Nonlinear Pricing and Oligopoly

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ABSTRACT. We consider the general problem of price discrimination with nonlinear pricing in an oligopoly setting where firms are spatially differentiated. We characterize the nature of optimal pricing schedules which in turn depends importantly upon the type of private information which the customer possesses – either horizontal uncertainty regarding brand preference or vertical uncertainty regarding quality preference. We show that as competition increases, the resulting quality distortions decrease, as well as price and quality dispersions. Additionally, we indicate conditions under which price discrimination may raise social welfare by increasing consumer surplus through encouraging greater entry.

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1 Introduction

Sorting customers with nonlinear pricing—a practice generally referred to as second-degree price discrimination—is a widespread phenomenon in most market economies. Since Dupuit’s [1849] early discussion of its manifestations in railroad pricing and Pigou’s [1920] later categorization of the phenomenon, it has been well studied by economists in the context of monopoly price setting. Following the work of Mussa-Rosen [1978], Maskin-Riley [1984], and Goldman-Leland-Sibley [1984], economists thoroughly understand that quality or quantity distortions are optimal for some customer groups in order to extract larger rents from other consumers with typically higher demands. More recently, inroads have been made in incorporating these models of monopoly nonlinear pricing into imperfectly competitive environments. Some notable examples are Katz [1984], Spulber [1989a, 1989b], Champsaur-Rochet [1989] and Wilson [1992, ch. 12].

Of particular interest is Spulber’s model of nonlinear pricing with spatial competition which considers competing firms that face consumers with unknown brand preferences. Specifically, the model supposes that the more preferred a particular firm’s brand, the greater the marginal utility the consumer derives from its consumption. This type of model is therefore one of horizontal preference uncertainty. The end result is that firms segment the market place via nonlinear pricing over quality. An alternative formulation which has not been directly studied is a model of vertical preference uncertainty. In such a setting, each consumer’s marginal valuation of quality is unknown by the competing firms, although the consumer’s brand preferences are complete information. Here, a high type consumer values marginal quality higher regardless of the firm; the high type does not necessarily have stronger brand preferences. Of fundamental importance, horizontal preferences are naturally incongruous across firms—a strong preference for one firm implies a weaker preference for the others. Vertical preferences, in contrast, are harmonious across firms—a customer with a

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1 See, for example, the excellent discussions contained in Carlton-Perloff [1990], Tirole [1988], Varian [1989] and especially Wilson [1992].

2 Briefly, Katz [1984] examines a particular form of second-degree price discrimination where there are two types of consumers: informed (with high quantity demands) and uninformed (with low quantity demands and random purchases). Katz’s model is very specialized and seeks to understand welfare consequences of price discrimination in a model where there is no social benefit to more firms serving the market—only losses through fixed costs of production. Champsaur-Rochet [1989] analyze a two-stage duopoly model where firms commit to quality intervals and then choose optimal nonlinear prices; such an equilibrium involves a splitting of the market into a low-quality firm and a high-quality firm. Spulber [1989a, 1989b] analyzes a model where the unknown parameter is the degree of consumer closeness in the product spectrum (e.g., brand loyalty). Wilson [1992, ch. 12] considers more general settings of nonlinear pricing and imperfect competition, but provides explicit solutions for only a few specific utility functions using numerical methods. In related papers, Oren, et al. [1983], consider competitive nonlinear pricing in a “Cournot” game with restrictive conjectures in which each firm takes the market share of its competitors as given. The effect of competition on third-degree price discrimination is studied by Borenstein [1985] and Holmes [1989]. Lastly, Hamilton and Thisse [1994] also analyze a model largely the same as Spulber [1989a], but which further considers the incentives of consumers to reveal location information.
high marginal valuation of quality for one firm will have similar preferences for other firms as well; all firms prefer these customers.

This paper makes two contributions to the literature by studying this dichotomy. First, it carefully introduces the notion of competition into the framework of nonlinear pricing with vertical preference uncertainty and considers the resulting implications. Second, and more fundamental, it studies the importance of the nature of the uncertainty in such competitive environments. Utilizing Spulber’s model, the two types of uncertainty are compared in terms of competitive responses to entry, efficiency properties, and empirical implications for quality.

There are several reasons for considering these two variations of preference uncertainty. First, analyzing this dichotomy provides a direct framework to glean insights into more general, but far less tractable, models of uncertainty. Ideally, we would like to consider quite large, multi-dimensional preferences with few a priori restrictions. But obtaining characterizations of the economic nature of optimal nonlinear prices in such models is exceptionally difficult. Focusing on just two aspects simplifies the problem to the point where we can obtain economic insights while still providing a rich information structure. Moreover, these two polar cases of preference uncertainty are economically significant and capable of describing a broad array of economic environments.

As a motivating example, consider the desktop computer market. In the setting of horizontal preference uncertainty, a customer’s “brand” preference for personal computers and their operating systems (e.g., Windows versus Apple OS) may be unknown by each firm but nonetheless strongly influence the ultimate purchase of the consumer. A strong preference for Windows, for example, implies a weaker preference for competing non-Windows computers and their operating systems. Additionally, a consumer who purchases his most preferred brand will plausibly have a higher marginal valuation of quality than he would if he consumed the less satisfactory product. A purchaser who strongly desires a Windows-compatible computer will attach a higher marginal value to additional computing power and hardware features than he would if he purchased the competing Apple computer. Alternatively, consider the case of vertical preference uncertainty: brand preference uncertainty is small but the consumer’s marginal valuation of quality is unknown. In this case, a consumer who desires an exceptionally fast computer will be willing to pay more for high quality regardless of which brand of computer he ultimately decides upon. Firms’ preferences over consumer types are congruous: high valuation consumers are more profitable for any firm.

Examining each form of uncertainty provides a more general framework for addressing issues in competitive nonlinear pricing. Given the difficulty of determining closed-form solutions for optimal multi-dimensional nonlinear prices, we instead proceed first by focusing on each form of private information in isolation, illustrating the importance of the form of uncertainty when determining
the equilibrium pricing contracts. Once we determine the nature of equilibrium pricing contracts in this general model of oligopoly pricing, we can ask some interesting questions regarding the effects of competitors on pricing strategy. Specifically, how do distortions in quantity or quality react to the introduction of an additional firm in the marketplace? Does the equilibrium number of firms provide too much or too little differentiation from a social point of view? Finally, how does price discrimination compare in this model to a marketplace where only uniform pricing is allowed? The general tradeoffs involved with monopoly are well known, but with competition there is an additional factor: price discrimination alters a firm’s profits relative to uniform pricing, thereby affecting entry.

Section 2 constructs a simple model of price discrimination where consumers’ preferences vary along two dimensions: horizontal (brand) preferences and vertical (quality) preferences. The case of second-degree price discrimination over unknown horizontal preferences is examined in Section 3. We initially consider a simple model of duopoly on an interval of consumer preferences with one firm at each end of a linear market. The equilibrium contracts are derived and the effect of competition on the consumer’s purchasing decision is illustrated. Later, we embed these results from the unit interval in a more general marketplace where the number of firms arises endogenously. The resulting conclusions for endogenous entry and brand uncertainty closely mirror Spulber [1989a,1989b] and are provided for completeness. In Section 4, however, we turn to the technically more difficult problem of second-degree price discrimination over vertical preferences. Again, the simpler case of two firms competing on an interval is first considered, and then we place our results in a more general free-entry oligopoly setting. Section 5 reconsiders the well known welfare effects of second-degree price discrimination, but now in the context of competition. We find an additional and important effect due to the presence of free-entry competition. Issues of quality distortions and price dispersion are addressed for both types of uncertainty. Section 6 concludes with some further implications of this analysis for the design of optimal nonlinear price schedules as well as a consideration of some simple applications of the theory to other economic environments.

2 A Model of Second-Degree Price Discrimination with Oligopoly

To illustrate the interaction between second-degree price discrimination and competition we consider a model where consumer preferences differ over two dimensions. First, consumers have idiosyncratic horizontal preferences for differing brands of the product which we parameterize by a distance \( \theta_i \) – the distance from the consumer’s ideal brand to firm \( i \)'s brand offering. Initially, we assume that there are only two firms along a linear market, between which all consumers are lo-
cated. Let $\Delta$ be the distance between two firms in terms of product distance. Then for a consumer located at a point $\theta \in [0, \Delta]$ between the two firms, $\theta_l = \theta$ is the distance of the consumer from the left firm while $\theta_r = \Delta - \theta$ measures the distance from the right. Second, we assume that consumers have varying vertical preferences for product quality. We denote the intensity of these preferences by $\nu$ which lies in the interval $[\underline{\nu}, \overline{\nu}]$.

We suppose that the consumer purchases a single unit of a good but with variable quality, $q$, at price $p$, with no possibility for arbitrage with other buyers.\footnote{Alternatively, we could characterize this as a model where each consumer chooses to purchase from only one firm, but buys a variable quantity. Under the quality framework with unit demands (e.g., one car, etc.), it is plausible to restrict a consumer to purchasing from a single firm. Under the alternative quantity framework, we would want this choice to rise endogenously, which requires some additional technical restrictions. Economically, both are intimately related, and so we often speak of the quantity/quality distortion.} We can therefore denote a consumer’s utility from purchasing a good of quality $q$ from firm $i$ as

$$U = u(q, \theta_i, \nu) - p,$$

which we assume to be thrice differentiable and strictly concave in $q$. Such a utility function is naturally increasing in $q$ and $\nu$ and decreasing in $\theta_i$. We also normalize a consumer’s reservation value to zero.

We proceed by making a few standard “sorting” assumptions on our representative consumer’s utility function. First, in addition to a consumer with $\theta_i < \theta_j$ preferring the good of firm $i$, we assume that the marginal utility to the consumer from an increase in product quality is higher for a purchase from firm $i$ than from firm $j$. Second, we also suppose that a consumer who has a higher preference for quality, $\nu$, is someone who has a higher marginal valuation for quality. To capture these economic requirements, we assume

**Assumption 0 (Sorting Conditions):** for any $(q, \theta_i, \nu)$,

1. $u_{\theta q}(q, \theta_i, \nu) < 0$ and $u_\theta(q, \theta_i, \nu) < 0$;
2. $u_{\nu q}(q, \theta_i, \nu) > 0$ and $u_\nu(q, \theta_i, \nu) > 0$.

Finally, for simplicity, we will assume that the utility function satisfies $u_{\theta \nu} \leq 0$ and $u_{\nu \nu} \leq 0$. These restrictions simplify the proofs of theorems without being crucial to our results.

When $\theta$ is private information to the consumer, the firms know only the distribution of $\theta \in [0, \Delta]$, which we take to be represented by the density $f(\theta)$ which is positive and integrable with a distribution function, $F$. We will find it convenient at times to denote the density function, $f^i(\theta_i)$, and distribution function, $F^i(\theta_i)$, for firm $i$, rather than using $f$ and $F$. In such a case it is
understood that $f^i(\theta_i) = f(\theta_i)$ and $f^r(\theta_r) = f(\Delta - \theta_r)$. We also take $\nu$ to be distributed on $[\underline{\nu}, \overline{\nu}]$ according to the commonly known positive density function $g$ with distribution function $G$.

Each firm, $i$, has a per-customer profit function given by

$$\pi_i = p_i - c_i q_i,$$

where $c_i$ is the firm’s marginal cost of producing a unit of quality. We will occasionally write “$-i$” to mean firms other than firm $i$. Whenever we consider firms with differing marginal costs, $c_i \neq c_{-i}$, we suppose that the cost heterogeneity is not too large relative to the distance separating firms, $\Delta$, such that $\max_q u(q, 0, \nu) - c_i q > \max_q u(q, \Delta, \nu) - c_{-i} q > 0$ for any $c_i$ and $c_{-i}$. Such an assumption guarantees that firm $i$ is always more efficient than any competitor (firm $-i$) at serving customers whose brand preferences perfectly coincide with its own product offering.

Our approach is one of finding an equilibrium in a mechanism design game between several principals. Our notion of equilibrium is that of a Bertrand-Nash equilibrium in nonlinear price schedules between competing firms. To this end, we examine a Direct Revelation Mechanism game in which each consumer announces his type to each principal, and the principals implement their pricing contracts subject to incentive compatibility and participation constraints. Although this is equivalent to studying a game in which each principal offers a nonlinear pricing schedule, as indicated in Martimort and Stole [1994], this is potentially with some loss of generality. Nonetheless, we follow the traditional mechanism-design literature for various reasons. First, the literature has generally followed this paradigm, e.g., Mussa-Rosen [1978] and Maskin-Riley [1984], and so our work provides an interesting comparison when competition is introduced. More importantly, we generally see firms choosing nonlinear price schedules rather than employing more complex communication games.

Competition, of course, introduces an extra complication into the standard analysis. The presence of a competitor produces a countervailing incentives problem, as discussed by Lewis and Sappington [1989], in that the participation constraint is now a function of type. It turns out that in our model of competition, these problems have clean solutions which nicely illustrate the effects of competition on pricing schedules.

For tractability, we consider each type of uncertainty in isolation. This simplification allows us to study the important differences between the two domains of price discrimination. In the following section, we examine the case of horizontal uncertainty where all firms know each consumer’s

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4See, for example, Myerson [1979] and Harris-Townsend [1981]. For a thorough treatment of the mechanism design literature, consult Fudenberg-Tirole [1991].
preferences for quality, but preferences over product variety are unknown. In section 4, we consider
the other extreme where preferences over quality are private information, although the spatial lo-
cation of consumer preferences in the brand spectrum are known by all. Section 5 considers the
strategic implications and welfare consequences of the two types of uncertainty.

3 PRICE DISCRIMINATION OVER HORIZONTAL PREFERENCES

Consider a situation in which private information over preferences exists exclusively over the hor-
izontal (brand) dimension. That is, $\theta_i$ is unknown to firm $i$, while either $\nu$ is identical for all
consumers or each consumer’s $\nu$ is known and firms can perfectly price discriminate over this
dimension.\footnote{This situation is essentially the case considered by Spulber [1989a, 1989b]. For completeness, we present a
treatment of this form of uncertainty in the present section, although except for the generalization to asymmetric
firms ($c_i \neq c_{-i}$), no new results are presented here.} For example, in the personal computer market for a given and discernible customer
group such as students, the degree of uncertainty over preferences may be strongest in the “Macin-
tosh” versus “PC” dimension, rather than in the “faster” versus “slower” dimension. We recognize
that our assumption that brand preferences are unknown while preferences in non-brand dimen-
sions are known is a polar case, but such an assumption nicely illustrates the economic effects of
this type of uncertainty.

3.1 A SIMPLE MODEL OF TWO FIRMS COMPETING ON AN INTERVAL

We first focus attention on a linear interval of consumer tastes, $[0, \Delta]$, with one firm located at each
extreme. We will at times refer to the firm located at 0 as the left firm, $i = l$, and the firm at $\Delta$
as the right firm, $i = r$.

Given our direct-revelation mechanism-design approach, each firm’s contracts can be repre-
sented as an ordered pair: $\{q_i(\theta), p_i(\theta)\}$. Each firm maximizes profits by choosing the optimal
contract subject to incentive compatibility and individual rationality (participation) constraints.
For notational simplicity, we define indirect utility functions for given contracts:\footnote{Throughout this section we assume that $\nu$ is known by all firms, and so at times to economize on notation we do
not explicitly recognize the dependence of utility and contracts on $\nu$.}

$$U^i(\hat{\theta}_i, \theta_i) \equiv u(q_i(\hat{\theta}_i), \theta_i, \nu) - p_i(\hat{\theta}_i),$$

$$U^i(\theta_i) \equiv U(\theta_i, \theta_i).$$
The IR and IC constraints are given (respectively) by

\[ U^i(\theta_i) \geq \max\{U^{-i}(\Delta - \theta_i), 0\}, \]

\[ U^i(\theta_i) \geq U^i(\bar{\theta}_i, \theta_i). \]

As is standard with this approach, we proceed by characterizing the set of contracts which satisfy the incentive compatibility (IC) and individual rationality (IR) constraints for one firm, given the other firm’s contract. With duopoly, however, we must account for the strategic interaction between the contracts. We can state the following lemma. All results are proved in an appendix.

**Lemma 1** Given firm \(-i\)’s contract \(\{p_{-i}, q_{-i}\}\), a consumer buys from firm \(i\) and truthfully reports \(\theta_i\) if and only if \(\theta_i \leq \bar{\theta}_i\), where \(\bar{\theta}_i\) satisfies

\[ U^i(\theta_i) = U^i(\bar{\theta}_i) + \int_\theta^{\bar{\theta}_i} u_\theta(q_i(s), s, \nu)ds, \tag{1} \]

\[ U^i(\bar{\theta}_i) \geq \max\{U^{-i}(\Delta - \bar{\theta}_i), 0\}, \tag{2} \]

and \(q_i(\theta_i)\) is nonincreasing in \(\theta_i\).

With this lemma, we can be assured that each firm’s IR constraint, if it binds, binds at only one point. To understand this, consider the left firm’s problem. To attract consumer \(\theta\), it must offer at least \(\max\{U^r(\Delta - \theta), 0\}\). By individual optimization, a customer’s indirect utility function is always strictly decreasing in location. Since \(U^r\) is strictly increasing in \(\theta\), and since any incentive compatible contract which the left firm can offer will have \(U^i(\theta)\) strictly decreasing, the IR constraint will bind at a single point.

Continuing our focus on firm \(i\), define the following function:

\[ \Phi^i(q, \theta) \equiv u(q, \theta, \nu) - c_i(q) + \frac{F^i(\theta)}{f^i(\theta)} u_\theta(q, \theta, \nu). \]

Following Myerson [1979], \(\Phi^i(\theta_i)\) represents the virtual profit from sales to customers with type \(\theta_i\). Specifically, the first two terms in \(\Phi^i\) represent the social surplus from selling quality level \(q\) to a consumer at location \(\theta\); the third term represents the expected consumer rents which must be left to the consumer in any equilibrium in order to induce the correct self-selection. In this sense, \(\Phi^i\) is a direct measure of the firm’s expected profits for the consumer segment \(\theta\). We make the following assumption on \(\Phi^i\):
Assumption 1 (Φ-Regularity): For all \((q,\theta,\nu)\) and \(i,\)

\[
\Phi^i_{qq}(q, \theta) < 0, \quad \Phi^i_{q\theta}(q, \theta) < 0, \quad \Phi^i_{\theta\theta}(q, \theta) < 0,
\]

and \(\Phi^i\) has an interior maximum in \(q\).

This assumption on \(\Phi^i\) is standard in the literature and not especially restrictive.\(^7\) We are now prepared to state the main result in this section.

**Proposition 1** Suppose A0 and A1 are satisfied. Then there exists a unique Nash equilibrium in the competition game on the interval with length \(\Delta\) which is characterized by

\[
f^i(\theta_i)[u_q(q_i(\theta_i), \theta_i, \nu) - c_i] = -F^i(\theta_i)u_{q\theta}(q_i(\theta_i), \theta_i, \nu), \quad \forall \theta_i \leq \theta_i,
\]

(3)

where either there is

**Local Monopoly**, in which case \(\theta_i + \theta_r < \Delta\), \(U^i(\theta_i) = 0\), and \(\bar{\theta}_i\) is determined by

\[
\Phi^i(q_i(\theta_i), \bar{\theta}_i, \nu) = 0,
\]

(4)

or **Competition**, in which case \(\theta_i + \theta_r = \Delta\) and \(U^i(\theta_i) = \bar{U}\), and \((\theta_i, \theta_r, \bar{U})\) are uniquely and jointly determined by

\[
f^i(\theta_i)[\Phi^i(q_i(\theta_i), \bar{\theta}_i, \nu) - \bar{U}] = -F^i(\theta_i)u_{q\theta}(q_{-i}(\bar{\theta}_{-i}), \bar{\theta}_{-i}, \nu), \quad i = l, r,
\]

(5)

\[
\theta_l + \theta_r = \Delta.
\]

(6)

In both cases \(U^i(\theta_i)\) is characterized by equation (1) of Lemma 1 and \(p_i(\theta_i)\) is characterized by the relation \(p_i(\theta_i) = u(q_i(\theta_i), \theta_i, \nu) - U^i(\theta_i)\).

Except for the additional generality of heterogeneous marginal costs and weaker assumptions on the primitives, this proposition was first discovered by Spulber [1989a] in the context of a model of monopolistic competition on a circular market, which we discuss below. To shed some light on

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\(^7\)It is satisfied, for example, whenever \(u_{q\theta} \leq 0, u_{\theta\theta} \geq 0\), and \(F^i\) satisfies the monotone hazard rate condition (MHRC). Specifically, \(\frac{d}{d\theta}\) is nondecreasing and \(\frac{d}{d\theta} \Phi^i\) is nonincreasing, which among others is satisfied by the uniform distribution used later in this paper. More generally, this assumption will hold whenever \(F\) satisfies MHRC, \(u_{q\theta}\) is sufficiently small, and the third derivatives of \(u\) are not too large relative to \(u_{qq}\) and \(u_{q\theta}\).
this result, let’s consider the position of the left firm. The proposition implies that quality will be
distorted away from the first-best for all customers whose preferences are not coincident with the
firm’s offering. Specifically, \( q \) will be set such that
\[
    u_q - c_t = -\int_{\theta}^{F} u_{q \theta} \geq 0,
\]
for any customer within radius \( \bar{\theta} \). Two possibilities exist. First, the left firm’s behavior could be
completely unconstrained by the presence of the right firm. This is a situation of multiple monopoly,
as each firm is an unfettered monopolist. More interestingly, consider the case of competition where
the left firm’s optimal contract offer explicitly takes into account the effects of outside competition
on reducing market share. Here, \( \bar{\theta} \) is chosen such that each duopolist is indifferent between stealing
some of her competitor’s market and increasing profits on those new customers on the one hand,
and indirectly raising the surplus for all customers served by raising the marginal customer’s surplus
on the other. This is equation (5).

Consider a simple numerical example. Let \( \theta \) be uniformly distributed on \([0, \Delta]\), \( \Delta = 1 \), \( c_l = 1 \),
\( c_r = 1.5 \), \( u(q, \theta, \nu) = 3q - \frac{1}{2}q^2 + (1 - \theta)q \). Here, the left firm has a cost advantage. Using the
proposition to characterize the equilibrium, we find that \( \bar{\theta}_l = \Delta - \bar{\theta}_r = 0.59 \) even though the socially
efficient partition occurs at \( \bar{\theta}_l = \Delta - \bar{\theta}_r = 0.75 \). The consumer utilities from both marginal cost
pricing (represented by * superscripts) and the Nash equilibrium in nonlinear pricing are illustrated
in Figure 1, as well as the resulting equilibrium qualities.

Figure 1 here.

Note that due to cost asymmetries there can be a quality discontinuity for the marginal cus-
tomer, here illustrated at 0.59. Also note that high-cost firms not only have lower average quality
and smaller market share than low-cost firms, but also have less quality dispersion (a direct con-
sequence of smaller market share over the location distribution).

3.2 A General Model of Circular Competition

Following Spulber [1989a] who used Salop’s [1979] model of monopolistic competition to study
nonlinear pricing with brand preference uncertainty, we can embed our simple description of duopoly
in a richer model with free entry on a circular product market. We now suppose that there are
many symmetric firms (endogenously determined) who choose whether to enter a circular market
with uniformly distributed customers for a fixed entry cost of \( K \). In accord with the principle of
maximal differentiation, we assume that the firms locate themselves at equidistant intervals on the
circular market.\textsuperscript{8} Entry will occur until expected profits from entry exceed $K$; for simplicity we will allow the number of firms to be a non-integer when considering the comparative statics arising from a change in $K$.

Along the lines of Kaldor [1935], we wish to focus our attention on local rather than global competition over customers. To this end, we assume that a firm’s potential market share is limited to the customers located between itself and its neighboring rivals. This assumption greatly simplifies the analysis by eliminating discontinuities in the best response functions of the firms while simultaneously concentrating on local interactions between rival firms. Using our result from Proposition 1, it is straightforward to solve the general unit circle model for the Nash equilibrium. A form of the result appears in Spulber [1989a].

**Proposition 2** Suppose $c_i = c$ for every $i$, and $u_{\theta\theta} \geq 0$. Then there exists a unique\textsuperscript{9} symmetric Nash equilibrium in the competition game on the unit circle in which each firm offers $\{q(\theta_i), p(\theta_i)\}$ for all types within $\theta$ distance from the firm where the contract satisfies

$$u_q(q(\theta_i), \theta_i, \nu) - c = -\theta_i u_{\theta\theta}(q(\theta_i), \theta_i, \nu), \quad \forall \theta_i \leq \theta,$$

(7)

and either there is

**Local Monopoly**, in which case $\theta < 1/2N$, $U^*(\theta) = 0$, and $\theta$ is determined by

$$\Phi(q(\theta), \theta, \nu) = 0,$$

or **Competition**, in which case $\theta = 1/2N$ and

$$U'(\theta) = \Phi(q(\theta), \theta, \nu) - \frac{1}{2N} u_{\theta\theta}(q(\theta), \theta, \nu).$$

In both cases, $U^*(\theta_i)$ is characterized by equation (1) of Lemma 1, $p_i(\theta_i)$ is characterized

\textsuperscript{8}Although this is not modeled formally in an extensive form game of location choice as in Economides [1989], one can imagine a similar three stage game with entry followed by simultaneous location choice and finally price-quality contracts. Under mild regularity conditions, there will be an incentive for firms to differentiate themselves at the second stage, and so firms will locate themselves equidistant from one another as a Nash equilibrium in the location subgame. Rather than devote attention to these subtleties, we will take the location decision as given, as in Salop [1979] so that we may directly focus our energies on the issues of entry.

\textsuperscript{9}Because of the arbitrariness of locations and firm numberings, “uniqueness” is defined up to the uniqueness of price-quality schedules and market shares.
by the relation \( p_i(\theta) = u(q_i(\theta), \theta, \nu) - U^i(\theta) \), and \( N \) satisfies

\[
2 \int_0^\theta \Phi(q(s), s)ds - \frac{1}{N} U(\theta) = K. \tag{8}
\]

The proposition is very similar to Proposition 1 except that now there is the standard free-entry condition in (8) and we have achieved the generality of an \( N \)-firm model at the cost of imposing symmetry. Interestingly, in the case of competition between all firms, as \( K \) decreases more firms enter with the result that \( \overline{\theta} = \frac{1}{2N} \) decreases. The nature of the distortionary contracts remains unchanged in the interval \( \theta_i \leq \overline{\theta} = \frac{1}{2N} \), but since the quality distortion is greatest at \( \overline{\theta} \), a reduction in \( K \) reduces quality distortions by increasing competition. In other words, distortions remain unchanged for a given market share, but since entry decreases market share, entry indirectly reduces quality distortions by providing substitute products. We will return to this analysis in Section 5.

4 Price Discrimination over Vertical Preferences

We now shift our attention to the case in which private information over preferences exists exclusively over the vertical (quality) dimension. Unlike the case of horizontal brand uncertainty, nonlinear pricing in an environment of vertical uncertainty and imperfect competition has not been studied.

We now suppose that \( \theta_i \) is known by all firms and that they can practice perfect price discrimination over brand preference, but firms do not know the quality preference parameter, \( \nu \). Airline pricing provides an important example where such assumptions appear roughly realistic. Brand preference in terms of non-stop versus routed flights is known and pricing can accommodate such characteristics. The marginal benefit of quality (in the form of fully refundable and unrestricted tickets), however, might be quite different between a business customer and a leisure traveler. Our assumption that there is no uncertainty with respect to brand preference is strong but underscores the crucial role that the specific type of private information plays.

4.1 A Simple Model of Two Firms Competing on an Interval

Return for the moment to our simple model of competition on the product interval, \([0, \Delta]\), in which we restrict attention to two firms: \( i = l, r \). Given our direct-revelation approach, we can characterize the set of offered contracts as an ordered pair: \( \{q_i(\theta_i, \nu), p_i(\theta_i, \nu)\} \). Again, each firm maximizes profits by choosing the optimal contract subject to incentive compatibility and participation con-
straints. We define the indirect utility functions as

\[ U^i(\theta, \nu) \equiv u(q_i(\theta, \nu), \theta, \nu) - p_i(\theta, \nu), \]

\[ U^i(\theta, \nu) \equiv U^i(\theta, \nu, \nu). \]

The IR and IC constraints (respectively) are

\[ U^i(\theta, \nu) \geq \max\{U^{-i}(\Delta - \theta, \nu), 0\}, \]

\[ U^i(\theta, \nu) \geq U^i(\theta, \nu, \nu). \]

Following standard techniques, we can characterize the incentive compatibility constraints by an integral condition and monotonicity.

**Lemma 2** A consumer who buys from firm \( i \) truthfully reports \( \nu \) if and only if

\[ U^i(\theta, \nu) = U^i(\theta, \nu) + \int_0^\nu u_q(q_i(\theta, t), \theta, t)dt, \tag{9} \]

and \( q_i(\theta, \nu) \) is nondecreasing in \( \nu \).

The IR constraints take more effort to simplify. Some definitions will be helpful to formalize the nature of the option. Define

\[ q_i^*(\theta, \nu) \equiv \arg\max_q u(q, \theta, \nu) - c_i q, \]

as the first-best quality level which would be purchased by a customer at \( \theta_i \) who could buy from firm \( i \) at marginal cost. The corresponding value of the outside option from choosing such a quality level is defined by

\[ \underline{U}^i(\theta, \nu) \equiv u(q_i^*(\theta, \nu), \theta, \nu) - c_i q_i^*(\theta, \nu). \]

By previous assumptions, \( q_i^* \) is uniquely defined and \( \underline{U}^i > 0 \) for any \( \theta \) and \( \nu \). We can now ask what a neighboring (but more distant) firm will offer a potential customer in any Nash equilibrium. The answer is marginal-cost pricing for precisely the same logic as that of Bertrand equilibrium.

**Lemma 3** In all Nash equilibria in which customer \((\theta, \nu)\) purchases from firm \( i \),

\[ p_{-i} = c_{-i} q_{-i}^*(\Delta - \theta_i, \nu). \]
As a consequence of Lemma 3, we know that a customer’s outside opportunity will be the utility he could receive by purchasing at marginal cost from a more distant firm. The IR constraint can now be restated as

$$U^i(\theta_i, \nu) \geq \max\{U^{-i}(\Delta - \theta_i, \nu), 0\},$$

since $U^{-i}(\Delta - \theta_i, \nu) = U^{-i}(\Delta - \theta_i, \nu)$. This combined with the envelope theorem implies the following preliminary result.

**Lemma 4** Suppose the IR constraint for firm $i$ is binding on the set $[\nu^-, \nu^+]$, then necessarily for all $\nu \in [\nu^-, \nu^+]$

$$u_\nu(q_i(\theta_i, \nu), \theta_i, \nu) \equiv u_\nu(q_{-i}^*(\Delta - \theta_i, \nu), \Delta - \theta_i, \nu). \quad (10)$$

As before with our private brand preference model, we will need some additional assumptions on the firm’s virtual profit function in order to guarantee our problem is well behaved. In this regard, we define

$$\Psi(q, \theta_i, \nu) \equiv u(q, \theta_i, \nu) - c_i q - \frac{1 - G(\nu)}{g(\nu)} u_\nu(q, \theta_i, \nu).$$

We assume the following regularity condition.

**Assumption 2** (**Ψ-Regularity**: For all $(q, \theta_i, \nu)$ and $i$,

$$\Psi_{qq}(q, \theta_i, \nu) < 0, \quad \Psi_{q\nu}(q, \theta_i, \nu) > 0, \quad \Psi_{\nu\nu}(q, \theta_i, \nu) > 0,$$

and $\Psi^i$ has an interior maximum in $q$.

Again, this assumption is not especially restrictive if one is prepared to assume that $\frac{1-G}{g}$ is nonincreasing in $\nu$ and both $u_\nu$ and the third derivatives of $u$ are relatively small. With this assumption, we are prepared to state the main result of the section.

**Proposition 3** Suppose A0 and A2 are satisfied, costs are symmetric, and demand is sufficiently great such that every customer is served. Then there exists a unique symmetric Nash equilibrium in the competition game on the interval with length $\Delta$. For the customer who is located such that $\theta_i \leq \frac{\Delta}{2}$, the equilibrium contract is characterized by

$$u_\nu(q_i(\theta_i, \nu), \theta_i, \nu) = u_\nu(q_{-i}^*(\Delta - \theta_i, \nu), \Delta - \theta_i, \nu), \quad \forall \nu \in [\nu, \tilde{\nu}], \quad (11)$$
and

$$g(\nu) [u_q(q_i(\theta_i, \nu), \theta_i, \nu) - c] = [1 - G(\nu)]u_{q\nu}(q_i(\theta_i, \nu), \theta_i, \nu), \quad \forall \nu \in [\hat{\nu}, \overline{\nu}],$$

(12)

where $\hat{\nu}$ is the unique value of $\hat{\nu} \in (\nu, \overline{\nu})$, if one exists, such that $q_i(\theta_i, \nu)$ defined above is continuous; otherwise, $\hat{\nu} = \nu$. Additionally,

$$U^i(\theta_i, \nu) = U^{-i}(\Delta - \theta_i, \nu) + \int_\nu^\nu u_\nu(q_i(\theta_i, t), \theta_i, t) dt,$$

$$p_i(\theta_i, \nu) = u(q_i(\theta_i, \nu), \theta_i, \nu) - U^i(\theta_i, \nu).$$

Proposition 3 is slightly less general than Proposition 1. Its hypotheses include that the firms are symmetric ($c_i = c$) and that all consumers are served (so that we are necessarily in the more interesting case of competition and not local monopoly).\textsuperscript{10} The results of the proposition, however, are fundamentally different from those of Proposition 1, thereby underscoring the importance of the form of the uncertainty involved. Again there is a distortion in quality which arises from the private information over $\nu$. In regions where the IR constraint does not bind, we have the familiar result from Mussa-Rosen [1978] and Maskin-Riley [1984] that

$$u_q - c = \frac{1 - G(\nu)}{g(\nu)} u_{q\nu} \geq 0,$$

and there is no distortion for the high type customer (just as there was no distortion for the close customer in Proposition 1). But where the IR constraint does bind, it typically binds in an interval and not at a point. Along the interval, quality is set in order to satisfy (11). Let $\tilde{q}_i$ be the level of quality which satisfies (11) and let $\tilde{q}_i$ be the level of quality which satisfies (12). $\tilde{\nu}$ is the unique value of $\nu$ such that $\tilde{q}_i(\theta_i, \tilde{\nu}) = \tilde{q}_i(\theta_i, \tilde{\nu})$. What is particularly interesting is that $\tilde{q}$ differs from the first-best level of quality only in so far as the consumer is closer to firm $i$ than to firm $-i$. As $\Delta$ shrinks, $\tilde{q}_i$ approaches the first best quality level and $\tilde{\nu}$ approaches $\overline{\nu}$.

Consider a simple numerical example. Let $\nu$ be distributed uniformly on $[1, 2]$ and $\Delta = 1$. Each firm’s marginal cost is zero for simplicity and $u(q, \theta_i, \nu) = \nu(1-\theta_i)q - \frac{1}{2}q^2$. First, let’s examine the equilibrium contract offered to a consumer who is at $\theta_i = 0.4$.

Figure 2 here.

\textsuperscript{10}One significant extension to Proposition 3 would weaken the symmetry requirement, possibly resulting in $\overline{\theta}_i$, depending upon $\nu$. That is, the market partition will not necessarily be $\Delta/2$, but may instead depend upon type.
Graphically, the quality levels as a function of \( \nu \) are depicted in Figure 2. The first-best quality level is the highest line. The upper envelope of the intersecting lines (specifically, \( q_i = \max\{\bar{q}_i, \hat{q}_i\} \)) gives the optimal second-best contract which firm \( i \) offers.

To more clearly understand the impact of the IR constraint, consider the contour graph of the second-best contract in Figure 3A, and the contour graph of the difference in quality between the first-best and second-best contracts in Figure 3B.

Figure 3 here.

The lowest difference contour line in Figure 3B runs along the \( \nu = 2 \) axis and then makes a sharp left bend along the \( \theta_i = 0.5 \) axis. These are areas where quality distortions are lowest: when \( \nu \) is high or when competition is vigorous (\( \theta_i \) is close to \( \theta_{-i} \)). The kinks in the contour graphs indicate the points where the IR constraint begins to bind. For low \( \theta_i \), the constraint is slack. As \( \theta_i \) increases, the IR constraint sweeps along the quality-preference spectrum (i.e., \( \tilde{\nu} \) increases toward \( \bar{\nu} \)) until the constraint is binding for every \( \nu \) at \( \theta_i = 0.5 \). At such a point there is perfect competition.

4.2 A General Model of Circular Competition

Consider a free-entry game as that in Section 3 on the unit circle. With little additional argument, we can show

**Proposition 4** Suppose \( c_i = c \) for each firm \( i \) and demand is sufficiently great such that every customer is served. Then there exists a unique symmetric Nash equilibrium in the competition game on the unit circle. For the representative consumer located between firms \( i \) and \( i + 1 \) and whose location satisfies \( \theta_i \leq \frac{1}{2N} \), the equilibrium contracts are characterized by

\[
    u_c(q_i(\theta_i, \nu), \theta_i, \nu) = u_c(q_{i+1}^*(1/N - \theta_i, \nu), 1/N - \theta_i, \nu), \quad \forall \nu \in [\underline{\nu}, \tilde{\nu}],
\]

and

\[
    u_q(q_i(\theta_i, \nu), \theta_i, \nu) - c_i = \frac{1 - G(\nu)}{g(\nu)} u_q(q_i(\theta_i, \nu), \theta_i, \nu), \quad \forall \nu \in [\tilde{\nu}, \bar{\nu}],
\]

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where $\bar{\nu}$ is the unique value of $\bar{\nu} \in (\nu, \bar{s})$, if one exists, such that $q_i(\theta_i, \nu)$ defined above is continuous; otherwise, $\bar{\nu} = \nu$. Additionally, 

$$U^i(\theta_i, \nu) = U^{i+1}(1/N - \theta_i, \nu) + \int_\nu^\nu u(p_i(q_i(t, \theta), \theta_i, t)dt,$$

$$p_i(\theta_i, \nu) = u(q_i(\theta_i, \nu), \theta_i, \nu) - U^i(\theta_i, \nu),$$

and $N$ satisfies

$$2 \int_0^{1/2N} \left\{ \int_\nu^\nu [\Psi(q(s, t), s, t) - U(s, \nu)] g(t)dt \right\} ds = K. \quad (15)$$

The nature of the equilibrium is similar to that under the duopoly game on $[0, \Delta]$. We now have a free-entry condition which demonstrates that as $K$ decreases, $N$ increases. Given that $\Delta = 1/N$ for the circular model, once again a reduction in entry costs lessens quality distortions. Unlike the case of lower entry costs under brand-preference uncertainty, here there are two clear competitive effects. First, lower entry costs result in greater entry and lesser market share, which indirectly reduce quality distortions. Second, fixing the cross-section for any $\theta$, more entry also enlarges the region in which the IR constraint binds, thereby reducing quality distortions directly on the lower types. This provides a testable implication that in markets with vertical price discrimination, we should see less distortion on the low end of quality consumption as competitors become close.

It is worth comparing our circular model of vertical uncertainty with the “finiteness property” of Shaked and Sutton’s [1982,1983] vertical differentiation framework. Recall that in Shaked and Sutton’s model, each firm can select only a single quality to offer for sale at a uniform price. Providing that the marginal cost of quality does not increase too quickly with quality, only a finite number of firms will enter to compete on the vertical market, regardless of entry costs. Because in the present model each firm is allowed to offer a continuum of qualities along the vertical spectrum, a single firm will emerge to capture the entire vertical spectrum at any given horizontal position. This occurs because for any horizontal segment, one firm (the closest) will have an absolute advantage over all others for all quality offerings. In a sense, we have a trivial finiteness property, with a single firm emerging in every quality offering. Of course, for arbitrarily low entry costs, an arbitrarily high number of firms enters, spreading themselves along the horizontal dimension of the market with each firm maintaining a constrained monopoly over the vertical spectrum at their horizontal position. As entry costs go to zero, the constraints on monopoly pricing effectively force each firm to price at marginal cost.
5 THE CHARACTER OF EQUILIBRIUM PRICE DISCRIMINATION:  
HORIZONTAL AND VERTICAL UNCERTAINTY COMPARED

In the previous sections we have developed two models of imperfectly competitive nonlinear pricing. Here, we focus more carefully on the character of the optimal price schedules. First, we consider the consequences of competition on entry and market efficiency. Although we can make general conclusions for both environments, we find that we can further identify an efficiency effect due to entry in the vertical uncertainty environment; this is not the case for the horizontal framework. Second, we highlight the differences in each firm’s offered product qualities that result from the fundamental differences between horizontal and vertical uncertainty. We find an additional and important competitive effect exists in the case of vertical uncertainty. Third, we address the question of whether competition can lead to increases in observed price or quality dispersion in our general framework.

5.1 ENTRY AND EFFICIENCY

As mentioned in Sections 3 and 4, the reduction in \( K \) increases competition on the unit circle and facilitates lower quality distortions. These intuitions are indeed correct as the next two propositions formalize. The first, Proposition 5, addresses the effects of competition on efficiency when the environment is one of horizontal uncertainty.

**Proposition 5** Suppose the conditions of Proposition 2 are satisfied and competition (rather than local monopoly) occurs in equilibrium. Let \( q(\theta, K) \) and \( U(\theta, K) \) be the symmetric equilibrium contract and utility, respectively, of consumer \( \theta \) in a market who buys from firm \( i \); let \( q^*(\theta, K) \) and \( U^*(\theta, K) \) be the contract and utility, respectively, of consumer \( \theta \) who buys from firm \( i \) at marginal cost. Then,

\[
E_\theta[U^*(\theta, K) - U(\theta, K)] \text{ increases in } K,
\]

\[
E_\theta[q^*(\theta, K) - q(\theta, K)] \text{ increases in } K,
\]

\[
CS = E_\theta[U(\theta, K)] \text{ decreases in } K.
\]

We use \( E_\theta[\cdot] \) to denote the expectation taken over \( \theta \). We must consider expectations as generally the symmetric introduction of another firm will only reduce the average \( \theta_i \), and not every consumer’s distance. This proposition states that a decrease in entry cost reduces (on average) quality distortions from the efficient level. As a consequence, average utility levels also increase toward the
perfectly competitive benchmark and consumer surplus rises. We make a similar statement about the vertical uncertainty environment in Proposition 6.

**Proposition 6** Suppose the conditions of Proposition 4 are satisfied. Let \( q(\theta_i, \nu, K) \) and \( U(\theta_i, \nu, K) \) be the symmetric equilibrium contract and utility, respectively, of consumer \((\theta_i, \nu)\) who buys from firm \(i\); let \( q^*(\theta_i, \nu, K) \) and \( U^*(\theta_i, \nu, K) \) be the contract and utility, respectively, of consumer \((\theta_i, \nu)\) who buys from firm \(i\) at marginal cost. Then,

\[
E_{\theta, \nu}[U^*(\theta_i, \nu, K) - U(\theta_i, \nu, K)] \text{ increases in } K,
\]

\[
E_{\theta, \nu}[q^*(\theta_i, \nu, K) - q(\theta_i, \nu, K)] \text{ increases in } K,
\]

\[
CS = E_{\theta, \nu}[U(\theta_i, \nu, K)] \text{ decreases in } K.
\]

Again, we take averages using \( E_{\theta, \nu}[\cdot] \) to denote the expectation taken over \( \theta \) and \( \nu \). The results are similar to the case of horizontal uncertainty.

Not surprisingly, the reduction of entry costs (i.e., the increasing of competitive pressures) leads to an increase in the number of firms, a corresponding reduction in quality distortions, and a movement towards marginal-cost pricing. A more interesting question lies in the comparison of second-degree price discrimination with uniform pricing. Here, we define uniform pricing as an offer from a firm to sell quality at a fixed price per unit, thereby generating a linear quality-price schedule. As is well known, replacing such uniform pricing with second-degree price discrimination produces two effects. First, inefficiencies are introduced as differing prices among consumers imply some degree of inter-consumer misallocation. Second, price discrimination may generally increase the amount of quality produced thereby increasing efficiency. Without rather restrictive conditions, it is difficult to say whether one effect dominates the other. With competition, however, comes a third effect: Entry may increase or decrease.\(^{11}\)

Fixing \( N \), if every firm but one practiced uniform pricing while the odd firm used nonlinear pricing, the odd firm must make at least as large profits as with uniform pricing. Unfortunately, if all firms are allowed to practice second-degree price discrimination, it is not clear that collectively profits rise or fall. As a direct effect, nonlinear pricing is more efficient at extracting consumer

\(^{11}\)Katz [1984] makes a related, but different, point in a model where production inefficiencies may increase due to the standard argument of Chamberlin [1920]. Using U-shaped average cost curves under the assumption that there is no gain from product variety and increases in entry arise from the presence of price discrimination, Katz shows that too much entry may result in social inefficiencies. In the present paper, we have taken a more general approach, explicitly characterizing the gains from variety (minimizing brand differences) as well as the costs of excessive industry capacity.
surplus and increasing profits. As an indirect effect, nonlinear pricing may severely reduce the profit of neighboring firms by raising the individual rationality constraint of the marginal consumer.

In our first model where horizontal preferences are private information, this ambiguity is manifest: allowing nonlinear pricing may significantly raise the marginal customer’s surplus, thereby lowering the profits of each firm. With lower profits in equilibrium, less entry may occur under price discrimination than under uniform pricing.\textsuperscript{12} In our second model where brand preferences are public information, the ambiguity does not exist. Under vertical discrimination, both uniform and nonlinear pricing models have identical outside options for the marginal consumer – marginal-cost pricing from the more distant firm. Thus, fixing the number of firms in a market, nonlinear pricing must increase profits. Therefore we have the following lemma for the quality-uncertainty environment.

\textbf{Lemma 5} \textit{Under the assumptions of Proposition 4 for the model of circular competition with vertical price discrimination, when nonlinear pricing is allowed there exists a unique equilibrium with }$N_{NL}$\textit{ firms entering with each making zero profits. When only uniform pricing is allowed, there exists a unique equilibrium with }$N_U$\textit{ firms entering with each making zero profits and where }$N_{NL} > N_U$\textit{.}

In the case of private information over vertical preferences, we can use this lemma to arrive at a conclusion regarding this third effect of price discrimination under competition. To focus on the effects of entry, we take as a point of reference the setting where consumer surplus is unchanged when moving from uniform pricing to nonlinear pricing \textit{for a fixed }$N$. That is, we assume that the first two effects (efficiency gains and distributional losses) are exactly offsetting. In a related setting, Katz [1984] suggests that welfare is reduced under nonlinear pricing as inefficient entry occurs dissipating the rents from price discrimination. In our setting, however, where there are gains from product variety, nonlinear pricing always increases social welfare. This is because some gains from the increased profitability of the market are passed along to the consumers through increased entry and beneficial product variety. In our setting, rents are decreased through free entry, but entry produces a positive externality on consumers. We can formalize this argument.

\textbf{Proposition 7} \textit{Let }$CS_{NL}(N)$\textit{ be the level of consumer surplus in a symmetric marketplace with }$N$\textit{ firms, each practicing nonlinear pricing over quality preferences. Likewise,}

\textsuperscript{12}Nonetheless, Spulber [1989a] demonstrates that for the specific case of quadratic utility and small fixed costs under horizontal uncertainty in the circular market model, social welfare is higher when firms practice nonlinear pricing than when they restrict their prices to be linear in }$q$.\n
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let $CS_U(N)$ be the level of consumer surplus in a symmetric marketplace with $N$ firms, each practicing uniform pricing. Suppose that $CS_U(N_U) = CS_{NL}(N_U)$. Then social welfare is improved by allowing such nonlinear pricing.

The hypothesis of the proposition is that the first two welfare effects of nonlinear pricing exactly cancel one another at the point where $N = N_U$. Because profits are zero under either form of pricing, nonlinear pricing is welfare enhancing due to the product variety effect of increased entry.

5.2 The Effect of Competition on Equilibrium Qualities

Increases in competition lower firm profits and (weakly) raise the quality consumed by any individual. We now consider the mechanisms by which the increase in quality occurs.

In the case of horizontal uncertainty, the introduction of competition (i.e., adding another firm to the marketplace) has the direct effect of shortening the average horizontal distance which consumers must traverse in order to consume their good. Because $\Delta$ decreases, so does the marginal consumer’s quality distortion. This can be seen directly from equation (7) in Proposition 2. The equation determining the distortion in $q$ depends only upon $N$ indirectly via its effect on $\theta_i$, the distance a consumer must travel to the nearest firm. Thus, a consumer located a distance $\theta$ from a firm will receive the same level of quality regardless how many firms are in the market. The effect of competition is that as $N$ increases, the size of $\theta_i$ decreases on average. Remarkably, this implies that a monopolist with $N$ brands located symmetrically around a circular market will sell the same qualities in equilibrium as $N$ separate firms. The only difference between monopoly and competition is that the prices paid by the consumer will generally be lower (as firms compete for the marginal customer) and there will generally be a greater number of brands (as firms enter until net profits are driven to zero).

In the case of vertical uncertainty, there is an additional competitive effect leading to increases in quality. As in the horizontal case, entry generates lower average distances between firms, which implies that qualities will increase on average. Additionally, even holding a consumer’s distance to a firm constant as one increases $N$, the average quality purchased will increase due to the tightening of some customers’ IR constraints. This is the effect in equation (14). The participation constraint for low vertical types (low $\nu$’s) requires that firms respond to increased competition by raising the quality for low $\nu$’s. As $N$ increases, both $\hat{\nu}$ and $\hat{q}_i$ also increase.

We now have a clear distinction between horizontal and vertical uncertainty with respect to quality. To illustrate this, suppose that firms offer only a finite number of products of varying quality. With the advent of increased competition, we expect firms in a horizontal-uncertainty
environment to respond by eliminating some of its low-quality offerings and concentrating on a smaller market segment. In contrast, firms in vertical-uncertainty environments will respond not only by dropping some low quality products, but by also raising the quality of it’s lower-end products that it continues to sell. In other words, a firm facing horizontal uncertainty does not change its offerings to consumers which it continues to serve, while a firm facing vertical uncertainty effectively compresses its quality offerings towards more efficient levels to those consumers it continues to serve. Product consolidation is therefore more drastic for firms serving customers with unknown vertical preferences.

5.3 Price and Quality Dispersion

In a recent study of airline pricing, an industry which most would regard is replete with second-degree price discrimination, Borenstein and Rose [1991] document that routes which are served by many airlines (measured by a Herfindahl index) tend to have higher price dispersion (as measured by a Gini coefficient of prices of tickets actually purchased). A natural question of the present paper is to consider its predictions regarding price and quality dispersion in a model of multi-product price discrimination with imperfect competition and compare them to the data.\(^\text{13}\)

It is clear following the analysis of Propositions 2 and 4 that the range of qualities offered by firms decreases as \(N\) grows. This fact, combined with our earlier lemmas regarding incentive compatibility, leads us to the unambiguous conclusion that the range of price dispersion must also decrease as the number of firms in the industry grows.

**Proposition 8** (i) Under the conditions of Proposition 2, for any \(\nu\) the ranges of price and quality over \(\theta_i\), \(q_i([0, 1/2N], \nu)\) and \(p_i([0, 1/2N], \nu)\), are decreasing in the number of firms in the industry. (ii) Under the conditions of Proposition 4, for any \(\theta\) the ranges of price and quality over \(\nu\), \(q_i(\theta_i, [\underline{\nu}, \overline{\nu}])\) and \(p_i(\theta_i, [\underline{\nu}, \overline{\nu}])\), are decreasing in the number of firms in the industry.

The proposition states that when firms compete on a market with either horizontal or vertical preference uncertainty, the range of prices and qualities offered to a consumer characterized by a particular \(\nu\) (\(\theta_i\)) decrease as \(N\) grows. This appears at odds with the empirical evidence on airline pricing.

One explanation for the observed price dispersion emerges if the price data is only observed for

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\(^{13}\)Dana [1992] provides an alternative story for such observed dispersion based on capacity constraints and demand uncertainty.
an aggregate group of all consumers, and not just consumers of a particular location (e.g., specific
time-route pairs, etc.) or of a particular quality-preference (e.g., business customers, etc.).\textsuperscript{14} It is
quite possible that the aggregation of the price ranges may indeed increase in size as competition
grows, which is consistent with the evidence. For example, if the lower prices for highly competitive
customer types and the higher prices for less competitive customers (those with some brand loyalty)
are aggregated, the total amount of price dispersion may increase due to competition over only a
particular consumer class.

To be concrete, consider a variation of the numerical example in Section 4 with vertical
uncertainty:\textsuperscript{15} \( u(q, \theta, \nu) = \nu(1 - \theta)q, \ C(q) = \frac{1}{2}q^2, \nu \) uniformly distributed over \([9, 10]\), and \( \Delta = 1 \).
At \( \theta = 0 \), a monopolist would produce a quality range of \([8, 10]\) and a price range of \([72, 91]\). At
\( \theta = \frac{1}{2} \), these ranges would shift to \([4, 5]\) and \([18, 20\frac{1}{2}]\), respectively. Aggregating these ranges across
the horizontal spectrum \( \theta \in [0, \frac{1}{2}] \) yields an aggregate quality range of \([4, 10]\) and an aggregate price
range of \([18, 91]\).

Now consider the duopoly setting. Supposing that the competition-induced quality constraint
does not bind for customers with the strongest brand preference, we have the duopolist behaving
as a monopolist at \( \theta = 0 \). At \( \theta = \frac{1}{2} \), however, marginal cost pricing ensues, yielding a quality
range of \([4\frac{1}{2}, 5]\) and a price range of \([4\frac{1}{2}, 5]\). Aggregating the ranges yields an aggregate quality
range of \([4\frac{1}{2}, 10]\) and an aggregate price range of \([4\frac{1}{2}, 91]\). Thus, although the quality dispersion
decreases from the presence of competition, the price range may increase due to aggregation
problems. Competition may reduce prices in only a few markets, while leaving other markets unaffected.
Aggregating prices over a partially competitive market may therefore suggest increased price disper-
sion. Nonetheless, the increased dispersion is only an aggregation phenomena within the confines
of the present model.

6 Conclusion

We have solved a generally posed problem of second-degree price discrimination in an oligopoly
setting where firms’ products are spatially differentiated and consumers’ preferences vary over hori-
zontal or vertical dimensions. The nature of optimal pricing schedules depends importantly on the
type of private information which the customer possesses (either information regarding brand pref-

\textsuperscript{14}This is related to the results of Borenstein [1985] and Holmes [1989] which show that price dispersion may increase
as one moves from monopoly to competition, although their results do not pertain to multiproduct second-degree
price discrimination. The numerical simulations presented in Appendix B of Borenstein and Rose [1994] are more
directly related.

\textsuperscript{15}Here, for simplicity we have assumed that costs are positive and quadratic. The strict convexity of the cost
function does not affect any of the propositions presented.
ference or information about quality preference) through its effect on the participation constraints. As competition increases, the quality distortions decrease in either case. Additionally, price discrimination may enhance social welfare by increasing consumer surplus through encouraging greater entry.

This paper provides a general framework for considering second-degree price discrimination in a competitive environment. A firm considering it’s choice of product line and prices in the presence of imperfect competition should be aware of the nature of preference uncertainty in the environment within which it is operating. It is, of course, difficult to give precise prescriptions to marketing managers as much depends upon elements outside this model. Furthermore, in this paper we have analyzed the two forms of uncertainty from the polar extremes where one exists and the other does not. A more satisfactory approach would combine both dimensions of uncertainty simultaneously in a multi-dimensional self-selection model, although such models are typically quite intractable and require numerical methods for solutions. Nonetheless, the result that the nature of uncertainty matters fundamentally when designing an optimal nonlinear pricing schedule is persuasive to both the manager and the economist studying the firm’s responses to competition.

Consider some of the marketing implications of the model when competition increases due to entry. The firms’ optimal brand consolidation response depends fundamentally on what kind of demand uncertainty it faces. First take the case where the uncertainty concerning the customer is primarily over the horizontal dimension. Recalling our example of consumer preferences of Windows versus Apple operating systems, a Windows-based computer company should focus its product line on those goods which cater most directly to the more brand sensitive customers. Typically, such a company would offer a line of computers of increasing quality, with price-cost margins for each increment of quality declining as quality increases. For example, companies like Compaq or IBM offer a range of computers with various features and levels of speed, memory and options. Higher quality computers (e.g., faster processor, more memory, etc.) have higher prices, but these higher prices reflect more than the additional costs associated with the higher quality – the prices also allow the firm to extract further profit from the market segment. This paper demonstrates that with the presence of competition, the optimal focus of the firm should be upon the higher end of this schedule of prices and quality of products, that is, the portion of the product line which is most attractive to brand-loyal customers.

\footnote{In Stole [1994], I have considered such a combination with the altered assumption that \( u_{\theta q} = 0 \) and the additional assumption that transportation costs are sunk upon reaching the firm. In such a setting, a tractable solution exists with no quality distortions for either \( \mathbf{\bar{v}} \) or \( \mathbf{\bar{y}} \), but distortions for all other types. More generally, this model is equivalent to the case where there is an unknown marginal preference parameter and the agent’s participation constraint is stochastic.}
Alternatively, take the case where the primary uncertainty over the customer’s preferences is along the vertical dimension. Returning to our example of the computer industry, it may be that the “brand preference” of an individual consumer is easily ascertainable (e.g., whether they prefer Apple or PC compatibility), but the marginal value they attach to increases in quality (e.g. speed, memory, etc.) is unknown. In such a setting (as in the horizontal environment), a monopoly computer firm should offer a menu of prices and qualities in an attempt to segment the market place into consumer groups of varying price sensitivity. In the presence of increased competition, a firm should continue to offer similar quality bundles for high valuation consumers, but it should increase the quality offered to the lower valuation consumers. An increase in quality allows the firm to more aggressively maintain the low-valuation consumer as its customer while still extracting a considerably higher price from the less elastic customer segment.

Although the results have been presented in the context of second-degree price discrimination, the analysis is applicable to larger range of economic settings in which nonlinear contracts and competition are prevalent. For example, a common question in the study of organizations is when a firm will find it optimal to hire its own sales force rather than contract with a common sales force; see, for example, the discussion in Stole [1991]. This paper makes inroads towards understanding the costs and benefits of exclusive agency by indicating how competing principals will affect the nature of the resulting exclusive agency employment contracts. This combined with a study of the costs and benefits of common agency employment contracts (i.e., the resulting contracts when multiple firms contract with the same principal) would complete the analysis. A related contractual application would be to examine how equilibrium requirements contracts are affected by the entry of additional downstream firms or additional upstream suppliers.

\footnote{Martimort [1992] and Stole [1991] independently study the related problem of intrinsic common agency under adverse selection. In their models, the presence of competition leads to a reduced distortion in the product market, but only in a model in which the agent (i.e., consumer) purchases from both firms. The present research considers consumers who wish only to buy from one firm and is therefore a closer approximation to actual consumer purchasing patterns.}
Appendix

Proof of Lemma 1: (Sketch) In the absence of an outside option, we could replace (2) with the more typical IR constraint of

\[ U^i(\vec{\theta}_i) \geq 0. \]

Proof of this simpler lemma is straightforward; see Fudenberg-Tirole [Chapter 7, 1991], for example. However, since the customer also has an option of buying from a rival firm, we must replace the simpler IR constraint with the requirements of (2). This condition is necessary. The proof that (1) and monotonicity of \( q_i \) is necessary and sufficient for incentive compatibility is standard. The incentive compatibility requirements in (1) guarantee that \( U^r(\Delta - \theta) \) is increasing in \( \theta \) and \( U^r(\theta) \) is decreasing in \( \theta \). This implies that (2) is also sufficient for participation. □

Proof of Proposition 1: Suppose that each firm serves customers within distance \( \vec{\theta}_i \), and that \( \vec{\theta}_l + \vec{\theta}_r \leq \Delta \). Firm \( i \)'s problem, taking \( U^{-i}(\cdot) \) as given, is to maximize

\[ E\Pi^i = \int_0^{\vec{\theta}_i} [p_i(s) - c_i q_i(s)] f^i(s) ds \]

subject to (1)-(2) and \( q_i \) nonincreasing in \( \theta_i \). From (1) and integration by parts,

\[ \int_0^{\vec{\theta}_i} U^i(s) f^i(s) ds = U^i(s) F^i(s) \bigg|_{0}^{\vec{\theta}_i} - \int_0^{\vec{\theta}_i} u_\theta(q_i(s), s, \nu) \frac{F^i(s)}{f^i(s)} f^i(s) ds. \]

Thus,

\[ \int_0^{\vec{\theta}_i} p(s) f^i(s) ds = \int_0^{\vec{\theta}_i} \left\{ u(q_i(s), s, \nu) + u_\theta(q_i(s), s, \nu) \frac{F^i(s)}{f^i(s)} - U^i(\vec{\theta}_i) \right\} f^i(s) ds. \]

Substituting and using our definition of \( \Phi^i \) and the IR requirement (2) from Lemma 1, we obtain

\[ E\Pi^i = \int_0^{\vec{\theta}_i} \left\{ \Phi(q_i(s), s, \nu) - \max\{U^{-i}(\Delta - \vec{\theta}_i), 0\} \right\} f^i(s) ds. \]

Since \( \Phi_{\theta \theta} < 0 \), the unique pointwise optimum over \( q_i \) of this expression is given by \( \Phi_{\theta q}(q_i, \theta_i, \nu) = 0 \). This is (3). Given \( \Phi_{\theta \theta} < 0 \), \( q_i \) is decreasing in \( \theta_i \). We need only check that it is optimal for firm \( i \) to choose to sell to the interval of customers within \( \vec{\theta}_i \) distance of the firm.

Differentiating \( E\Pi^i \) with respect to \( \vec{\theta}_i \) yields

\[ \Phi^i(q_i(\vec{\theta}_i), \vec{\theta}_i, \nu) - U^i(\vec{\theta}_i) + \delta(\vec{\theta}_i, \vec{\theta}_r) \frac{F^i(\vec{\theta}_r)}{f^i(\vec{\theta}_i)} u_\theta(q_{-i}(\vec{\theta}_{-i}), \vec{\theta}_{-i}, \nu) = 0, \]

where \( \delta = 1 \) if \( \vec{\theta}_l + \vec{\theta}_r = \Delta \), and 0 otherwise.

There are two possible cases to consider: (I) [Local monopoly] there exists an interval of customers, \( [\vec{\theta}_i, \Delta - \vec{\theta}_i] \), which is served by neither firm, and (II) [Competition], the market is covered. To distinguish the cases, define \( \theta^*_i \) as the solution to \( \Phi^i(q_i(\theta^*_i), \theta^*_i, \nu) = 0 \) if a solution exists; otherwise, set \( \theta^*_i = \Delta \). Given our assumptions on \( \Phi^i \), \( \theta^*_i \) is uniquely defined.

Case I: \( \theta^*_l + \theta^*_r < \Delta \). In such a case, \( \delta = 0 \) and these values of \( \theta^*_i \) represent the unique solution to the optimal choice of \( \vec{\theta}_i \). Thus, (4) holds. Hence, the IR constraint is 0 when both monopolists ignore the possibility that their customers may buy from their competitor and so this represents
the unique Nash equilibrium.

Case II: \( \theta_i^* + \theta_i^r \geq \Delta \). In such a case, the active element in the IR constraint (2) is the utility which a customer could obtain by buying from a competitor. Thus, \( \delta = 1 \), and a Nash equilibrium satisfies (5)-(6). To see that there is in fact an equilibrium, substitute out \( \tilde{U} \) and \( \tilde{\theta} \), to obtain the relation

\[
\Phi^l(q_i(\tilde{\theta}_i), \tilde{\theta}_i, \nu) - \Phi^r(q_r(\Delta - \tilde{\theta}_i), \Delta - \tilde{\theta}_i, \nu) + \frac{F^l(\tilde{\theta}_i)}{f^l(\tilde{\theta}_i)} u_\theta(q_r(\Delta - \tilde{\theta}_i), \Delta - \tilde{\theta}_i, \nu) - \frac{F^r(\Delta - \tilde{\theta}_i)}{f^r(\Delta - \tilde{\theta}_i)} u_\theta(q_i(\tilde{\theta}_i), \tilde{\theta}_i, \nu) = 0.
\]

Simplifying further,

\[
u(q_i(\tilde{\theta}_i), \tilde{\theta}_i) - c_i q_i(\tilde{\theta}_i) - u(q_r(\Delta - \tilde{\theta}_i), \Delta - \tilde{\theta}_i) + c_r q_r(\Delta - \tilde{\theta}_i) = 0.
\]

This expression is continuous and strictly decreasing in \( \tilde{\theta}_i \). Given our assumptions on \( u \) and \( c_i \) at the extremes of the market, the equation is positive at \( \tilde{\theta}_i = 0 \) and negative at \( \tilde{\theta}_i = \Delta \). Thus, there exists a unique Nash equilibrium. □

**Proof of Proposition 2:** (Sketch) The proposition is essentially a corollary of proposition 1 with \( \Delta = 1/N \). Profits are increasing in \( \Delta \), and therefore decreasing in \( N \). Condition (8) follows from the free-entry, zero profit condition, where we have allowed for \( N \) to take on non-integer values. □

**Proof of Lemma 2:** Standard. See Fudenberg-Tirole [Chapter 7, 1991], for example. □

**Proof of Lemma 3:** Since the customer does not purchase from firm \(-i\), it is weakly optimal to offer a price equal to marginal cost. If price were above marginal cost in equilibrium, firm \( i \) could offer a more profitable contract to the marginal customer who is located between firm \( i \) and firm \(-i\). But then firm \(-i\) would prefer to lower price slightly (closer to marginal cost) to steal the customer. A simultaneous Nash equilibrium must have inactive firms offering marginal-cost pricing. □

**Proof of Lemma 4:** If the IR constraint for firm \( i \) binds on the set \([\nu^-, \nu^+]\), Lemma 2 implies

\[
\tilde{U}^{-i}(\theta_i, \nu^-) + \int_{\nu^-}^{\nu^+} u_\nu(q_i(\theta_i, t), \theta_i, t) dt = \tilde{U}^{-i}(\Delta - \theta_i, \nu).
\]

Differentiating and using the envelope theorem on the expression on the right yields (10). □

**Proof of Proposition 3:** Given symmetric costs and since there exists a comparative advantage in serving local customers, the market will be partitioned at \( \tilde{\theta}_i \). Define \( \tilde{q}_i(\theta_i, \nu) \) and \( \tilde{q}_i(\theta_i, \nu) \) by

\[
u(\tilde{q}_i(\theta_i, \nu), \theta_i, \nu) = \nu(q^*_i(\Delta - \theta_i, \nu), \Delta - \theta_i, \nu),
\]

and

\[
u(q(\tilde{q}_i(\theta_i, \nu), \theta_i, \nu) - c = \frac{1 - G(\nu)}{g(\nu)} u_\nu(q(\tilde{q}_i(\theta_i, \nu), \theta_i, \nu),
\]

respectively.

Given A0 and A1, as well as our initial assumptions about \( u \), it is straightforward to show that

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(i) for \( \theta < \frac{1}{2} \), \( \tilde{q}_i(\theta, \nu) > \hat{q}_i(\theta, \nu) \); (ii) if \( \tilde{q}_i = \hat{q}_i \), it occurs only at a unique point, \( \nu \); and (iii), both \( \tilde{q}_i \) and \( \hat{q}_i \) are increasing in \( \nu \).

Consider the candidate solution given in the proposition: \( q = \max\{\tilde{q}_i, \hat{q}_i\} \). This solution is continuous by construction and monotonic by the regularity conditions in A2. In addition, this solution satisfies the sufficient optimality conditions of Seierstad and Sydsaeter [1977], Theorem 6, for the following Hamiltonian:

\[
\mathcal{H} = [u(q_i, \theta_i, \nu) - cq_i - U_i] + \lambda(\nu)u_\nu(q_i, \theta_i, \nu) + \mu(\nu)[U_i(\nu, \theta_i) - \mathcal{U}^{-1}(\Delta - \theta_i, \nu)],
\]

where \( \mu(\nu) > 0 \quad \forall \nu \in [0, \tilde{\nu}] \) and \( \mu = 0 \) otherwise. Therefore, the candidate solution is in fact the firm’s optimal quality schedule. \( U_i \) and \( p_i \) are chosen consistent with Lemma 2, which completes the proof. \( \square \)

**Proof of Proposition 4:** Set \( \Delta = 1/N \) in Proposition 3. Profits are monotone in \( \Delta \), and therefore, \( N \). Condition (15) follows from the free-entry, zero profit condition, where we have allowed for \( N \) to take on non-integer values. \( \square \)

**Proof of Proposition 5:** Since efficient quality is provided to the customer located at \( \theta_i = 0 \) and profits are zero in equilibrium, it is sufficient for all three statements to show that \( \frac{d\theta_i}{dK} > 0 \). Because the equilibrium number of firms is strictly decreasing in \( K \) and converges to \( \infty \) as \( K \) goes to \( 0 \), and \( \bar{\theta}_i = \frac{1}{2N} \), the result is immediate. \( \square \)

**Proof of Proposition 6:** Using the definitions from the proof of Proposition 3 and our assumption that \( u_{\theta\nu} \leq 0 \), note that \( \tilde{q}_i - q^*_i \) decreases pointwise for any \( (\theta_i, \nu) \) as \( \Delta \) decreases. Thus, \( q^*_i - \max\{\tilde{q}_i, \hat{q}_i\} \) increases pointwise as \( \Delta \) decreases. Since \( N \) goes to \( \infty \) as \( \Delta \) goes to 0) as \( K \) decreases to 0, this is sufficient for all three statements. \( \square \)

**Proof of Lemma 5:** Follows from the fact that under both uniform pricing and second-degree price discrimination, the reservation utility of the consumer is unchanged. Thus, profits must be higher with second-degree price discrimination for a given \( N \), and hence in equilibrium, \( N_{NL} \geq N_U \). \( \square \)

**Proof of Proposition 7:** Profits are zero in equilibrium any free-entry equilibrium. Consumer surplus under either regime must necessarily be increasing in \( N \) as product variety increases welfare. Using Lemma 5, welfare is improved with second-degree price discrimination when initially \( CS_U(N_U) = CS_{NL}(N_U) \). \( \square \)

**Proof of Proposition 8:** Quality dispersion decreases in \( N \) directly from the proofs of Propositions 5 and 6. Price dispersion is directly related in quality dispersion from the truth-telling constraints which imply for the case of horizontal uncertainty:

\[
p_i(0, \nu) - p_i(1/2N, \nu) = -\int_0^{1/2N} u_q(q_i(s, \nu), s, \nu) \frac{\partial q(s, \nu)}{\partial \theta} ds.
\]

The integrand is always negative, and so price dispersion increases in \( \frac{1}{2N} \). For the case of vertical uncertainty,

\[
p_i(\theta, \nu) - p_i(\theta, \nu) = \int_{\nu}^{\tilde{\nu}} u_q(q_i(\theta, t), \theta, t) \frac{\partial q(\theta, t)}{\partial \nu} dt,
\]
or alternatively,

\[
p_i(\theta_i, \nu) - p_i(\theta_i, \hat{\nu}) = \int_{\nu}^{\hat{\nu}} u_q(\hat{q}_i(\theta_i, t), \theta_i, t) \frac{\partial \hat{q}(\theta_i, t)}{\partial \nu} dt \\
- \int_{\nu}^{\hat{\nu}^{(N)}} \left[ u_q(\hat{q}_i(\theta_i, t), \theta_i, t) \frac{\partial \hat{q}(\theta_i, t)}{\partial \nu} - u_q(\hat{q}_i(\theta_i, t), \theta_i, t) \frac{\partial \hat{q}(\theta_i, t)}{\partial \nu} \right] dt.
\]

The second integral in the equation above is positive since in the region \([\nu, \hat{\nu}]\) both \(\hat{q} \geq \hat{q}\) and \(\frac{\partial \hat{q}(\theta_i, \nu)}{\partial \nu} > \frac{\partial \hat{q}(\theta_i, \nu)}{\partial \nu}\). Because the size of the second integral increases in \(\hat{\nu}\), which itself increases in \(N\), price dispersion must decrease in \(N\). \(\square\)
REFERENCES


Figure 1: First-Best versus Nash Equilibrium Utilities and Qualities
Figure 2: Quality Contracts for Customer $\theta_i = 0.4$
Figure 3: (A) Optimal Second-Best Contracts for Firm i  
(B) Quality Differences Between the First and Second-Best