegration). For the sake of exposition, each entity can be thought of consisting of a single machine. We will assume there exists beneficial joint production between the entities. That is, if the entities, together with their employees, employ their machines in a coordinated manner

[3]Stole and Zwiebel (1996b) derives the bargaining game in its most general setting and considers general implications for bargaining theory and game theory. Stole and Zwiebel (1996a) focuses on applying the outcome of this bargaining game to a wide range of labor, technological choice and organizational design decisions of the firm.

[4]See Grossman and Hart (1986), Hart and Moore (1990) and Hart (1995) for a comprehensive treatment of this approach.

[5]For example, Posner [1986, p. 306] states, "employment at will is the usual form of [the] labor contract". As we have argued elsewhere (see Stole and Zwiebel (1996a)), while there are some notable limitations under which courts have held employees responsible for damages for violating long-term labor obligations, such examples in practice are quite specialized. Consequently, the assumption of nonbinding labor contracts is employed frequently throughout the labor and contract theory literature.

[6]All that is needed for our purposes is that firms cannot contract on how many employees other firms will hire *that are qualified for specific tasks*. For this, it would suffice either that a firm cannot observe the identity of all the employees in another firm or that courts cannot verify the qualifications of employees. For example, the courts may not be able to tell how many "real" managers a firm has when the title of manager can be bestowed by a firm both on those qualified and those that are not.

they will be more productive than they could be individually. Such joint production could take place either within a firm or across two firms using market transactions. A firm negotiates with its own workers, and with the other firm if the two entities are not integrated.

Formally, in our model a merger has several consequences. First, it combines the two bargaining units into one. Under nonintegration, any production that used both machines would require the consent of both firms (as well as whatever employees were necessary). Under merger, the consent of only the one merged firm (and its employees) is required. We will refer to this as the "bargaining effect", and will see that it generally implies that holding all else fixed, employees capture a greater share of the surplus when jointly producing divisions are combined into a merged firm.[7]

This bargaining difference between merger and nonintegration is exactly what merger means in Hart and Moore (1990) as well. Nonetheless, the implications that follow from the "bargaining effect" in our analysis are often precisely the opposite of the implications of Hart and Moore. Intuitively, this is because we focus on a different form of inefficiency (an inefficiency in the bargaining game due to at-will labor as opposed to their inefficiency in ex-ante investments), and because we take the perspective that ownership is determined by firm profit maximization, rather than the Hart-Moore efficiency criterion. In Section 5, we discuss in some detail the justification for this change in perspective, and why it alters many results of Hart and Moore.

This "bargaining effect" provides one answer to Williamson's classic question of selective intervention.[8] In particular, Williamson asks why, upon merger, the combined firm cannot undertake all activities of the nonintegrated firms, intervening selectively only when beneficial. Such logic would suggest that it is always weakly beneficial for two firms to merge. However, in our setting, by combining divisions, the firm owner can no longer credibly negotiate with employees as though the units are separate. Only a single approval is needed for access to the assets of both divisions, and all residual profits accrue to one firm rather than two. This changes threat points in labor negotiations, and therefore wages. Thus, while it may be true that total surplus (firms' and workers') would increase under merger, the firms' surplus from merger may still fall.

A second consequence of merger, given our assumption about the non-contractibility of employment across firms, is that the employment decision of the two divisions can be coordinated in a manner that is not possible with two nonintegrated firms. One can think of employment decisions

[7]There is widespread empirical evidence confirming that larger firms pay higher wages, even after controlling for natural firm and worker characteristics and unionization. See, for example, Lester (1967), Masters (1969), Mellow (1982), Brown and Medoff (1989) and Brown, Hamilton and Medoff (1990). Our analysis can be seen as confirming a reduced form bargaining division assumed in Perotti and Spier (1993). In their examination of the consequences of capital structure on employee negotiations, they presume that when debtholders' consent is necessary (in addition to that of shareholders) to realize a surplus, labor's share of this surplus falls.

[8]See for example Williamson (1975) and (1985).

as a noncontractible residual right of control. We will call this the "coordination effect", and will see that such an ability to coordinate the number of employees hired in each division may yield both productive and bargaining benefits to the merged firm.[9]

Finally, at times we will assume that a merger may change the production possibilities (either positively or negatively). In order to draw theory-of-the-firm implications, we will not want to stack the deck by assuming such an exogenous benefit or cost of merger. Consequently, much of the analysis will be concerned with a setting where there is no such change. Nonetheless, our model will have interesting things to say about how such an exogenous change in production compares with our other effects, and therefore, we will at times consider such changes as well.

To fix ideas and clarify our motivation, consider recent labor negotiations at U.S. automakers, in which outsourcing was reported to be among the most contentious issue in negotiations.[10] In these negotiations, it is generally understood that the automakers can undertake certain manufacturing at much lower wages outside the firm.[11] The reason for this disparity in wages inside and outside the firm is not immediately obvious. One might expect that certain groups of manufacturing employees within the firm would be able to extract excess wages due to their ability to hold up a wide range of production by failing to produce their needed parts. However, one could ask why external employees making the same parts can't extract the same considerations from the automaker by a similar threat.

One answer might be that perhaps an automaker can commit not to negotiate directly with these

[9]Milgrom and Roberts (1990) discusses bargaining costs to nonintegration, including costs to coordinating across firms. In their conception, this cost to nonintegration is countered by increased influence activity within an integrated firm. Holmström (1996) also discusses the ability of an integrated firm to effect worker assignments not possible over the market. Furthermore he argues that a crucial aspect of a firm is its ability to internalize externalities of incentive design under informational imperfections. However, he also states that integration is likely to strengthen the firm's bargaining power. Here he states that, "It is readily seen that in the Shapley value formula (which Hart and Moore use in the multi-person case), a coalition gets more as a collective bargaining unit than separated." In general this is clearly not true, and in particular, it does not hold for simple production games, where firm's assets are essential in order to realize valuable production (as we consider here). While the characteristic function underlying many cooperative games is taken to be superadditive, this by no means implies that different parties would increase their combined Shapley values by merging.

[10]**** Need to update this ****** For example, the 1998 strike at GM's stamping plants, or the 1996 strike at GM's Dayton brake plants. Outsourcing was also the central issue in the 2002 strike by machinists at the Marietta, Georgia assembly plant of Lockheed Martin Co., and labor negotiations between Machinists and Boeing in 2002; see, for example, *Wall Street Journal* articles "Machinists' Strike at Lockheed May Stall U.S. Fighter-Jet Order," 3/12/02; "Boeing Union Seeks Mediator in Contract Talks," 8/16/02.

[11]**** Need to update ***** For example, see the following *Wall Street Journal* articles, among many others: "GM Talks Hurt by Plans to Close Plant," 7/9/98; "Canadian Auto Workers Target Outsourcing Trend," 7/20/98; "UAW Officials Warn that Problems with GM Also Exist at Ford, Chrysler," 7/20/98; "For GM, a Hard Line on Strikes Has Become a Matter of Necessity," 6/12/98. All these articles discuss, in part, GM's desire to purchase more parts from outside suppliers that pay lower wage rates as a important factor underlying the current dispute. As just one example, the last of these listed articles states, "GM executives know that Ford and Chrysler have in large part already accomplished the kind of streamlining GM is pushing to do, including farming work out to outside suppliers whose labor rates are much lower." [p.A4, 6/12/98]

external workers, but rather, negotiate only with their firm.[12] However, this answer is not altogether satisfying. Any such external firm should presumably still be able to hold up the automaker, and these employees should in turn be able to hold up their own firm. In fact, it is simple to show in our bargaining model that this indirect route of hold-up yields employees the same wages that they would realize if they could negotiate directly with automaker as well.[13]

Instead, what is critical in our setting is that under integration workers need to only reach an agreement with the automaker in order to be valuable. If the automaker were instead to spin off a division, both this new firm, owning the necessary machines, and the automaker would be essential for the employees' production to be valuable. Consequently, any surplus of production that requires the workers input, combined with the spun-off equipment, to make output used specifically for automaker's cars, would be divided between employees and *both* firms. Our analysis will indicate that this will generally lessen the hold-up power of these employees.

Such a spin-off has costs to the automaker as well, however. Both for productive reasons and for bargaining reasons, the automaker may benefit from coordinating the employment levels across its divisions. For example, redundant workers at one plant could perhaps be re-deployed to other similar activities elsewhere, in the event of a production shock, or if workers at another plant threatened to strike. If such coordination is not contractually feasible across firms, there will be a corresponding loss to such a spin-off.

Overall, our analysis yields several interesting and original implications regarding mergers, employment and the scope of the firm. Suppose that the production possibilities are the same under merger as under joint production over the market. In such a setting, for any fixed number of employees, merging implies that wages for employees with holdup power increases. In general, the merged firm may choose to either increase or decrease the number of employees hired, and merging may either increase or decrease firm profits. A number of examples suggest that a merger will yield higher firm profits when joint surplus derives from very unequal labor inputs from the two divisions relative to their bargaining power (i.e., when coordination externalities are large). Conversely, nonintegration will be more profitable when labor inputs from the two divisions necessary to realize joint surplus are similar (coordination externalities are small), and when gains from joint production are large.[14] Along similar lines, the merged firm hires more than the nonintegrated firms when joint

[12] For example, if the automaker cannot verify who is working for its partners, or what role each employee plays, it can credibly refuse to negotiate with anyone who claims to be such an employee.

[13] In particular, as footnote 30 below discusses, allowing employees to negotiate across firms as well as within their own firm will have no effect on the outcome of our bargaining game. What is of course relevant, is our technological assumption that an employee is only valuable in combination with a particular machine for which she is suited to work.

[14] This final statement is not trivial: we reemphasize that we are assuming here that output from joint production is realized equally over the market or in a merger. Rather, it follows from the outcome of our bargaining game, whereby

surplus follows from very unequal labor inputs from the two divisions and when it is beneficial for negotiations to hire a lot of redundant labor. The merged firm hires less when labor inputs necessary to realize joint surplus are similar for the two divisions and gains are not sufficient for firms to hire much redundant labor.[15]

Several important caveats are in order. First, at times firms might be able to circumvent contractual restrictions on other firms' hiring decisions by contracting on output instead. Thus, Firm A may be able to contract for Firm B to "deliver the intermediate goods necessary for our joint production that three employees can produce." This may mitigate our analysis to some extent. However, in our setting, firms may benefit from the hiring of unproductive redundant workers by the other firm, for the role they play in bargaining. Insofar as such additional hires do not contribute to output, contracts written on output but not employment levels will not be able to account for such employees.[16] And second, we do not claim our model applies to all mergers. There are many mergers where bargaining and more generally, employee related issues seem to play at most a tangential role. However, we contend that the effects we analyze are likely to be important in industries where a high fraction of firms' value derives from employee-specific human capital.[17]

The paper is organized as follows. In Section 2 we develop the model under which we analyze the consequences of mergers, in the process reviewing our underlying framework of firm-employee negotiations that is adopted from Stole and Zwiebel (1996a, 1996b). Section 3 addresses the case of exogenously given employment levels. In this setting, several simple and powerful results emerge regarding the attractiveness of mergers in bargaining environments. Section 4 considers a series of examples under the more complex setting in which employment is endogenous. Section 5 briefly

firms capture a greater fraction of this joint production relative to their employees when they remain separate entities.

[15]There is a large empirical finance literature on market reactions to mergers, generally documenting large gains (on the order of 30-40%) for targets, and much smaller average market reactions (slightly negative in a number of studies) for acquirers. See, for example, Jensen and Ruback (1983), Jarrell, Brickley and Netter (1988), and Schwert (1996). Also related is the literature on the diversification discount, which explores the link between conglomerates and firm performance, using Tobin's q as a measure of firm performance. See, for example, Lang and Stulz (1994) and Berger and Ofek (1995) which document lower q's for conglomerates, and Chevalier (1999) and Campa and Kedia (1999) for evidence that conglomerates exhibit similar underperformance prior to undertaking diversifying acquisitions, thereby calling the methodogy employed by this literature into question.

Our paper tangentially relates to both of these literatures, insofar as it gives a normative theory for labor-related costs and benefits of a merger, even under the assumption that the merger does not affect the underlying production set. However, without a positive theory for which mergers will occur, or for why value decreasing mergers may take place, which is outside of the scope of our analysis, our model does not yield direct predictions for market reactions (or q) over the set of mergers that occur. Our model does, however, yield predictions for which firms are likely to merge, under the assumption that value increasing mergers are more likely than value decreasing mergers.

[16]Alternatively, one might imagine that at times Firm A might be willing to hire employees to work in combination with Firm B's assets, at its own expense. However, in many cases, whether Firm B employs these employees as specified is likely to be unverifiable. For example, if firm A would benefit by redundant employees at firm B, and sends employees over to firm B for this purpose, firm B could reduce the number of its own employees, thereby defeating A's purpose. The key point here is that the employment decision of a firm is likely to be noncontractible in many settings, and as such, is properly identified as a residual right of ownership.

[17]Examples of such industries include the computer industry, investment banking, and those where professional partnerships are common (such as accounting, consulting and law).

compares our model with Hart and Moore (1990) and Section 6 concludes.

2 The Model

2.1 The Basic Setting

We consider two firms, A and B, operating in the market. For simplicity, one can think of each of these firms consisting of a single machine, machine A and B, respectively. We presume there is some benefit to these firms interacting with one another, which could take many forms, including: an upstream and downstream firm, a joint venture, or coordination on R&D. The fact that there is a productive motivation for the firms to interact by no means implies they should merge; we will be concerned with comparing the consequences of such interaction over the market with that when done within a merged firm. If the two firms merge, we will call the combined firm "AB", and will refer to divisions A and B within this new firm.

Firms A and B hire type A and B employees respectively, and for simplicity, this is the only variable input into their production. The firms' technologies, *absent any interaction with the other firm*, are represented by the production functions $F^A(\cdot)$ and $F^B(\cdot)$, whose arguments are labor. We will take labor inputs to be discrete (integers) throughout this paper. Type A and B employees can be thought to be employees qualified to work on the machines that A and B respectively own. All employees within each type are productively identical, and the two types of agents have outside options of $\underline{w}_A$ and $\underline{w}_B$ respectively.[18]

If firms A and B have n_A and n_B employees, respectively, and they choose to interact with one another over the market, we presume they can together produce $F^J(n_A, n_B) \geq F^A(n_A) + F^B(n_B)$. We will denote this productivity "synergy" recognized over the market by the function S^J, defined by $S^J(n_A, n_B) \equiv F^J(n_A, n_B) - \big(F^A(n_A) + F^B(n_B)\big)$. The inequality $S^J(n_A, n_B) \geq 0$ simply reflects the notion of productive selective intervention; the firms could choose no joint interactions, obtaining what they produce independently. Provided that this inequality is strict, one would expect firms A and B to always interact with one another, and, trivially, they will do so in our setting. Our focus is hence not on whether or not A and B choose to interact, but whether they will do so over the market or within one merged firm AB, and the consequences that this decision has on employment, wages and profits.

[18]Stole and Zwiebel (1996b) demonstrate that it is easy to generalize these assumptions in our bargaining setting, allowing for continuous labor, multiple inputs to production, reversible capital, a mix of both replaceable and irreplaceable employees and heterogeneous employee within firms. See Stole and Zwiebel (1996a) for more on this hiring decision, including a training-cost justification of employees' temporary irreplaceability in a setting with an *ex-ante* pool of identical workers.

Alternatively, firms A and B may choose to merge, forming firm AB. We let the production function of this merged entity be represented by $F^M(n_A, n_B)$ when there are n_A type A employees in division A and n_B type B employees in division B. We will define the *additional* synergy associated with the merger, as opposed to that which follows simply from joint production over the market, by $S^M(n_A, n_B) \equiv F^M(n_A, n_B) - F^J(n_A, n_B)$. S^M may be either positive or negative, though for much of the paper we will take $S^M \equiv 0$; that is, we will assume the merged firm is productively identical to joint production over the market. It is important to emphasize that in such an event, production by the merged firm yields the same joint benefits S^J as does production over the market; S^M is the *excess* production realized by the merged firm. The more general case where S^M may differ from 0, however, will yield further interesting implications for how the inframargins of S^M are allocated among the parties.[19]

For any given integration and hiring decision by the firms, employees negotiate with their respective firms over wages according to the intrafirm negotiation setting of Stole and Zwiebel (1996a, 1996b). For analytic simplicity, we will confine ourselves to the simplest case where workers and the firm have even bargaining power; that is, bilateral surplus between any worker and her firm is evenly split, where bilateral surplus is measured relative to the parties' respective payoffs if they fail to reach an agreement.[20] Results from this bargaining model that we employ here and some brief intuition are provided in the following subsection. We assume that integration decisions are decided ex-ante by the firms, in a manner that maximizes their combined profits, correctly anticipating future negotiations. We discuss this assumption in further detail in Section 5, noting that this differs from ex-ante efficiency, insofar as employees welfare is not taken into account. In Section 3 we analyze the case where employment levels are fixed when comparing merger and nonintegration.[21] In Section 4 we consider a series of examples where firms choose their employment level optimally, conditional on the integration decision. For this employment decision, if A and B are not integrated, they each choose their own employment levels as a Nash equilibrium, taking into account both the other firm's decision and future bargaining with their employees. If the firms merge, the hiring decision for both divisions is instead made by the single combined firm AB.

[19] In accord with our property rights approach, S^M could be interpreted as the gain (or loss) from merging due to the incentives that such a change in ownership structure provides for ex-ante investments as in Grossman and Hart (1986). More generally, one could interpret S^M as one of the many costs and benefits proposed to accompany mergers.

[20] Stole and Zwiebel (1996b) demonstrate that bargaining results generalize cleanly to any uneven bargaining split, including those where bargaining power of each worker depends on the number of other workers present.

[21] We analyze the fixed employment case both because it provides a clearer setting for a number of insights, and because it vastly simplifies the analysis. Furthermore, in some settings, hiring or training new employees after an integration decision may be a slow process, thereby justifying a focus on fixed employment. See Wolinsky (1997) for a model extending the intrafirm bargaining framework to a dynamic setting where opportunities to hire new employees arrive stochastically over time, and see de Fontenay and Gans (2003) for an analysis this framework when a finite pool of outside replacement workers exists ex-post as well as ex-ante.

During negotiations, employees take into account any joint surplus their firm may realize due to (correctly) anticipated joint production. This, of course, affects the margins for which the employees hold-up their respective firm. Our model of negotiation is described in further detail in the following subsection. After reaching agreements with employees, or terminating those for which agreement is not reached, production takes place. If the divisions have merged, output is given by F^M. If the firms remain separate entities, they produce F^J jointly over the market. In the latter case, each firm receives the output it could realize on its own, plus its split of the joint surplus. We presume this division of joint surplus is divided equally by the two firms; for further justification, see footnote 26 below.

There is no uncertainty in our bargaining game, and all participants are fully informed about valuations. Consequently, all correctly anticipate the equilibrium outcome throughout negotiations, and the bargaining game is *ex-post* efficient. The only source of inefficiency is in the number of employees hired, both because firms will overhire to lessen employees' negotiating positions, and because each firm does not take into account the externality that its hiring decision imposes on the other firm.

2.2 Intrafirm Bargaining of the At-Will Firm

Here, we give a brief overview of the intrafirm bargaining framework developed in Stole and Zwiebel (1996a and b). For simplicity here, we present results for a pared-down version of the bargaining setting analyzed there, only considering aspects of the game that will be relevant for our present analysis. For more details, an interested reader should consult these papers.

In this work, a firm and its employees negotiate over wages. Labor contracts are assumed to be at-will, thus, renegotiation is possible at any time prior to production. Employees have some hold-up power; in particular, at the time of negotiation, they are presumed to be (at least partially) irreplaceable.

Stole and Zwiebel (1996b) demonstrate the equivalence of three alternate formulations to our wage-negotiation game. Here we primarily describe the most simple of these formulations. Given our nonbinding labor contracts assumption, we look for a wage and profit profile that is immune to any renegotiations between employees and the firm. We call such a profile *stable*. Thus, in a stable wage and profit profile, no employee stands to gain from renegotiating with the firm, and the firm cannot gain from renegotiating with any employee. In any such renegotiation, bilateral surplus, relative to the two parties' outside options (defined immediately below), is split according to the parties' relative bargaining powers.[22]

[22]Insofar, as in the present paper we will consider the simple case of even division of surplus, this condition becomes

Crucial to our conception is the definition of outside options that determines bilateral surplus between two parties. We assume that if any negotiation between an employee and a firm ends in disagreement, the employee leaves the firm (and industry) without returning. Thus, the employee's outside option if she fails to reach an agreement with the firm is just her exogenously given reservation wage. For the firm, however, it is profits that would ensue in equilibrium in our bargaining game, with one less worker present, as renegotiations over wages will occur. As such, one can think of the firm's profits being determined iteratively; profits with three workers present depends on the outside option of profits with two workers, which in turn depends on the threat point of profits with one worker, etc.... Given this, it should not be surprising that in equilibrium, the firms' profits, and the employees wages, depend on inframargins of production as well as the final margin. We show that this renegotiation-proof stability criterion yields a unique wage and profit outcome for employees and the firm. Specifically, we find the following results:[23]

First consider the case with homogeneous labor (only one type of employee), and equal bargaining power. Let Δ represent the first difference operator, $\underline{w}$ the employees' reservation value and $\pi^C(n)$ the firm's classical (non-bargaining) profit function (i.e., $\pi^C(n) \equiv F(n) - \underline{w}n$). Then, if there are n employees in the firm, the unique stable wage and profit profile is given by,

$$w(n) = \frac{1}{n(n+1)} \sum_{i=0}^{n} i\Delta F(i) + \frac{1}{2}\underline{w}; \tag{1}$$

$$\pi(n) = \frac{1}{n+1} \sum_{i=0}^{n} \pi^C(i). \tag{2}$$

Note that the weights on $\Delta F(i)$ and $\underline{w}$ in equation (1) and on $\pi^C(i)$ in equation (2) sum to 1. Thus, wages are given by a weighted average of marginal and inframarginal production and the employees' reservation wage. Firm profits also depend on such inframargins.[24] Equation (2) states the rather striking result that profits can be expressed as a simple average of classical (non-bargaining) profits, averaged over all labor configurations from 0 to the actual number of employees n.[25]

Now suppose there are two different types of employees within the same firm. Then we find that if there are n_A type A employees and n_B type B employees, firm profit in the unique stable

simply the balanced contribution condition of Myerson (1980).

[23]These results depend on regularity conditions which we do not state here. For derivations and details, see Stole and Zwiebel (1996a or 1996b).

[24]As a simple motivating example, consider the derivation of $\pi(1)$. To this end, first note that at $n = 0$, we have $\pi(0) = \pi^C(0) = F(0)$ and $w(0) = 0$. Next, consider $n = 1$. The firm's surplus from retaining the worker is $\pi(1) - \pi(0)$, while the worker's surplus is $w(1) - \underline{w}$. Equating these, we have $\pi(1) - \pi(0) = w(1) - \underline{w}$, or simplifying, $F(1) - w(1) - F(0) = w(1) - \underline{w}$. Rearranging, $w(1) = \frac{1}{2}(\Delta F(1) + \underline{w})$ and $\pi(1) = \frac{1}{2}\pi^C(1) + \frac{1}{2}\pi^C(0)$.

[25]When bargaining power is not equal, this expression generalizes to a *weighted* average over the same classical profits.

profile is given by:

$$\pi(n_A, n_B) = \frac{1}{n_A + n_B + 1} \sum_{i_1=0}^{n_A} \sum_{i_2=0}^{n_B} \frac{\binom{n_A}{i_1}\binom{n_B}{i_2}}{\binom{n_A+n_B}{i_1+i_2}} \pi^C(i_1, i_2). \tag{3}$$

Hence, once again firm profits are given by a weighted average of classical profits, where the average is taken over all labor configurations from $(0,0)$ to (n_A, n_B).

One important feature to note from equation (1) is that the more inframarginal the margin of production (the further from being the last margin produced), the less impact it has on employees' wages. In particular, margin i has weight $\frac{i}{n(n+1)}$. This should not be surprising; the more inframarginal a margin of production, the less impact an employee can have on this margin by threatening to quit. Consequently, presuming the production function F eventually exhibits decreasing returns, there will be a benefit to the firm in overhiring. By doing so, high margins are pushed back further, lowering the fraction of these margins captured by employees, and therefore, lowering wages. We show that given such intrafirm bargaining, all firms overhire relative to a firm that does not negotiate with employees. This will be true for the firms in our present analysis.[26]

While we have chosen to emphasize the renegotiation-proof stable-profile interpretation of our bargaining setting here, it is worth noting that the identical outcome follows from two other equivalent formulations examined in Stole and Zwiebel (1996b).

First, one can show this outcome is the unique subgame perfect equilibrium of the following extensive form game: The firm negotiates with employees in any pre-specified order. In each negotiation session, the firm and the employee bargain according to the Binmore, Rubinstein and Wolinsky (1986) bargaining game with breakdown. Employees observe whether bargaining sessions that do not include them yield an agreement or breakdown, but do not observe the wage that was reached under such an agreement. In the event of negotiation breakdown between any employee and the firm, that employee leaves the firm, and all remaining employees (starting from the beginning of the order) have the opportunity to renegotiate in a new bargaining session.

Furthermore, when bargaining power is equal, our outcome is also given by the cooperative game solution concept of Shapley value applied to a closely corresponding cooperative game. (If bargaining power is not equal, we instead obtain weighted Shapley value.) In particular, a cooperative game is defined by its characteristic function, which indicates how every coalition fares on its own. The natural characteristic function to associate with our production function is one where any collection of workers without the firm just obtains the sum of their outside options, while any collection together with the firm can realize the outcome of the firm's production function evaluated

[26]Of course, in our present setting with discrete employees (as opposed to a continuum), bargaining firms only weakly overhire. They may hire the same number of employees as their classical counterpart.

at the number of employees in the coalition. Thus, consider the case of the integrated firm AB. Let $i_1(S)$ and $i_2(S)$ be the number of type A and B workers in any subset S of the collection of all employees and the firm. Then, we would define the characteristic function v, as follows: For any S,

$$v(S) = F^M(i_1(S), i_2(S)) \qquad \text{if } AB \in S; \tag{4}$$

$$v(S) = \underline{w}_A i_1(S) + \underline{w}_B i_2(S) \qquad \text{if } AB \notin S. \tag{5}$$

Stole and Zwiebel (1996b) demonstrate that the Shapley value of this cooperative game is given by our intrafirm bargaining equilibrium. It frequently turns out that this cooperative game formulation makes for the easiest manner to find the equilibrium to our intrafirm bargaining game, and thus we will employ this result as a tool throughout this paper.

Summarizing, our intrafirm bargaining framework for employee-firm negotiations yields expressions (2) and (3) for profits of a firm with one or two divisions respectively. For firm AB, equation (3) directly gives profits, while profits for firms A and B are given by equation (2) plus each firm's respective share of joint surplus. For the division of joint surplus, we once again apply our balanced contributions criterion. The agreement of both firms are necessary in order to realize this joint surplus, and therefore, balanced contributions implies that they should share in it equally. We presume that employees correctly anticipate such joint production (given that it is always mutually beneficial in our setting), and therefore correctly take into account their contributions to joint surplus when negotiating.[27]

[27] As a more formal justification of this division of surplus between the firms, consider the following variant to the extensive form game described above in the text. First the firms negotiate with one another on how surplus will be split (i.e., on a payment from one to the other) conditional on joint production occurring. Then each firm negotiates with its employees as before. Also, as before, all negotiation sessions take the form of the Binmore-Rubinstein-Wolinsky (1986) bargaining game with breakdown. In the event of negotiation breakdown between any employee and the firm, that employee leaves the firm, the firms renegotiate with each other, and then all remaining employees (starting from the beginning of the order within each firm) have the opportunity to renegotiate. If instead there is a negotiation breakdown between the two firms, each firm proceeds to bargain with its own employees, and each firm produces individually without the benefit of joint production. Agents not in a bargaining session observe whether or not an agreement was reached, but not the terms of the agreement.

Note that in this game, when firms are bargaining over the division of their joint surplus S^J, they take into account the effect their threat (of not producing jointly) has on their wage bills. This formulation is more consistent with our notion of employee bargaining – i.e., employees take into account the effect their leaving has on other employees wages in determining their firm's outside option. It is, however, straightforward to show that the unique subgame perfect equilibrium to this game is the same division of surplus employed in our analysis here – that is, balanced contributions between every worker and her firm, and between the two firms. This can be shown with a rather simple extension to the proof to Theorem 2 in Stole and Zwiebel (1996b) (which demonstrates that within one firm, the pairwise bargaining game yields the unique stable wage and profit profile). Furthermore, as this proof demonstrates, the timing of when the firms negotiate relative to when employees negotiate does not matter; what is important is that all bargaining outcomes are renegotiable if any pairwise meeting results in breakdown.

3 Fixed Employment

In this section, we analyze the implications our model has for mergers holding the number of employees in each division fixed. More generally, upon merging, a firm might choose to either add or remove employees prior to bargaining, and we will consider such changes in Section 4. Here we show that even holding employment fixed, employee bargaining yields a number of interesting implications for mergers.

It will be useful for our analysis and interpretation to define multi-dimensional margins of production. Margins for firms A and B producing independently, without joint production, are defined in the standard manner. That is, we define $f^i(n) = F^i(n) - F^i(n-1)$, for $n \geq 1$, $i = A, B$. Given benefits to joint and merged production, $S^J(n_A, n_B)$ and $S^M(n_A, n_B)$, respectively, we define associated joint *margins* of production $s^J(n_A, n_B)$ and $s^M(n_A, n_B)$ as follows: For $i = J, M$,

$$\begin{aligned} s^i(0,0) &\equiv S^i(0,0), \\ s^i(n_A,0) &\equiv S^i(n_A,0) - S^i(n_A-1,0), \quad \forall n_A \geq 1 \\ s^i(0,n_B) &\equiv S^i(0,n_B) - S^i(0,n_B-1), \quad \forall n_B \geq 1 \end{aligned}$$

$$s^i(n_A,n_B) \equiv S^i(n_A,n_B) - S^i(n_A-1,n_B) - S^i(n_A,n_B-1) + S^i(n_A-1,n_B-1), \forall n_A, n_B \geq 1. \quad (6)$$

Note that this definition of margins is just the standard univariate definition along the axes when either n_A or n_B is 0.[28] Equation (6), which defines the margins in the interior of S^i, can be extended to cover this definition along the boundaries as well, provided that $S^i(-1, n_B)$ and $S^i(n_A, -1)$ are defined to be 0. Note that equation (6) is simply the discrete analog of the cross-partial of S^i. The rationale for calling s^i a margin comes from noting that that the sum of all preceding margins yields S^i; that is, they satisfy a discrete multidimensional analog to the fundamental theorem of calculus. Specifically, given any production functions F^A,F^B,S^J,S^M, it is straightforward to show that the margins of production f^A, f^B, s^J, s^M, as defined above are the unique functions such that, $\forall n, x, y$,

$$F^I(n) = \sum_{i=0}^{n} f^I(i), \quad I = A, B; \quad (7)$$

$$S^I(x,y) = \sum_{i_1=0}^{x} \sum_{i_2=0}^{y} s^I(i_1, i_2), \quad I = J, M. \quad (8)$$

We now turn to comparing profits of the nonintegrated firms A and B with the merged firm AB. Profits for AB, under our bargaining game, are simply given by expression (3), which gives the

[28] In many settings S^J and S^M are likely to equal 0 along these axes. That is, there are no gains to joint production (whether over a market or under a merger) if one of the units has no employees. While we by no means rule out this case, we allow for joint gain deriving from workers of one firm employing the machine of the other.

Shapley value of this firm for the corresponding cooperative game with a characteristic function given by equations (4) and (5). Let $\pi_A(N_A)$ and $\pi_B(N_B)$ represent the stand-alone profits under our employee bargaining game to firm A and firm B with respectively N_A and N_B employees. Also, let $N \equiv N_A + N_B$. Then we can express the merged firm's profits as follows.[29]

Theorem 1 *Profits for the merged firm AB are given by:*

$$\pi^M_{AB}(N_A, N_B) = \pi_A(N_A) + \pi_B(N_B) + \sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \mu^M(i_1, i_2)\left(S^J(i_1, i_2) + S^M(i_1, i_2)\right), \tag{9}$$

where,

$$\mu^M(i_1, i_2) \equiv \frac{1}{N+1}\left(\frac{\binom{N_A}{i_1}\binom{N_B}{i_2}}{\binom{N}{i_1+i_2}}\right) > 0 \quad \text{and} \quad \sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \mu^M(i_1, i_2) = 1. \tag{10}$$

There are a couple of features worth noting in this result. First, the merged firm's share of nonjoint output is identical to that which would be realized without any interaction between the two firms. Intuitively, if divisions A and B are productively independent, then merging does not affect the bargaining outcome with employees in any manner. Second, joint production S^J and S^M is shared with employees. Furthermore, AB's share of this joint production is given by a weighted average of the total joint product S^J and S^M evaluated at all inframarginal levels of labor. Note that the weights μ^M can be properly thought of as a density function, as its values are nonnegative and sum to 1. This same density μ^M gives the weights that determine AB's share of both S^J and S^M.

Turning now to the case of nonintegration, the following result gives profits realized by firms A and B.

Theorem 2 *Profits of the nonintegrated firms A and B are given as follows: For $k = A, B$,*

$$\pi^J_k(N_A, N_B) = \pi_k(N_k) + \sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \frac{1}{2}\mu^J(i_1, i_2) S^J(i_1, i_2), \tag{11}$$

where,

$$\mu^J(i_1, i_2) \equiv \frac{1}{N+1}\left(\frac{\binom{N_A}{i_1}\binom{N_B}{i_2}}{\binom{N}{i_1+i_2}}\right)\frac{2(i_1+i_2+1)}{N+2} \quad \text{and} \quad \sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \mu^J(i_1, i_2) = 1. \tag{12}$$

Once again, a few comments are in order. As with the merged firm, producing jointly over the market does not affect the firms' shares of the production F^A or F^B that would be realized

[29] Proofs are in the Appendix.

independent of one another. Note, however, that the fraction of output S^J captured by the firms operating independently over the market differs from that of the merged firm. Summing the profits of the two firms under joint production, the two firms' share of joint production is once again given by a weighted average of S^J evaluated at inframarginal levels of labor, where now the relevant density is μ^J. This density μ^J differs from the relevant density μ^M for the merged firm by the factor $\frac{2(i_1+i_2+1)}{N+2}$ for all (i_1, i_2). This factor is less than 1 for low values of $i_1 + i_2$, but greater than 1 when $i_1 + i_2$ exceeds $\frac{N}{2}$. Thus, the nonintegrated firm has more weight on S^J terms evaluated at high values of $i_1 + i_2$ and less weight on S^J terms evaluated at low values. This suggests that if S^J is increasing, the nonintegrated firms are likely to capture more of S^J than the merged firm in bargaining with employees. This is what we have labelled the bargaining effect. Formally, we state this result in the following theorem.[30]

Theorem 3 *Suppose $S^M \equiv 0$, and $S^J(i_1, i_2)$ is weakly increasing in i_1 and i_2. Then for any employment level (N_A, N_B), provided that $S^J(N_A, N_B) > S^J(0,0)$, it follows that,*

$$\pi_A^J(N_A) + \pi_B^J(N_B) > \pi_{AB}^M(N_A, N_B).$$

Thus, if production possibilities are not altered by merger ($S^M \equiv 0$), provided that joint production S^J is increasing in labor and the number of employees is fixed, nonintegrated firms capture more of this output (and employees less) than under a merger. This may, of course, be offset by positive output S^M that can by definition only be realized under a merger. Additionally, the merged firm will have a greater ability to coordinate its employment decisions, a subject we turn to in the following section.

It is insightful to restate Theorems (1) and (2) in terms of margins s^J and s^M. Substituting equation (8) into equations (9) and (11), and rearranging terms immediately yields the following result.

Theorem 4 *Profits for the merged firm AB can be written:*

$$\pi_{AB}^M(N_A, N_B) = \pi_A(N_A) + \pi_B(N_B) + \sum_{i=0}^{N_A} \sum_{j=0}^{N_B} \bar{\Phi}^M(i,j) \left(s^J(i,j) + s^M(i,j) \right). \tag{13}$$

[30] While it might appear that the restricted bargaining channels available to employees under joint production also play a role in Theorem 3, this intuition is not accurate. While an employee of firm A does not bargain directly with an employee of firm B, the bargaining that occurs between these two firms is enough to ensure that bilateral surplus between a firm and the employees of the *other* firm are split evenly. Thus, for our characteristic function (which implies that employees are valuable only in conjunction with their own firm's machine), allowing employees to negotiate additionally with the firm that does not employ them yields the same result. Thus, the crucial distinction between the merged and the nonintegrated firms is that in the former case only firm AB must consent to realize joint margins s^J, whereas, under joint production, both A and B are essential. See Section 5 for further discussion of this point, and its relationship to Hart and Moore (1990).

where $\bar{\Phi}^M(i,j)$ are coefficients given by,

$$\bar{\Phi}^M(i,j) = \sum_{i_1=i}^{N_A} \sum_{i_2=j}^{N_B} \mu^M(i_1, i_2) \qquad \forall (i,j). \tag{14}$$

The sum of the profits of the nonintegrated firms A and B can be written as:

$$\pi_A^J(N_A, N_B) + \pi_B^J(N_A, N_B) = \pi_A(N_A) + \pi_B(N_B) + \sum_{i=0}^{N_A} \sum_{j=0}^{N_B} \bar{\Phi}^J(i,j) s^J(i,j). \tag{15}$$

where $\bar{\Phi}^J(i,j)$ are coefficients given by,

$$\bar{\Phi}^J(i,j) = \sum_{i_1=i}^{N_A} \sum_{i_2=j}^{N_B} \mu^J(i_1, i_2) \qquad \forall (i,j). \tag{16}$$

Note that $\bar{\Phi}^M(\cdot,\cdot)$ and $\bar{\Phi}^J(\cdot,\cdot)$ can be thought of as "upper cumulative density functions" for the densities μ^M and μ^J respectively. That is, they are defined by summing all weight under these densities above their given argument. If we let $\underline{\Phi}^M(\cdot,\cdot)$ and $\underline{\Phi}^J(\cdot,\cdot)$ represent the (standard) CDFs for μ^M and μ^J, then at any point (x,y) and for $I = M, J$, the upper and standard CDFs are related by,

$$\bar{\Phi}^I(x,y) = 1 - \underline{\Phi}(N_A, y-1) - \underline{\Phi}(x-1, N_B) + \underline{\Phi}(x-1, y-1). \tag{17}$$

Theorem 4 indicates that firm profits under either nonintegrated or merged production can be written as profits without any joint production, plus, a fraction of each of the subsequent margins of joint (and merged) production. The fraction of these margins captured by the firms, as opposed to the employees, is given by $\bar{\Phi}^M(i,j)$ under merger, and $\bar{\Phi}^J(i,j)$ under joint production. Comparing the distributions μ^M and μ^J allows us to make a strong statement ordering $\bar{\Phi}^M$ and $\bar{\Phi}^J$.

Theorem 5 *For any $(x,y) \neq (N_A, N_B)$ and $(x,y) \neq (0,0)$, it follows that*

$$\underline{\Phi}^J(x,y) < \underline{\Phi}^M(x,y); \qquad \bar{\Phi}^J(x,y) \geq \bar{\Phi}^M(x,y). \tag{18}$$

On the boundaries $(0,0)$ and (N_A, N_B) we have:

$$\underline{\Phi}^J(0,0) < \underline{\Phi}^M(0,0); \qquad \bar{\Phi}^J(N_A, N_B) > \bar{\Phi}^M(N_A, N_B);$$

$$\bar{\Phi}^J(0,0) = \bar{\Phi}^M(0,0) = 1; \qquad \underline{\Phi}^J(N_A, N_B) = \underline{\Phi}^M(N_A, N_B) = 1.$$

Thus, this theorem indicates that the nonintegrated firms' combined share of every margin s^J exceeds that of the the merged firm AB. Recall that Theorem 3 above indicated that nonintegrated firms realize a greater share of S^J than a merged firm when S^J is an increasing function. If $s^J \geq 0$, equation (8) indicates that this condition is satisfied immediately. Identically, we can immediately observe the same result by employing the inequality $\bar{\Phi}^J(x,y) \geq \bar{\Phi}^M(x,y)$ from theorem 5 in theorem 4.

Summarizing, when operating jointly over the market, the share of joint production S^J that A and B realize together can be written as a share, given by $\bar{\Phi}^J(i,j)$, of each of the margins of joint production. Each firm realizes half of this total. A firm's share of the output that it can realize without recourse to the other firm remains unaltered by joint production. If the firms instead choose to merge, AB's share of the joint margins of production that are realizable over the market falls, from $\bar{\Phi}^J(i,j)$ to $\bar{\Phi}^M(i,j)$. Hence the firm loses share $\sum_{i=0}^{N_A}\sum_{j=0}^{N_B}(\bar{\Phi}^J(i,j) - \bar{\Phi}^M(i,j))s^J(i,j)$ of the already realized joint surplus in the process of merging. However, insofar as the merged firm's marginal production possibilities differ from joint production possibilities by s^M, the firm gains (loses) a share of any such new (foregone) production. In particular, the firm gains (loses) $\sum_{i=0}^{N_A}\sum_{j=0}^{N_B}\bar{\Phi}^M(i,j)s^M(i,j)$. The split of surplus between a firm and its employees from production that can be realized without recourse to the other firm remains unaltered by merged production.

We end this section by presenting an example to give some intuition for these theorems. For functions $s^J(i_1,i_2)$, $s^M(i_1,i_2)$, $\bar{\Phi}^J(i_1,i_2)$, and $\bar{\Phi}^M(i_1,i_2)$, we define the matrices $\mathbf{s^J}$, $\mathbf{s^M}$, $\bar{\mathbf{\Phi}}^{\mathbf{J}}$, and $\bar{\mathbf{\Phi}}^{\mathbf{M}}$ respectively such that the (i,j)th element of the matrix is given by its corresponding function evaluated at $(i-1, j-1)$.

Example 1 Let $N_A = 2$, $N_B = 2$. Let $\mathbf{s^J}$ and $\mathbf{s^M}$ be given by:

$$\mathbf{s^J} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & k^J & l^J \\ 0 & m^J & n^J \end{vmatrix};$$

$$\mathbf{s^M} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & k^M & l^M \\ 0 & m^M & n^M \end{vmatrix}.$$

Recall that defining s^J and s^M uniquely defines S^J and S^M, and that Theorems 1 and 2 tell us that stand-alone production functions F^A and F^B are irrelevant to the merger decision.

Calculating $\bar{\mathbf{\Phi}}^{\mathbf{J}}$ and $\bar{\mathbf{\Phi}}^{\mathbf{M}}$ from Theorem 4 for $N_A = 2$, $N_B = 2$ gives

$$\bar{\mathbf{\Phi}}^{\mathbf{J}} = \frac{1}{30}\begin{vmatrix} 30 & 25 & 15 \\ 25 & 22 & 14 \\ 15 & 14 & 10 \end{vmatrix};$$

$$\bar{\mathbf{\Phi}}^{\mathbf{M}} = \frac{1}{30}\begin{vmatrix} 30 & 20 & 10 \\ 20 & 16 & 9 \\ 10 & 9 & 6 \end{vmatrix}.$$

Note that, as Theorem 5 indicates, $\bar{\Phi}^J(i,j) > \bar{\Phi}^M(i,j)$, for all $(i,j) \neq (0,0)$, and $\bar{\Phi}^J(0,0) = \bar{\Phi}^M(0,0) = 1$. It follows from equation (15) that if the two firms produce jointly over the market, their combined split of joint production S^J will be given by,

$$\frac{1}{30}(22k^J + 14l^J + 14m^J + 10n^J).$$

If instead the two firms merge, equation (13) indicates that their combined split of joint product S^J and merged product S^M is given by,

$$\frac{1}{30}\left(16(k^J + k^M) + 9(l^J + l^M) + 9(m^J + m^M) + 6(n^J + n^M)\right).$$

Taking the difference of these two expressions, it follows that the two firms profit more operating jointly over the market than by merging if and only if

$$6k^J + 5l^J + 5m^J + 4n^J \geq 16k^M + 9l^M + 9m^M + 6n^M. \tag{19}$$

There are several features worth noting from this example. In particular:

1. Higher margins s^J make joint production relatively more attractive than mergers even though such output is produced equally by the merged firm and the nonintegrated firms. The merged firm AB must share more of each of these margins with employees. Conversely, higher margins s^M make mergers relatively more attractive, as such margins are (trivially, by definition) only realized under merger.

2. The more inframarginal the margin, the more this margin adds to the firms' profitability under either joint or merged production. This is not surprising given intrafirm bargaining; equations (2) and (3) indicates that for any fixed manner of production, firms engaged in bargaining with employees capture more of a margin of production the more inframarginal it is.

3. Additionally, the more inframarginal a margin s^J, the more of it is foregone to employees when changing from nonintegrated production to merged production. For example, in equation (19), the firms lose $\frac{6}{30}$th of margin k^J (the $(1,1)$ margin of joint production) if they choose to merge, while they lose $\frac{5}{30}$th of margins l^J and m^J (the $(1,2)$ and $(2,1)$ margins of joint production) and $\frac{4}{30}$th of margin n^J (the $(2,2)$ margin of joint production).

4. The share of the margins s^J lost by merger is less than the share of the corresponding margin s^M gained by a merger, with the ratio of the former share lost to the latter share gained falling the more inframarginal the margin. Thus, the ratio of the coefficients on k^J and k^M in equation (19) is $\frac{3}{8}$, which is less than the corresponding ratio of $\frac{2}{3}$ for n^J and n^M.

4 Changes in Employment Levels

We now allow firms to change their employment levels conditional on the integration decision. In order to focus on this employment choice and its consequences, throughout this section we set $S^M \equiv 0$. That is, we suppose that joint production over the market is productively identical to production in a merged firm. As the analysis in the previous section indicated, when $S^M \equiv 0$ and S^J is strictly increasing, for any fixed number of workers, firm profits will be higher under nonintegration than for a merger. However, when employment can be altered, the merged firm has an advantage in coordinating employment across the divisions. Under our assumption that firms cannot contract on employment levels, nonintegrated firms will in general not be able to coordinate on optimal employment, because each firm does not fully capture its own contribution to joint surplus S^J and the impact its hiring decision has on the other firm's wages. For example, if workers in the two firms are good substitutes for one another, excess workers in one firm may serve to lessen the threat of a worker leaving the other firm. However, in overhiring, a firm will only capture a fraction of this positive bargaining externality when it negotiates with its counterpart.

In general, the analysis when employment is variable becomes rather complicated and admits many possibilities. Nonetheless, we can illustrate the general economic effects of merger through a series of examples.

Example 2 – The Bargaining Effect

Let $F \equiv F^A = F^B$, with $F(0) = 0$, $F(n) = 1 \quad \forall n \geq 1$; $S^J(i,j) = S$ if $i \geq 2$ and $j \geq 2$, and $S^J(i,j) = 0$ otherwise; and $\underline{w}_A = \underline{w}_B = .75$. Thus, if divisions A and B do not interact at all, the first employee for each division generates revenue of 1, whereas all future employees add nothing. Under joint production, if there are two or more workers both in division A and in division B, an extra product of S is realized. The following table depicts the total output $F^A(i) + F^B(j) + S^J(i,j)$ that A and B can therefore produce jointly (either over the market or within a merged firm) with i and j employees respectively:[31]

	$j=0$	$j=1$	$j=2$	$j=3$
$i=0$	0	1	1	1
$i=1$	1	2	2	2
$i=2$	1	2	2+S	2+S
$i=3$	1	2	2+S	2+S

It follows from equation (2) that if the two divisions did not produce jointly, each would choose to hire one employee, and realize profits of $\frac{1}{8}$. (The worker's outside wage is $\frac{3}{4}$, and she also captures

[31] We only give $S^J(i,j)$ for $i,j \leq 3$ in this table. We will take it to be understood here and in further examples that there is no further gain to production beyond this range we explicitly depict in the table.

half of the remaining surplus of $\frac{1}{4}$, leaving $\frac{1}{8}$ for hte firm.) While hiring a second employee would lower wages, this would not be sufficient to compensate for the lack of production. However, under joint production, there is an extra benefit of S to be realized provided that each firm hires at least two employees. We will let S vary from 0 to 2 in this example. Employing Theorems 1 and 2, it follows that equilibrium employment as a function of S is as follows:

Merged Production:

If $0 \leq S \leq 2$, AB chooses the employment profile of (1,1) and realizes profits of $\frac{1}{4}$.

Nonintegrated Production

If $0 \leq S < \frac{5}{4}$, the unique Nash equilibrium in the nonintegrated firms' employment decision is the employment profile of (1,1), yielding profits of $\frac{1}{8}$ for each of the firms.

If $\frac{5}{4} \leq S \leq 2$, both the employment profiles of (2,2) and (1,1) are Nash equilibrium in the nonintegrated firms' employment decision. The (2,2) employment equilibrium Pareto dominates the (1,1) equilibrium for the firms, yielding profits of $\frac{1}{12}(2S-1)$ for each firm.

We presume that A and B can coordinate on a Pareto dominating equilibrium in the case where one exists. Then, comparing the outcome under merged and nonintegrated production, when $s \geq \frac{5}{4}$, employment falls from (2,2) to (1,1) upon merger. This is due to the effect analyzed in the previous section: the merged firm concedes more of the margin S to employees than the nonintegrated firms do. Consequently, when S exceeds $\frac{5}{4}$, AB does not have sufficient incentive to hire employees to produce this margin while firms A and B do. Over this range, the combined profits from A and B exceeds that of the merged firm. If instead $0 \leq S < \frac{5}{4}$, employment is (1,1) in both cases, and here, profits are the same.

In Example 2 wages and the return on labor were not high enough to induce any of the firms to hire redundant labor. That is, none of the firms overhired productively inefficient labor, for the leverage it would provide in employee negotiations. Given that such redundant labor will generally exert a positive externality on the wage bill of a jointly producing firm, one might suspect that the merged firm would have greater incentive to hire redundant labor than the nonintegrated firms. This intuition is supported by scaling up the benefits from production in this previous example, while holding wages fixed. By increasing benefits sufficiently, all firms will have an incentive to overhire, as now the gain this yields in bargaining outweights the cost of hiring the redundant workers.

Example 3 – Coordination Effect With Bargaining Externalities

Consider the same technology as in example 2 above, except that now we will consider a much larger gain to joint production. In particular, let $S = 6$. Then, equilibria are as follows:

Merged Production:

Firm AB chooses the employment profile of (4,4) and realizes profits of 1.51.

Nonintegrated Production

Pure-strategy Nash equilibria in the joint firms' employment decision are given by the employment profiles of (3,3), (2,2) and (1,1). Profits for each of the two firms in these three equilibria are given, respectively, by 1.34, .917 and .125.

Note that for the Pareto dominant nonintegration equilibrium, the combined profits of A and B is significantly higher than that of AB. As S increases, the gain to nonintegration from what we have labeled the bargaining effect also increases. A and B would do even better if they each hired 4 employees each like the merged firm; this would yield profits of 1.41 for each of them. However, such a profile is not an equilibrium; each of the firms would have an individual incentive to cut hiring back to 3 employees. Thus, the merged firm overhires more than the nonintegrated firms because it internalizes the beneficial effects that hiring redundant employees in one division may have for the wage bill of the other division.

Example 3 suggests that one setting under which merger increases employment is when there is sufficient incentive to hire redundant employees. The merged firm internalizes the coordination externalities faced by the nonintegrated firms. However, this example also indicates when such externalities exist (i.e., when gains to joint production are high), the bargaining effect (of example 2) is likely to have an even larger effect. Consequently nonintegration is more profitable; an integrated firm could increase value by splitting apart. While the subsequent firms would then lessen their respective employment in a productively inefficient manner, this would be more than compensated for by the lower pay that their remaining employees received.

Another manner in which a merger can lead to an increase in employment is when the division of joint surplus is not divided by the firms in proportion to the hiring they must undertake in order to realize this surplus. Such a production externality is also internalized through a merger. Here, the merged firm may be more profitable than its nonintegrated counterparts combined. We choose production to be very similar to that in the above examples, in order to highlight the distinction.

Example 4 – Coordination Effect With Production Externalities

Let F^A, F^B, $\underline{w}_A$ and $\underline{w}_B$ be as in example 2 above. Let $\mathbf{S^J}$ be given by $S^J(i,j) = S$ if $i \geq 2$

and $j \geq 1$, or if $j \geq 2$ and $i \geq 1$, and $S^J(i,j) = 0$ otherwise. Thus, this example differs from example 2 only in that the margin S can now be realized with one worker in one division and two in the other, instead of requiring two in each division. The following table depicts the total output $F^A(i) + F^B(j) + S^J(i,j)$ that A and B can produce jointly in this case with i and j employees respectively:

	$j=0$	$j=1$	$j=2$	$j=3$
$i=0$	0	1	1	1
$i=1$	1	2	2+S	2+S
$i=2$	1	2+S	2+S	2+S
$i=3$	1	2+S	2+S	2+S

We will consider S between 0 and 2. Once again employing Theorems 1 and 2, it follows that equilibrium employment as a function of S is as follows:

Merged Production:

If $0 \leq S < \frac{5}{6}$, AB chooses the employment profile of (1,1) and realized profits of $\frac{1}{4}$.

If $\frac{5}{6} \leq S \leq 2$, AB chooses either the employment profile of (2,1) or (1,2) and realized profits of $\frac{S}{4} + \frac{1}{24}$.

Nonintegrated Production

If $0 \leq S < \frac{25}{24}$, the unique Nash equilibrium in employment decisions is the employment profile of (1,1), yielding profits of $\frac{1}{8}$ for each of the firms.

If $\frac{25}{24} \leq S \leq 2$, the employment profiles of (2,1) and (1,2) are the only pure-strategy Nash equilibria in the nonintegrated firms' employment decision. In these equilibria, profits are $\frac{S}{5} + \frac{1}{8}$ for the firm hiring one employee, and $\frac{S}{5} - \frac{1}{12}$ for the firm hiring two employees.

There are several things to note about this example. While the equilibria for the merged firm and the nonintegrated firms correspond when $0 \leq S < \frac{5}{6}$ and when $\frac{25}{24} < S \leq 2$, over the range of $\frac{5}{6} \leq S < \frac{25}{24}$ the merged firm hires more than the nonintegrated firms. Furthermore, while we know from Section 2 that over the range where the hiring is the same, the combined profits for A and B exceed those for AB,[32] profits for AB are higher when $\frac{5}{6} \leq S < \frac{25}{24}$ and the employment profiles diverge.

Intuitively, the sum of the profits of the nonintegrated firms would be higher when $\frac{5}{6} \leq S < \frac{25}{24}$

[32]When the employment profile is (1,1) under merger or joint production, this inequality only holds weakly (i.e., profits are the same), as S^J is 0 over the relevant range. When instead the employment profile in both settings is (1,2) or (2,1), profits under joint production strictly dominate that of the merged firm.

if one of them were to hire a second employee, but the firm doing so would be individually worse off. If the two firms could write a contract on one another's employment level, a side transfer from the firm hiring one employee to the firm hiring two employees would ensure the better outcome. However, given our assumption that employment is noncontractible, the two nonintegrated firms cannot coordinate employment decisions as the merged firm can.

At the risk of belaboring a point, it is worth noting the difference between examples 2 and 4. In example 2 both firms needed to hire a second employee in order to realize benefit S. The gain to each firm if they both did so would exceed the cost of hiring this additional employee. (Note that the gain consists of both sharing in S and in lowering the wage for the first employee that each firm hires.) In contrast, in example 4 only one firm needed to hire a second employee for the joint margin S to be realized. Both firms would then share equally in S as they are both essential to its realization. Consequently, each firm would prefer the other firm hire the second employee, and therefore, the merged firm which internalizes this cross-firm externality performs better.

Examples 2 and 4 suggest a crucial distinction between production technologies where firms must both contribute labor to realize joint surplus and where the contribution of labor by one firm may yield joint surplus to be shared by both firms. A particularly interesting example of the latter is when labor hired by one firm can serve as a substitute for labor hired by the other. Our final example considers such a case.

Example 5 – Substitute Labor

Let F^A, F^B, $\underline{w}_A$ and $\underline{w}_B$ be as in example 2 above. Let $\mathbf{S^J}$ be given by $S^J(i,j) = S$ if $i \geq 2$ and $j = 0$, or if $j \geq 2$ and $i = 0$, and $S^J(i,j) = 0$ otherwise. In this example (as in the previous examples), the first worker at each firm can produce 1. A further worker is beneficial under joint production, but only if the other firm does not have its first employee. That is, an employee from firm A can substitute for a firm B employee, yielding production of S in place of 1. Consistent with the interpretation of imperfect substitutes, we consider $0 \leq S \leq 1$,[33] The following table depicts the total output $F^A(i) + F^B(j) + S^J(i,j)$ that A and B can produce jointly in this case with i and j employees respectively:

	$j=0$	$j=1$	$j=2$	$j=3$
$i=0$	0	1	1+S	1+S
$i=1$	1	2	2	2
$i=2$	1+S	2	2	2
$i=3$	1+S	2	2	2

[33] Note that this example is a simple technology where S^J is not an increasing function of labor.

Equilibrium in this example is as follows: *Merged Production:*

If $0 \leq S \leq 1$, AB chooses the employment profile of (1,1) and realizes profits of $\frac{1}{4}$. (If $S = 1$, AB could also realize these same profits by choosing the employment profiles of (0,2) or (2,0).)

Nonintegrated Production

If $0 \leq S \leq 1$, there are three pure-strategy Nash equilibria in the nonintegrated firms' employment decisions, given by the employment profiles of (1,1), (2,0) and (0,2). The (1,1) equilibrium yields profits of $\frac{1}{8}$ for both firms, while the (2,0) and (0,2) equilibria yield profits of $\frac{S}{4}$ for the firm employing two workers and profits of $\frac{S}{4} - \frac{1}{12}$ for the firm employing no employees. Consequently, for $0 \leq S < \frac{2}{3}$ the Nash equilibrium of (1,1) yields the highest combined profits, given by $\frac{1}{4}$. When instead $\frac{2}{3} < S \leq 1$ profits are higher in the (0,2) and (2,0) than the (1,1) equilibrium, but profits in these asymmetric equilibria only Pareto dominate the (1,1) equilibrium when $\frac{5}{6} < S \leq 1$.

Note that when $\frac{2}{3} < S \leq 1$, the nonintegrated firms do better combined under the less efficient production profile. A (0,2) or (2,0) profile instead of a (1,1) profile lowers production from 2 to $1 + S$. However, this is mitigated by the fact that now both firms are essential to realize S. In contrast, under the production profile of (1,1), only one firm is essential for each margin of production. By making both firms essential, they are able to extract more of the surplus in negotiations with employees, and this in turn makes the inefficient (0,2) or (2,0) profiles optimal to the firms if S is high enough. Under merged production, there is no such bargaining effect to offset the lost production under the (0,2) or (2,0) profile, and consequently employment of (1,1) is optimal. Whenever in equilibrium the nonintegrated firms produce under the (2,0) or (0,2) profiles, their combined profits exceed those of the merged firm.

Taken together, these examples are indicative of several general tendencies. In particular:

1. For margins of joint production that require labor inputs of each firm proportional to the firms' division of gains from these margins, nonintegrated firms will generally produce more and profit more than the merged firm. Intuitively, the nonintegrated firms capture more of these joint margins than the merged firm in negotiations with employees, and the externalities they impose on one another are less severe when their labor inputs are similar to their division of surplus.

2. In contrast, if margins of joint production instead require labor inputs of each firm that are very different to the firms' division of gains from these margins, the merged firm will generally

produce more than the nonintegrated firm, and may realize higher profits. While production of a margin would increase the sum of the profits for the nonintegrated firms if it increases profits for the merged firm, it may not be individually rational for the firm with the higher labor requirement. Profits for the merged firm will be higher if this gain from coordination is large enough to exceed the bargaining effect.

3. If the production function is such that firms have incentives to hire redundant labor to facilitate employee bargaining,[34] the merged firm will tend to hire more redundant labor than the nonintegrated firms. Redundant labor imposes a positive bargaining externality on the other firm which is internalized by a merger. While this extra hiring is beneficial to the merged firm, technologies that brings about redundant hiring (high margins relative to wages) yields higher profits to nonintegrated firms through the bargaining effect, and therefore, generally makes nonintegration optimal in such circumstances.

Several of these conditions may of course be present simultaneously. For example, in example 2 a margin S was realized under equal labor inputs from both firms, leading to less hiring for the merged firm. However, increasing S, as in example 3, makes redundant employees beneficial, which yields more excess hiring by the merged firm than the nonintegrated firms.

5 Comparison with Hart and Moore (1990)

It is instructive to compare the bargaining effect of nonintegration in our model with the consequences of integration in Hart and Moore (1990). In order to focus on this distinction, we presently hold the number of employees fixed (no coordination effect) and assume $S^M = 0$ (no productive difference between merger and nonintegration).

First note that the manner that surplus is divided in our model is essentially the same as that employed by Hart and Moore.[35] Nonetheless, the two models yield very different implications for the scope of the firm, and in particular, for the ownership of complementary assets. We now discuss the reason for this.

Hart and Moore presume contracts are incomplete due to the inability to contract on ex-

[34]Stole and Zwiebel (1996a) show that the extent of overhiring is captured by a property of the production function (at any given point of production) denoted the *frontload factor*. The frontload factor can, loosely speaking, be thought of as a generalization of concavity to a complete ordering of functions.

[35]Hart and Moore start with the primitive of the characteristic function, and directly assume that surplus is divided according to Shapley values. We instead start with the underlying production functions, together with the noncooperative notion of a wage and profit profile that is immune to bilateral renegotiations. However, as discussed above, when we simplify our setting to considerations of equivalent bargaining power, as we do throughout our analysis in this paper, our solution is given by the Shapley value for a natural corresponding characteristic function.

ante investments. This in turn yields inefficiencies in such investments: parties do not invest optimally, due to future hold-up of such investments. Additionally, they presume that ownership is chosen ex-ante in a manner that minimizes inefficiencies that arise from this noncontractibility. In such a setting, Hart and Moore find that strictly complementary assets should be owned together (Proposition 8 in their paper). Intuitively, inefficient investment is minimized when as few parties as possible are able to hold-up productive benefits that derive from the investment of others. If a pair of machines is always employed together in a manner that interacts with the ex-ante investment of others, putting both machines in one agent's hands rather than two will always weakly lessen the number of party's necessary to realize the gains from this investment.[36] This in turn implies that when the two strictly complementary assets are owned together, all investors will realize weakly larger gains from their investments, lessening investment distortions. A number of their other implications for ownership follow from similar logic.

Our results differ due to two important differences in the setting. First, we consider an alternative form of contractual incompleteness: instead of noncontractible ex-ante investments, we assume that the labor relationship is at-will and consequently noncontractible. Consequently, our inefficiency takes a different form from that of Hart and Moore. In our setting, the threat of employee hold-up and renegotiation leads firms to distort their labor decisions.

A second difference between our model and Hart and Moore is that we adopt the perspective that ownership structure is determined by ex-ante firm profit maximazation, rather than overall efficiency. The difference between efficiency and profit maximization is of course equivalent to the welfare of the noncorporate sector – in our setting, the employees. In our setting, firms make an integration decision based on how the they will will fare relative to employees in future negotiations. An ex-ante efficiency perspective would instead argue for including employees welfare in such a calculation, on the grounds that employees could bribe the firm to undertake the efficient outcome instead of maximizing profits.

At least in our setting, we would argue that there are several good reasons to adopt a profit maximizing notion of integration rather than an efficiency maximizing notion. First, employees often may not acquire hold-up power until they are trained for specific tasks in the firm, or are locked-in in some other manner. Consequently, there may be no representative to negotiate on behalf of future employees when firms consider mergers.

Second, even if all affected employees are already locked-in when the integration decision is made, it might not be possible for firms to commit contractually to a particular ownership structure.

[36] Weakly, because the second party that no longer owns the other machine might be essential for other reasons. For example, it may be the party that has undertaken the investment in question.

In many settings, future decisions to combine or break apart one's assets is an inalienable residual right of ownership. Nor could employees, in our setting, agree to accept lower future wages, on the condition that the firms remain integrated, as it is precisely the non-enforceability of wage agreements that underlie employees hold-up power in our bargaining model. Finally, liquidity constraints and coordination problems among employees are likely to constrain employee ownership of assets.

Our manner of contractual inefficiency, combined with our presumption of a profit maximization criterion of ownership reverses many standard Hart-Moore implications. For example, instead of complementary assets being owned together, our bargaining effect indicates a gain to separating the assets. Intuitively, in both papers, separating ownership implies that two firms rather than one will be essential for (at least some fraction of) the employee's production to be beneficial. In Hart and Moore this decreases overall efficiency as it lowers an employee's incentive to invest. In our setting, however, there are no ex-ante investments, and furthermore, such a split yields higher firm profits, as it lessens employee hold-up power and therefore employee wages. Moreover, the bargaining effect in our model reverses many of the other Hart and Moore implications that rely on a similar complementarity reasoning.

6 Conclusion

We have explored the implications for mergers and the scope of the firm in a sensible incomplete contracting environment. In particular, we restrict contracts only by presuming labor contracts are nonbinding (at-will) and therefore subject to renegotiation, and that employment levels cannot be contracted across firms. Employing a multilateral bargaining framework to analyze this environment yields a rich set of implications for mergers and the scope of the firm, and can serve to rationalize commonly observed merger-related renegotiations.

Among other results, our analysis indicates that if production possibilities are not altered by a merger, for any fixed number of employees, merging will yield higher employee wages. Consequently, in such a setting, profits will be higher under nonintegration. This may be offset, however, by an advantage that the merged firm has in coordinating its employment decisions across its divisions. Considering such changes in employment, our analysis suggests that profits will be higher under a merger when relative labor inputs necessary for joint production across firms differs substantially from the relative bargaining power of these firms. In such a setting, the merged firm is likely to hire more than its nonintegrated counterparts. Conversely, nonintegrated firms will hire more and will be more profitable when gains from joint production (realizable both over the market or under integration) are large.

A final remark on the approach taken in this paper is perhaps in order. Starting with the property rights view that the boundary of the firm is determined by contractual incompleteness, we have identified a source of incompleteness that we feel is well-motivated in practice, yet differs from that which is typically modeled in the literature. In particular, while there is a huge literature that considers the implication of ex-ante inefficient investments on ownership, relatively little work has considered the implications other plausible manners of contractual incompleteness on the same questions. We have then analyzed the consequences this manner of contractual incompleteness for mergers and the boundary of the firm. While we believe the manner of contractual incompleteness on which we focus is quite important for firms with a high level of human specific capital, we by no means believe this is the only important alternative to the standard noncontractable ex-ante investments assumption. The program of carefully indentifying other common forms of contractual incompleteness and exploring their implications for the boundary of the firm, strikes us as a rich area for further research.

Appendix

Proof of Theorem 1: AB's profits are given by equation (3), its Shapley value in the cooperative game defined by characteristic function (4) and (5). Shapley values can be expressed as the expected contribution according to the characteristic function of an agent to the set preceding agents, under an equal weighting of all possible player orderings.[37] From this characteristic function, it follows that, for a given ordering, if the preceding set of players consists of i_1 type A workers and i_2 type B workers in S, then the firm AB's contribution to this coalition is given by,

$$F^A(i_1) + F^B(i_2) + S^J(i_1, i_2) + S^M(i_1, i_2) - \underline{w}_A i_1 - \underline{w}_B i_2.$$

Averaging over all possible orderings yields,

$$\pi^M_{AB}(N_A, N_B) = \frac{1}{N+1} \sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \left(\frac{\binom{N_A}{i_1}\binom{N_B}{i_2}}{\binom{N}{i_1+i_2}} \right) \left(F^A(i_1) + F^B(i_2) + S^J(i_1, i_2) + S^M(i_1, i_2) - \underline{w}_A i_1 - \underline{w}_B i_2 \right).$$

With some work, one can show that $\forall i_1 \leq N_A$, the following equality holds:[38]

$$\sum_{i_2=0}^{N_B} \left(\frac{\binom{N_A}{i_1}\binom{N_B}{i_2}}{\binom{N}{i_1+i_2}} \right) = \frac{N_A + N_B + 1}{N_A + 1}.$$

Consequently,

$$\begin{aligned} \pi^M_{AB}(N_A, N_B) &= \frac{1}{N+1} \sum_{i_1=0}^{N_A} \frac{N+1}{N_A+1} (F^A(i_1) - \underline{w}_A i_1) + \frac{1}{N+1} \sum_{i_2=0}^{N_B} \frac{N+1}{N_B+1} (F^B(i_2) - \underline{w}_B i_2) \\ &+ \frac{1}{N+1} \sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \left(\frac{\binom{N_A}{i_1}\binom{N_B}{i_2}}{\binom{N}{i_1+i_2}} \right) \left(S^J(i_1, i_2) + S^M(i_1, i_2) \right). \end{aligned}$$

The desired result follows immediately by substituting in $\pi_A(N_A)$ and $\pi_B(N_B)$. $\|$

Proof of Theorem 2 For firms A and B, profits are given by profits that they would realize if they produced in isolation from one another and negotiated with their employees accordingly, given by $\pi_A(N_A)$ and $\pi_B(N_B)$ respectively, plus an even split of the share of joint surplus S^J that they realize. Thus, the result follows if it is shown that the fraction of S^J realized by the two firms combined is given by $\sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \mu^J(i_1, i_2) S^J(i_1, i_2)$. Given the manner that we have imposed balanced contributions, we have shown elsewhere that S^J is split by employees and the two firms in

[37] See, for example, Owen Chapter 10.

[38] One manner of proof is by induction over (N_A, N_B).

accord with Shapley values from the associated cooperative game with the characteristic function given as follows:[39] For any S,

$$\begin{aligned}
v(S) &= F^J(i_1(S), i_2(S)) && \text{if } A, B \in S; \\
v(S) &= F^A(i_1(S)) + \underline{w}_B i_2(S) && \text{if } A \in S, B \notin S; \\
v(S) &= F^B(i_2(S)) + \underline{w}_A i_1(S) && \text{if } B \in S, A \notin S; \\
v(S) &= \underline{w}_A i_1(S) + \underline{w}_B i_2(S) && \text{if } A, B \notin S.
\end{aligned}$$

As noted the preceding proof, Shapley values can be expressed as an agents' expected contribution to the set of preceding agents, under an equal weighting of all possible orderings. Additionally, from above, we only need to show that this implies that S^J is divided as stated in the theorem; thus, we need only concern ourselves with terms in the firms' profits containing S^J. Given the characteristic function above, for any ordering of employees and the firm, only the contribution of the second firm in the ordering will depend on S^J. In particular, for a particular ordering $\mathcal{P}$, let $i_1(\mathcal{P})$ and $i_2(\mathcal{P})$ represent the number of type A and type B workers, respectively, that precede the second firm in $\mathcal{P}$. Then if firm A follows firm B in $\mathcal{P}$, A's marginal contribution to the set preceding it in $\mathcal{P}$ is given by:

$$S^J(i_1(\mathcal{P}), i_2(\mathcal{P})) + F^J(i_A(\mathcal{P})) - \underline{w}_A i_A(\mathcal{P}).$$

Therefore, taking the expectation of $S^J(i_1(\mathcal{P}), i_2(\mathcal{P}))$ over an equal weighting of all possible orderings $\mathcal{P}$ will give the share of S^J realized by firms A and B combined.

Now consider all possible orderings of firm A, firm B, N_A type A employees and N_B type B employees. The probability, over equal weighting of all orders, that exactly i_1 type A employees and i_2 employees will precede firm A is given by $\mu^M(i_1, i_2)$. Furthermore, conditional on firm A being preceded by exactly i_1 type A employees, and i_2 type B employees, the probability that firm A will also precede firm B is given by $\frac{(i_1+i_2+1)}{N+2}$. Multiplying these together implies that the probability firm A will follow firm B in an ordering where A is also be preceded by i_1 type A employees and i_2 type B employees is given by $\frac{1}{2}\mu^J(i_1, i_2)$. Symmetry implies an identical probability for firm B. Combining these two and taking expectations over all possible values of (i_1, i_2) immediately gives the desired result. $\|$

Proof of Theorem 3: It is useful for both this proof and the following one to define for any configuration of type A and B employees, (i, j) such that $0 \leq i \leq N_A$, $0 \leq j \leq N_B$, its *mirror point* by $(N_A - i, N_B - j)$, that is, its reflection across the diagonal from $(0, N_B)$ to $(N_A, 0)$. The

[39] To see this, simply employ the proof of Theorem 4 in Stole and Zwiebel (1996b) to the present setting. (That is, show that balanced contributions holds for all pairs of players and that the payoff structure is efficient.)

relationship defines a symmetric 1-1 correspondence; we will call a point and its mirror point a mirror pair. Also define $g(i,j) \equiv \frac{2(i+j+1)}{N+2}$; thus $\mu^M(i,j) = \mu^J(i,j)g(i,j)$.

Mirrored pairs will be useful for profit comparisons due to the the following two properties that hold for all such pairs:

$$\mu^J(i,j) = \mu^J(n_A - i, n_B - j); \tag{20}$$

$$g(i,j) + g(n_A - i, n_B - j) = 2. \tag{21}$$

From equations (9)-(12), it follows that when $S^M \equiv 0$,

$$\pi_A^J(N_A) + \pi_B^J(N_B) - \pi_{AB}^M(N_A, N_B) = \sum_{i_1=0}^{N_A} \sum_{i_2=0}^{N_B} \mu^M(i_1,i_2)\left(g(i_1,i_2) - 1\right) S^J(i_1,i_2). \tag{22}$$

Consider summing the argument on the right hand side of equation (22) over a mirror pair. Using (20) and (21), one obtains,

$$\mu(i,j)(1 - g(i,j))\left(S(N_A - i, N_B - j) - S(i,j)\right). \tag{23}$$

Since S is increasing in both its arguments and $g(i,j) \geq 1$ if and only if $i + j \geq \frac{N_A + N_B}{2}$, it follows if $i \leq \frac{N_A}{2}$ and $j \leq \frac{N_B}{2}$, or if both these inequalities are simultaneously reversed, then expression (23) will be positive for this pair. A difficulty arises, however, because no such simple statement can be made for off-diagonal mirrored pairs, such as when $i < \frac{N_A}{2}$, $j > \frac{N_B}{2}$. This is handled by grouping the sum in equation (22) into sums over pairs of mirrored pairs, where we match on-diagonal and off-diagonal pairs in a manner that ensures that the positive contribution of the on-diagonal pair dominates any possible negative contribution of its respective off-diagonal pair.

In particular, it follows from equations (22) and (23) that,[40]

$$\begin{aligned}
&\pi_A^J(N_A) + \pi_B^J(N_B) - \pi_{AB}^M(N_A, N_B) = \\
&\quad \sum_{i=0}^{\frac{N_A}{2}} \sum_{j=0}^{\frac{N_B}{2}} \big(\mu^M(i,j)\left(g(i,j) - 1\right) S^J(i,j) + \mu^M(N_A - i, N_B - j)\left(g(N_A - i, N_B - j) - 1\right) S^J(N_A - i, N_B - j) \\
&+ \quad \mu^M(i, N_B - j)\left(g(i, N_B - j) - 1\right) S^J(i, N_B - j) + \mu^M(N_A - i, j)\left(g(N_A, j) - 1\right) S^J(N_A - i, j)\big) \\
&= \quad \sum_{i=0}^{\frac{N_A}{2}} \sum_{j=0}^{\frac{N_B}{2}} \big(\mu^M(i,j)(1 - g(i,j))(S^J(N_A - i, N_B - j) - S^J(i,j)) \\
&+ \quad \mu^M(i, N_B - j)(1 - g(i, N_B - j))(S^J(N_A - i, j) - S^J(i, N_B - j))\big).
\end{aligned} \tag{24}$$

We will now show that each term in the final double summation in (24) is weakly positive. Consider any (i,j) such that $i \leq \frac{N_A}{2}$, $j \leq \frac{N_B}{2}$. Since we know from above that for such (i,j),

[40] Note that if both $\frac{N_A}{2}$ and $\frac{N_B}{2}$ are integers, then the $(\frac{N_A}{2}, \frac{N_B}{2})$ term will be quadruply counted in this summation. However, for this term $g(\frac{N_A}{2}, \frac{N_B}{2}) - 1 = 0$, and therefore this will not alter the sum.

$\mu^M(i,j)(1-g(i,j))(S^J(N_A-i,N_B-j)-S^J(i,j))>0$, it is sufficient to show that

$$\left|\mu^M(i,j)(1-g(i,j))(S^J(N_A-i,N_B-j)-S^J(i,j))\right| \geq$$
$$\left|\mu^M(i,N_B-j)(1-g(i,N_B-j))(S^J(N_A-i,j)-S^J(i,N_B-j))\right|. \quad (25)$$

To demonstrate this, first note that since S^J is weakly increasing in both arguments, it follows that

$$\left|(S^J(N_A-i,N_B-j)-S^J(i,j))\right| \geq \left|(S^J(N_A-i,j)-S^J(i,N_B-j))\right|.$$

Furthermore, $0 < g(i,j) \leq g(i,N_B-j)$. Finally, note that $\mu^M(i,j)$ and $\mu^M(i,N_B-j)$ only differ by the binomial term in the denominator. It follows for $i \leq \frac{N_A}{2}$, $j \leq \frac{N_B}{2}$, that $\binom{N}{i+j} \leq \binom{N}{i+N_B-j}$, and therefore, $\mu(i,j) \geq \mu^M(i,N_B-j) > 0$. Putting these results together immediately implies that inequality (25) must hold. Furthermore, since $S^J(N_A,N_B) > S^J(0,0)$ and $g(0,0) < g(0,N_B)$, this inequality is strict for $(i,j)=(0,0)$. Hence it follows from equation (24) that $\pi^J_A(N_A)+\pi^J_B(N_B)-\pi^M_{AB}(N_A,N_B)>0$, completing the proof. $\|$

Proof of Theorem 5: The boundary conditions are immediate from the construction of the distribution functions.

Now consider the transformation from joint to merged production. Equations (20) and (21) imply that,

$$\mu^J(i,j)+\mu^J(n_A-i,n_B-j) \equiv \mu^M(i,j)+\mu^M(n_A-i,n_B-j),$$

for all (i,j). Importantly, however, measure is transferred from the mirror point with the lower coordinate sum to that of the higher coordinate sum. If each has the same coordinate sum, the transformation has no effect on either point; two symmetry points will have the same coordinate sum if and only if N is even and the points lie on the locus $i+j=\frac{N}{2}$.

Choose some interior point $(x,y) \neq (n_A,n_B)$. Consider the sets of points

$$\mathcal{X} \equiv \{(i,j)|i \leq x, \ \ j \leq y\} \quad \text{and} \quad \mathcal{X}^c \equiv \{(i,j)|i \geq x^c, \ \ j \geq y^c\},$$

where (x^c,y^c) is the mirror point of (x,y). By construction, the mirror relationship forms a bijection from $\mathcal{X}$ to $\mathcal{X}^c$. These sets may or may not intersect, and the coordinate sum $x+y$ may be greater or smaller than x^c+y^c. We consider each case in turn.

Case 1: $\mathcal{X} \bigcap \mathcal{X}^c = \emptyset$ and $x+y \leq \frac{N}{2}$. The condition $x+y \leq \frac{N}{2}$ is implies that each point in $\mathcal{X}$ has a lower coordinate sum than its mirror point. (Except for (x,y) which may have the same coordinate sum as (x^c,y^c) if $x+y=\frac{N}{2}$.) Hence, the sum of the measures of $\mathcal{X}$ and $\mathcal{X}^c$ is unaffected by our transformation, but the transformation strictly raises the total measure of $\mathcal{X}^c$ and strictly lowers the total measure of $\mathcal{X}$. (The strict inequalities follow directly from noting that as long as $(x, y) \neq (n_A, n_B)$, and therefore $(x^c, y^c) \neq (0, 0)$, the sets $\mathcal{X}$ and $\mathcal{X}^c$ contain a distinct set of points.) Hence, the cumulative distribution functions are ordered accordingly.

Case 2: $\mathcal{X} \bigcap \mathcal{X}^c = \emptyset$ and $x + y > \frac{N}{2}$. This case is the same as in Case 1, except for the fact that some points in $\mathcal{X}$ necessarily have a higher coordinate sum than their mirror points in $\mathcal{X}^c$. Consider the Figure below.

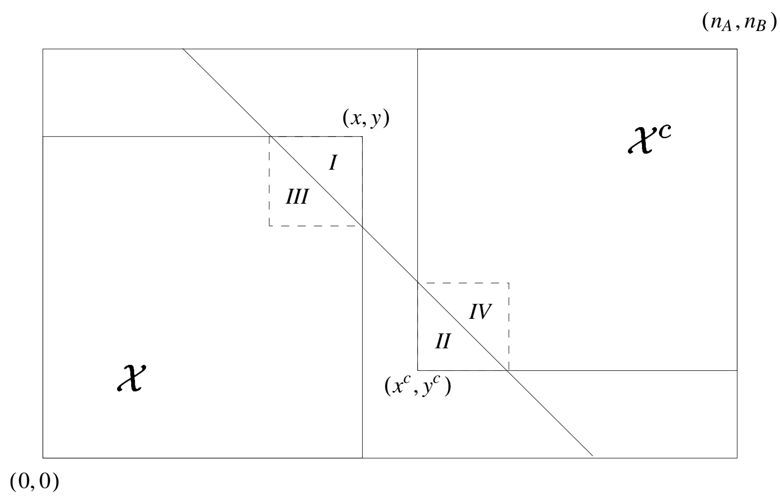

Figure 1: Case 2.

The points in region I in $\mathcal{X}$ have a higher coordinate sum than the corresponding mirror points in region II of $\mathcal{X}^c$. Thus, mass is shifted from $\mathcal{X}^c$ to $\mathcal{X}$ by our transformation. Nonetheless, regions III of $\mathcal{X}$ gains an equivalent amount of mass from region IV of $\mathcal{X}^c$, so the net change in mass over these regions is zero. In the remaining (necessarily non-empty) regions, however, a positive amount of mass is shifted from $\mathcal{X}$ to $\mathcal{X}^c$, providing (as in Case 1, above), thus maintaining our CDF ordering result.

Case 3: $\mathcal{X} \bigcap \mathcal{X}^c \neq \emptyset$. If the sets only intersect at the vertices (i.e., $x = x^c$ and $y = y^c$), then the masses are unchanged on the vertices and otherwise Case 1 applies. Suppose instead that there is an intersection containing more than one point. We necessarily have a subset of points within $\mathcal{X}$ in which each element has a higher coordinate sum than its mirror point in $\mathcal{X}^c$. As in Case 2, we proceed by pairing off these mass shifts against others, leaving a net change. To this end, consider the following figure.

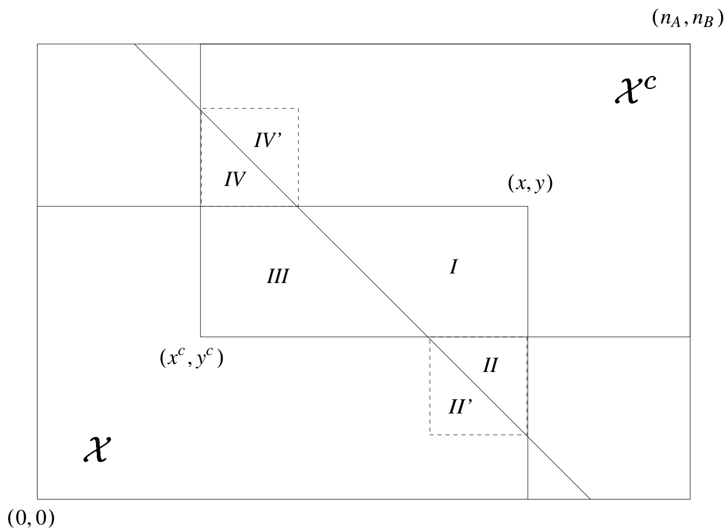

Figure 2: Case 3.

Regions I and II of $\mathcal{X}$ have higher coordinate sums than their mirror points in regions III and IV, respectively, in $\mathcal{X}^c$. First consider the rectangle defined by regions I and III. Within this rectangle, the measure before and after the transformation is unchanged because every point within the intersection has its unique mirror point also within the intersection; the transformation only serves to move the measure around within the centered rectangle, which is of no consequence for evaluating the cumulative distribution functions. Now consider regions II and IV. The net effect over these regions is for positive measure to be moved from $\mathcal{X}^c$ to $\mathcal{X}$. The regions, II' and IV', on the other hand, represent exactly the reverse measure shift, and so the net effect over regions II, II', IV, and IV' is zero. The remaining area outside of the marked regions is non-empty if $(x, y) \neq (n_A, n_B)$, and so a positive measure on net is transferred from $\mathcal{X}^c$ to $\mathcal{X}$, hence providing CDF ordering. $\|$

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