Work costs and nonconvex preferences in the estimation of labor supply models

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Abstract

Works costs have not been adequately handled in labor supply estimation, likely due to their complexity. We show that, if work costs are not accounted for in the budget and time constraints in a structural labor supply model, they will be subsumed into the data generating preferences. Even if underlying preferences over consumption and leisure are convex, the presence of unobservable work costs can make these preferences appear nonconvex. However, we show that, under plausible conditions, policy relevant calculations, such as estimates of the effect of tax changes on labor supply and deadweight loss measures, are not affected by the fact that estimated preferences incorporate work costs.

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1. Introduction

In empirical studies, economists nearly always assume that preferences are convex. For example, when estimating labor supply models using local linearization methods (Hall, 1973), the assumption of convex preferences is used to reduce the global labor supply decision down to a marginal decision that is determined by the after tax wage and nonlabor income associated with the budget constraint segment on which the individual is observed. In Hausman (1985), convex preferences yield a computationally easy method of identifying the utility maximizing point on the nonlinear budget constraint and facilitate the straightforward setup of the likelihood function. Finally, in

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the method proposed in MaCurdy et al. (1990), strictly convex preferences yield an implicit function that can be used to solve for optimal hours on a differentiable approximation to the budget constraint which is then inverted and used in the likelihood function.

As we argue in Heim and Meyer (2003b), a possible reconciliation of the findings in previous studies, which often found estimates of labor supply parameters either inconsistent, or constrained to be consistent with economic theory, is that the data used in the various estimation methods were generated by the maximization of nonconvex preferences. We further show that the standard methods used to estimate labor supply in this setting cannot be adapted to allow for the estimation of parameters consistent with nonconvex preferences and suggest a method that can. Why, then, should one consider the possibility that preferences may be nonconvex in the setting of labor supply?

One reason is that underlying preferences over consumption and leisure may be nonconvex. Individuals may simply prefer either no work or significant hours to an intermediate level of work. Preferences that are nonconvex may still satisfy a number of other weaker assumptions, including being complete, reflexive, transitive, continuous, monotonic and locally nonsatiated. It may be that preferences simply do not satisfy convexity even if they satisfy other conditions.

Another reason is that the time frame over which the data are collected is not sufficiently long for convexity of preferences to apply. As noted in Mas-Collel et al. (1995) and Varian (1992), the standard justification for the assumption of convex preferences is that, although one may not want to consume two goods together at the same time, one would prefer a mix of goods over a longer period of time. In the case of most consumption goods, the time frame necessary for this averaging argument to apply is probably short. However, in the case of consumption and hours of work choice, the time frame needed for the averaging argument to apply may be quite long, perhaps even a lifetime. As a result, it may be that in the monthly or yearly time frame that is conventionally used in labor supply estimation, convexity of preferences does not hold.

Finally, in this paper, we show that, even if underlying preferences over consumption and leisure are convex in the period of analysis, what we call data generating preferences or observable preferences in a structural labor supply model may be nonconvex because they may be composed of more than just an individual’s underlying preferences. An individual’s consumption and leisure usually cannot be observed, and so they must be inferred from monetary outlays (or income) and hours of work. Because work costs lead the observed variables used in estimation to differ from the ideal variables in the structural model, preferences may appear nonconvex.

The usual assumption is that all income or outlays are devoted to consumption, and that all noncompensated hours are leisure time. Contrary to this assumption, however, individuals face both time and monetary costs of work when making their choice of labor supply including commuting time, transportation costs, child care, clothing, costs associated with the stress of work and the time preparing for and recovering from work.

These work costs can be sizable, with time work costs likely comprising the larger share. For example, estimates of mean commuting time in several countries range from about 7% to 10% of market work time (Juster and Stafford, 1991).
Furthermore, these costs of work may vary in a complex way with the number of hours that the individual works. Monetary transportation costs (gas, subway fares, parking, etc.) may consist of a fixed cost and costs linear in the number of days worked, or there may be volume discounts available (e.g., monthly transit or parking passes), making such costs a concave function of the number of hours worked. Child care and clothing costs are also likely to be concave in annual hours worked.

There may be economies to working a schedule similar to other people. When this is done, car pools may be used, less expensive child care is available, etc. This suggests that costs of work are greater if one works a number of hours away from full- or part-time work, again implying concavity of costs to an individual over at least some range of hours.

Finally, there are large time costs that are incurred in preparing for, and recovering from work, as well as costs of having the additional responsibilities and complications in one’s life that often come with work. The stress and emotional costs of work figure prominently in the psychology and sociology literatures, for example, in Lee and Ashforth (1996) and Morris and Feldman (1996). Furthermore, Hobfoll (1989) conceptualizes stress as leading to a loss of resources including time and money. These types of costs are likely to be large relative to explicit money costs, and it is likely that these costs are concave in the number of monthly or annual hours that individuals work due to the increased ease in dealing with work if one has a daily and weekly routine. In addition, the presence and likely substantial magnitude of such costs also illustrates that a large part of the costs of work could be thought of as either time costs or as a feature of preferences. Thus, conceptually, it is hard to distinguish time costs of work from preferences.

Given the above discussion, if one is using monthly or yearly data in structural labor supply estimation, ignoring costs of work or employing a fixed cost specification will likely result in a poor approximation to the actual choice set that individuals face. Despite this, work costs have either been left out of the model or treated in a simplistic manner. However, even the studies that include a fixed cost of work in their empirical specifications have tended to find a marked effect on estimated parameters, suggesting that the treatment of work costs in labor supply estimation may be an important element of a model’s specification.

This paper proceeds as follows. In Section 2, we show that even if underlying preferences are convex, the presence of unobservable work costs can make observable preferences nonconvex. A necessary condition for nonconvex observable preferences is that time or money costs of work be concave over some range. Nevertheless, in Section 3, we show that if work costs are incorporated into estimated preferences, some policy relevant calculations, such as deadweight loss calculations and estimates of the effect of tax changes on labor supply, may still be performed under plausible conditions. Section 4 concludes.

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1 See, for example, Hausman (1980), Blau and Robins (1988), Bourguignon and Magnac (1990), Ribar (1992), Hoynes (1996), and Gong and van Soest (2002).

2 In Heim and Meyer (2003a), we also show that if work costs are allowed to be subsumed into observable preferences, joint identification of the work costs and utility functions is not possible if we only make weak assumptions about the shapes of these functions.
2. How work costs can alter estimated preferences

In this section, we demonstrate the observational equivalence between a model in which work costs are specified as part of the budget constraint, and an alternative model in which work costs are specified as altering the shape of indifference curves. We then examine the effect that the incorporation of work costs into preferences will have on the shape of those preferences.

2.1. Incorporation of work costs into utility functions

The following proposition demonstrates that, given a problem in which the consumer maximizes utility over consumption and leisure subject to a budget constraint that incorporates tax laws and monetary costs of work and a time constraint that incorporates time costs of work, there exists a problem involving the maximization of a function over outlays and hours of work that incorporates preferences and time and monetary work costs subject to only the tax law generated budget constraint, and for which the optimal hours of work is the same.\(^3\) We will refer to this function as a composite utility function and to the preferences it represents as observable preferences.

First, let an individual’s utility function be denoted \(U(C,L)\), where \(C\) denotes consumption, and \(L\) denotes leisure. Let the individual’s after tax budget constraint, ignoring the costs of work, be \(f(W,Y,h,\theta)\), where \(W\) denotes their gross wage \(Y\) denotes nonlabor income, and the number of hours they work is denoted as \(h\). Monetary costs of work are given by \(F_1(h)\), and time work costs are given by \(F_2(h)\). Finally, let \(O\) denote total monetary outlays, the sum of outlays on the composite consumption good and monetary costs of work.

**Proposition 1.** For every consumer problem in which a utility function, \(U(C,L)\), is maximized subject to an arbitrary budget constraint that incorporates monetary costs of work, \(F_1(h)\), and a time constraint that incorporates time costs of work, \(F_2(h)\), there exists an equivalent problem in which a composite utility function, \(\tilde{U}(O,h)\), that incorporates preferences, and time and money work costs is maximized subject to only the budget constraint and for which the optimal hours choice is the same.

**Proof.** Consider a consumption–leisure choice problem subject to a general budget constraint that incorporates money costs of work, and an hours constraint that incorporates time costs of work,

\[
\begin{align*}
\max_{C,L,h} & \quad U(C,L) \\
\text{s.t.} & \quad C + F_1(h) \leq f(W,Y,h,\theta) \\
& \quad h + F_2(h) + L \leq \tilde{H}
\end{align*}
\]

where \(\theta\) denotes tax parameters, \(\tilde{H}\) the time endowment, and all other variables are as defined previously.

\(^3\) These results are analogous to the results in Feenstra (1986), Fuhrer (2000), and others in showing how some element of an individual’s budget constraint could be equivalently viewed as affecting the individual’s preferences, or vice versa.
Define \( O = \text{money outlays} = C + F_1(h) \). Using \( C = O - F_1(h) \), and substituting the time constraint in for \( L \), we can rewrite Eq. (1) as

\[
\max_{O,h} U(O - F_1(h), \bar{H} - h - F_2(h))
\]

s.t. \( O \leq f(W, Y, h, 0) \).

(2)

Define \( \tilde{U}(O,h) = U(O - F_1(h), \bar{H} - h - F_2(h)) \). Then, we have

\[
\max_{O,h} \tilde{U}(O,h)
\]

s.t. \( O \leq f(W, Y, h, 0) \).

(3)

Because the problems are equivalent, if \((C^*, L^*)\) solves Eq. (1), then \((O^*, h^*)\), where \( O^* = C^* + F_1(h^*) \) and \( h^* + F_2(h^*) = \bar{H} - L^* \), solves Eq. (3).

The above proposition also holds if the worker faces only monetary (or only time) costs of work. To see this, simply set \( F_2(h) \) [or \( F_1(h) \)] to 0.4

The following proposition demonstrates that the converse of the above proposition is also true.

**Proposition 2.** For every consumer problem in which utility over outlays and hours of work that incorporates the time and money costs of work, \( \tilde{U}(O,h) \), is maximized subject to a budget constraint, there exists an equivalent problem in which utility over consumption and leisure, \( U(C,L) \), is maximized subject to a budget constraint that incorporates monetary costs of work and a time constraint that incorporates time costs of work, and for which the hours choice is the same.

**Proof.** Using the notation above, start with

\[
\max_{O,h} \tilde{U}(O,h)
\]

s.t. \( O \leq f(W, Y, h, 0) \).

(4)

Using that \( O = C + F_1(h) \), and \( L = \bar{H} - h - F_2(h) \), define \( g(h) = F_2(h) + h \). Then, \( \bar{H} - L = g(h) \Rightarrow h = g^{-1}(\bar{H} - L) \). Thus, Eq. (4) now becomes

\[
\max_{C,L,h} \tilde{U}(C + F_1(g^{-1}(\bar{H} - L)), g^{-1}(\bar{H} - L))
\]

s.t. \( C + F_1(h) \leq f(W, Y, h, 0) \)

\( L = \bar{H} - h - F_2(h) \)

(5)

Defining \( \bar{U}(C,L) = \tilde{U}(C + F_1(g^{-1}(\bar{H} - L)), g^{-1}(\bar{H} - L)) \) yields the result.

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4 The setting in the proposition above is clearly a simplified one, in which \( F_1(h) \) and \( F_2(h) \) are exogenous to the workers choice, whereas workers obviously have some choices to make regarding how they get to work, what type of training they take, what form of child care their children receive, etc. However, it is possible to generalize this proposition to a model in which individuals can choose among various modes of transportation, child care, etc., with each mode having its own time and money cost schedules. Details are in Heim and Meyer (2003a).
Because the problems are equivalent, if \((O^*,h^*)\) solves Eq. (4), then \((C^*,L^*)\), where 
\[ C^* = O^* - F_1(h^*) \]
and 
\[ L^* = \bar{H} - h^* + F_2(h^*) \]
solves Eq. (5).

Because these two maximization problems are equivalent, a data generating process involving the maximization of a utility function subject to budget and time constraints that incorporate work costs has an equivalent data generating process, in which individuals maximize a composite utility function which subsumes the work costs subject only to a tax law generated budget constraint.\(^5\) Thus, if work costs are present in actuality but ignored or greatly simplified in the specification of a structural labor supply estimation method, the unaccounted for work costs will be subsumed into the estimated preferences.\(^6\)

Two main issues arise when one allows work costs to be subsumed into estimated preferences. First, doing so may affect the proper specification of heterogeneity because heterogeneity in time and money costs of work may now cause there to be additional dimensions of heterogeneity in observable preferences,\(^7\) possibly invalidating the one-dimensional specification of heterogeneity that is typically used. Secondly, doing so may affect the assumptions that can be made about the shape of such preferences, possibly affecting the choice of the estimation method itself. We address this concern in the following section.

2.2. Nonconvexity of observable preferences due to work costs

In this section, we demonstrate that, when work costs are subsumed into observable preferences, it is possible that the resulting preferences would be nonconvex even when the underlying preferences are convex. We then discuss the effects of such an occurrence on the choice of an estimation method.

The following proposition demonstrates a necessary condition on the monetary costs function, denoted as \(F_1(h)\), and time costs function, denoted as \(F_2(h)\), for observable preferences, denoted as \(\tilde{U}(O,h)\), to be nonconvex.

Proposition 3. **Strict concavity of either \(F_1(h)\) or \(F_2(h)\) over some range of \(h\) is a necessary condition for observable preferences \(\tilde{U}(O,h)\) to be nonconvex.**

Proof. Suppose not, that \(F_1(\alpha h + (1 - \alpha) h') \leq \alpha F_1(h) + (1 - \alpha) F_1(h') \) and \(F_2(\alpha h + (1 - \alpha) h') \leq \alpha F_2(h) + (1 - \alpha) F_2(h')\) for all \(h' \neq h\) and \(\alpha \in [0,1]\), but that \(\tilde{U}(O,h)\) is nonconvex. Then

\[
\tilde{U}(\alpha O + (1 - \alpha) O', \alpha h + (1 - \alpha) h')
\]

\(^5\) If work costs are partially observable, it is a simple extension of the propositions above to show that if the budget constraint is specified using the tax law generated budget constraint and the observable work costs, then the unobservable work costs will be incorporated into the estimated preferences.

\(^6\) Furthermore, because the shape of observable preferences are affected both by the shape of underlying preferences and by the shape of work cost functions, any characteristic of the observable preferences (such as nonconvexity or nonmonotonicity) could have resulted from the properties of any of the component functions. This aspect of the propositions presented above is similar in spirit to a result in Browning (1997).

\(^7\) For a formal demonstration of this point, see Heim and Meyer (2003a).
\[
U = U\left(aO + (1 - \alpha)O' - F_1(xh + (1 - \alpha)h'), \quad \tilde{H} - (xh + (1 - \alpha)h') - F_2(xh + (1 - \alpha)h')\right).
\]

Because \(F_1(xh + (1 - \alpha)h') \leq \alpha F_1(h) + (1 - \alpha)F_1(h')\) and \(F_2(xh + (1 - \alpha)h') \leq \alpha F_2(h) + (1 - \alpha)F_2(h')\), and \(U(C, L)\) is monotonic in both arguments, we have

\[
\begin{align*}
\alpha & \geq U\left( aO - F_1(h), \frac{\partial F_1}{\partial h'} \right), \\
& \quad \left( 1 - \alpha \right) \left[ \frac{\partial F_1}{\partial h'} \right] - F_2(xh + (1 - \alpha)h') \right)
\end{align*}
\]

By the quasiconcavity of \(U(C, L)\),

\[
\begin{align*}
\geq & \min \left\{ U(O - F_1(h), \frac{\partial F_1}{\partial h'}), \\
& \quad U(O' - F_1(h'), \frac{\partial F_1}{\partial h'}) \right\} \\
= & \min \{ \tilde{U}(O, h), \tilde{U}(O', h') \}.
\end{align*}
\]

Hence \(\tilde{U}(O, h)\) is quasiconcave; observed preferences are convex, and we have a contradiction. \(\square\)

Obviously, the sufficient condition for \(\tilde{U}(O, h)\) to be nonconvex is, for some \(O \neq O'\) and \(h \neq h'\),

\[
\begin{align*}
U & \left( aO + (1 - \alpha)O' - F_1(xh + (1 - \alpha)h'), \quad \tilde{H} - [xh + (1 - \alpha)h'] - F_2(xh + (1 - \alpha)h') \right) \\
< & \min \left\{ U(O - F_1(h), \frac{\partial F_1}{\partial h'}), \\
& \quad U(O' - F_1(h'), \frac{\partial F_1}{\partial h'}) \right\}.
\end{align*}
\]

Essentially, this condition requires that \(F_1(h)\) or \(F_2(h)\) be sufficiently concave for observable preferences, \(\tilde{U}(O, h)\), to be nonconvex.

If one further makes the assumption that all functions are continuous and twice differentiable, it can be shown that the sufficient condition for observable preferences to be nonconvex becomes

\[
U_1 \frac{\partial F_1}{\partial h_2} + U_2 \frac{\partial F_2}{\partial h_2} + \frac{1}{U_1^2} \left( 1 + \frac{\partial F_2}{\partial h} \right)^2 (-U_1^2 U_{22} + 2U_1 U_2 U_{12} - U_2^2 U_{11}) \leq 0,
\]

where \(U_i\) denotes the partial derivative of \(U\) with respect to its \(i\)th argument. To interpret this, under the assumption that underlying preferences are monotonic and convex, we know that \(U_1\) and \(U_2\), as well as the entire last term, are positive. Hence, the sufficient condition amounts to requiring that the second derivatives of either or both of the work
cost functions be sufficiently negative so that the sum of the first two terms is larger in absolute value than the final term.

Because work costs vary in a complex manner with the number of hours worked and may be concave in the number of hours or even decrease over a range of hours given the conditions above, it is a distinct possibility that observable preferences over outlays and hours of work will exhibit nonconvexities. As a result, if one uses a method that relies on the assumption that preferences are convex while specifying the budget constraint as the budget constraint resulting from tax laws, then the model may be misspecified.8

If this is the case, it drastically alters the researchers’ set of possible estimation methods. In Heim and Meyer (2003b), we showed that all of the usual methods of estimating labor supply parameters, including local linearization, the Hausman method, and the MaCurdy method, cannot be modified to allow for the estimation of observably nonconvex preferences, but that alternate methods based on estimating parameters of a direct utility function may be applied in this case.

3. Irrelevance of composition of estimated preferences to some policy and welfare analyses

In this section, we examine whether estimating a composite utility function and not knowing or estimating the work cost functions will affect one’s ability to perform certain policy analyses.

Obviously, without data on or estimates of work costs, some calculations cannot be performed, such as examining the labor supply effects of implementing a tax credit for child care costs. We show, however, that the lack of knowledge about work cost functions does not preclude one from making some of the most common policy relevant calculations. Namely, we show that the results of the most common policy and welfare calculations are invariant to whether the shape of estimated preferences arises solely from the shape of underlying preferences, or some combination of underlying preferences and work costs. Furthermore, these results hold whether or not estimated preferences are nonconvex. The key to these propositions is that the proposed policy change must not affect the shape of the work cost functions. As such, work costs must not be treated differently in the tax code than leisure time or consumption.9

Suppose that we are interested in the effect of a change in the tax law generated budget constraint, from $f(W,Y,h,\theta_1)$ to $f(W,Y,h,\theta_2)$, on an individual’s labor supply. Using the

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8 In Heim and Meyer (2003b), we find both theoretical and empirical support for the contention that, using an estimation method that relies on the assumption that preferences are convex (such as in Hall (1973), Hausman (1981), or MaCurdy et al. (1990)) when data generating preferences are actually nonconvex, can lead estimated parameters to exhibit wrongly signed compensated wage effects. Because compensated wage effects were either wrongly signed or constrained to be of the correct sign in a number of studies (see, for example, MaCurdy et al. (1990), Blomquist and Hansson-Brusewitz (1990), Colombino and Del Boca (1990), and Triest (1990)), it may be that not taking account of the complex form of costs of work in the estimation method led to the puzzling results in these studies.

9 One should also note that we are assuming there are no general equilibrium effects of the tax policy change, or the tax code, that change the magnitude of work costs.
notation of Section 3, consider an estimated (possibly composite) utility function $\hat{U}(O,h)$, which may consist of work costs subsumed into observable preferences, or may consist solely of underlying preferences. Let $h_1$ be the hours of work that maximize this function on the budget constraint $f(W,Y,h,\theta_1)$, and $h_2$ be the hours that maximize this function on the budget constraint $f(W,Y,h,\theta_2)$. Note that, given Proposition 1, the hours that maximize underlying utility on the two budget constraints would be $h_1$ and $h_2$, respectively, regardless of whether $\hat{U}(O,h)$ consists solely of preferences or consists of preferences augmented by work costs. Hence, the estimate of the labor supply effect of the change in the tax generated budget constraint is the same in either case. As a result, given estimates of $\hat{U}(O,h)$, we can examine the effect of such a policy change as if the estimated preferences consisted solely of underlying preferences.

A similar equivalence holds for deadweight loss calculations. Again, this result comes with the caveat that work costs must not be treated differently than consumption (or leisure time) in the tax code. The following proposition, then, demonstrates that the calculation of the deadweight loss of an income tax that does not affect work costs is invariant to whether estimated preferences have monetary or time work costs contained within them.

Firstly, consider a case in which observable, possibly nonconvex, preferences over consumption and leisure are represented by the utility function $\bar{U}(C,L)$, which in the absence of costs of work could also be represented as $\bar{U}(C,h) = \bar{U}(C,H - h)$. Secondly, consider another case in which the underlying preferences over consumption and leisure are represented by $\hat{U}(C,L)$. However, suppose that due to monetary costs of work, $F_1(h)$, and time costs of work $F_2(h)$, we observe preferences $\hat{\hat{U}}(O,h)$, where $\hat{\hat{U}}(O,h) = \hat{U}(O - F_1(h),H - F_2(h) - h)$. Finally, let $\bar{U}(a,b) = \hat{U}(a,b)$, so that both sets of observable indifference curves over $O$ (which equals $C$ in the absence of time costs) and $h$ have the same form, and hence are observationally equivalent if we cannot observe the costs of work.

The following propositions demonstrate that the calculation of the deadweight loss of an income tax that does not affect work costs is invariant to whether preferences have monetary or time work costs contained within them.

**Proposition 4.** The deadweight loss from imposing an arbitrary tax schedule, $f(W,Y,h,\theta)$, on an agent with possibly nonconvex preferences $\bar{U}(C,L)$, which may be represented in the absence of costs of work as $\bar{U}(C,h) = \bar{U}(C,H - h)$, equals the deadweight loss from imposing this same tax schedule on an agent with underlying preferences $\hat{U}(C,L)$ and unobservable monetary and time work costs, $F_1(h)$ and $F_2(h)$, yielding possibly nonconvex observable preferences $\hat{\hat{U}}(O,h) = \hat{U}(O - F_1(h),H - F_2(h) - h)$, where $\bar{U}(a,b) = \hat{U}(a,b)$.

**Proof.** See Appendix A.

For a sketch of the proof, consider Figs. 1 and 2 which illustrate this proposition for the case of only monetary work costs. Fig. 1 demonstrates the calculation of deadweight loss when the underlying preferences are nonconvex. In this case, the leisure the individual consumes is $L^*_0$ which corresponds to working hours $h^*_0$. Letting $e(W,\theta,u)$ denote the generalization of the expenditure function to the setting of an arbitrary budget.
constraint, defined as the unearned income required to afford a bundle that provides utility, $u$, given wage, $W$, and tax parameters, $\theta$, we have that $e(W,\theta,u_0) = Y$. Let $\theta_0$ denote tax parameters in which all tax rates are set to zero, and no taxes are collected. In the

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Fig. 1. Graphical demonstration of Proposition 4—nonconvex inherent preferences.

Fig. 2. Graphical demonstration of Proposition 4—nonconvex observable preferences.
absence of taxes, the individual then could have reached the same level of utility with 
unearned income \( e(W, \theta_0, u_0) = \tilde{C}_0 - \tilde{W}_0 \). The amount of income tax the government collects is \( R_0 = Y + Wh_0^* - C_0^* \), and hence, the deadweight loss of the tax system is

\[
DWL_0 = e(W, \theta, u_0) - e(W, \theta_0, u_0) - R_0
\]

\[
= Y - [\tilde{C}_0 - \tilde{W}_0] - [Y + Wh_0^* - C_0^*].
\]

In Fig. 2, the indifference curve is only observably nonconvex because of the presence of the costs of work. However, the observable indifference curve, \( \hat{U} = (O_h) \), is exactly the same shape as in the previous figure. Thus, the individual consumes the same amount of leisure, \( L_1^* = L_0^* \), and works the same number of hours \( h_1^* = h_0^* \). Consumption is lower in this figure, but the total amount of outlays in this figure, \( O_1^* = C_1^* + F_1(h_1^*) \), equals the amount of consumption, \( C_0^* \), in Fig. 1.

Therefore, to calculate the deadweight loss in this case, we first note that at the optimal consumption and leisure bundle in the presence of the tax, unearned income must be \( e(W, \theta_0, u_0) = Y \). If the tax were not in place, the individual could have reached the same level of utility with unearned income \( e(W, \theta_0, u_0) = \tilde{C}_1 + F_1(\hat{h}_1) - \tilde{W}_1 \). The amount of revenue that the government collects is \( R_1 = Y + Wh_1^* - C_1^* - F_1^*(h_1^*) \), and so the deadweight loss of the tax system in this figure is

\[
DWL_1 = e(W, \theta, u_0) - e(W, \theta_0, u_0) - R_1
\]

\[
= Y - [\tilde{C}_1 + F_1(\hat{h}_1) - \tilde{W}_1] - [Y + Wh_1^* - C_1^* - F_1^*(h_1^*)]
\]

\[
= Y - [\tilde{O}_1 - \tilde{W}_1] - [Y + Wh_1^* - O_1^*].
\]

Finally, because \( O_1^* \), \( \tilde{O}_1 \), \( h_1^* \) and \( \hat{h}_1 \) in Fig. 1 are the same amounts as \( C_0^* \), \( \tilde{C}_0 \), \( h_0^* \) and \( \hat{h}_0 \), respectively, in Fig. 2, the two deadweight losses are the same.

Thus, if we calculate the deadweight loss explicitly accounting for the fact that observable preferences have work costs embedded within them, we get the same quantity as when we calculate deadweight loss using a utility function whose indifference curves have the same shape. As such, given estimates of preferences that may or may not subsume work costs, we can proceed calculating the deadweight loss as if the estimated preferences consist solely of underlying preferences.

Because the proposition above dealt with an arbitrary budget constraint, linear and piecewise linear taxes are obviously special cases of the budget constraint specification, and so the result above applies in these familiar cases as well.

The intuition behind this result is straightforward. As was noted above, the tax distortion on the consumption-leisure choice is unaffected by the source of the shape of the indifference curve, so long as the items that influence that shape of the indifference curves (the monetary and time costs of work) are not treated differently in tax law.\(^{11}\)

\(^{11}\) These propositions also hold if some work costs are observable and accounted for in the budget constraint, and other work costs are unobservable and subsumed into estimated preferences. In addition, a similar proposition applies to goods or activities other than labor supply, when the consumption of such a good or activity involves the expenditure of time and/or money. Proofs of these claims are presented in Heim and Meyer (2003a).
Thus, the question becomes whether the costs of work are actually treated differently by tax law. Clearly, time costs of work are not affected by tax law. Because we believe time costs outside of the workplace are the main work costs, our results should largely apply. Money costs of work are more likely to be deductible, particularly child care costs. In such situations, our DWL propositions are less applicable. However, child care costs are often not deductible in the U.S. Furthermore, in the U.S. and elsewhere, the regular costs of travel to or from work are not deductible. Overall, given the preponderance of time costs of work, most work costs are not likely to be differentially treated under tax systems, and the above propositions should be largely applicable.

4. Conclusion

Work costs have been generally ignored or incorporated in a simplistic way in structural labor supply models. We show that if one ignores work costs in formulating an estimation approach, then work costs will be incorporated into observable preferences. We then show that the incorporation of work costs into observable preferences may yield preferences that are nonconvex. Finally, we show that estimated preferences that incorporate work costs can still be used to simulate the labor supply effects of changes in tax policy and to estimate the deadweight loss of the income tax, as long as the tax code treats work costs like other expenditures of time and money.

Because a realistic explicit incorporation of the costs of work is often infeasible, these results imply that one should be wary of making the assumption that preferences are convex when work costs are specified in a simple manner. The results provide a rationale for the contention in Heim and Meyer (2003b) that puzzling findings in the literature, where estimated labor supply functions violate basic economic assumptions, may be due to estimation methods being used on data generated by individuals with nonconvex (or observably nonconvex preferences), contrary to the assumed data generating process.

Whether estimated preferences are actually nonconvex, of course, is an empirical issue. This paper, however, provides a theoretical rationale as to why researchers may want to allow for potentially nonconvex preferences and provides guidance about the policy analyses that may safely be performed with such estimates.

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Appendix A

The following proposition demonstrates the deadweight loss equivalence for an arbitrary budget constraint.

**Proposition 5.** The deadweight loss from imposing the nonproportional tax schedule, \( f(W,Y,h,\theta) \), on an agent with possibly nonconvex preferences, \( \bar{U}(C,L) \), which may be represented in the absence of costs of work as \( \bar{U}(C,H-h) \), equals the deadweight loss from imposing this same tax schedule on an agent with underlying preferences, \( \bar{U}(C,L) \), and unobservable monetary and time work costs, \( F_1(h) \) and \( F_2(h) \), yielding possibly nonconvex observable preferences \( \bar{\bar{U}}(O,h) = \bar{U}(O - F_1(h), H - F_2(h) - h) \), where \( \bar{U}(a,b) = \bar{\bar{U}}(a,b) \).

**Proof.** Consider a choice of consumption and hours of work,

\[
(C_0^*, L_0^*) = \arg \max_{C,L} \{ \bar{U}(C,L) : C \leq f(W, Y, h, \theta), h = H - L \},
\]

where \( W \) is the gross wage, \( Y \) is nonlabor income, \( \theta \) are tax parameters, \( f(W,Y,h,\theta) \) is the after tax income from working \( h \) hours, \( H \) is the time endowment, and the price of consumption is normalized to \( I \). This may be written in an equivalent way as

\[
(C_0^*, h_0^*) = \arg \max_{C,h} \{ \bar{U}(C,h) : C \leq f(W,Y,h,\theta) \}.
\]

Let

\[
u_0 = \bar{U}(C_0^*, L_0^*) = \bar{U}(C_0^*, h_0^*).
\]

Let \( e(W,\theta,u) \) denote the generalization of the expenditure function to the setting of an arbitrary budget constraint, defined as the unearned income required to afford a bundle that provides utility, \( u \), given wage, \( W \), and tax parameters \( \theta \). Using the duality between the utility maximization problem and the expenditure minimization problem, we have the value of the expenditure function evaluated at \( u_0 \),

\[
e(W,\theta,u_0) = Y.
\]

Now, let

\[
(C_0, L_0) = \arg \min_{C,L} \{ C - Wh : \bar{U}(C,L) \geq u_0, h = H - L \},
\]

which also has an equivalent formulation as

\[
(C_0, h_0) = \arg \min_{C,h} \{ C - Wh : \bar{U}(C,h) \geq u_0 \}.
\]

Let \( \theta_0 \) denote tax parameters in which all tax rates are set to zero, and no taxes are collected. Clearly, by the definition of the expenditure function,

\[
e(W,\theta_0,u_0) = C_0 - Wh_0.
\]

Finally, let the taxes collected by the government be characterized by \( R_0 \), where

\[
R_0 = Y + Wh_0^* - C_0^*.
\]
By the definition of deadweight loss,

\[
\text{DW } L_0 = e(W, \theta, u_0) - e(W, \theta_0, u_0) - R_0.
\] (26)

Substitution of Eqs. (21), (24) and (25) into Eq. (26) yields

\[
\text{DW } L_0 = Y - [\tilde{C}_0 - WH] - [Y + WH_0^* - C_0^*].
\] (27)

Now, let

\[
(C_1^*, L_1^*, h_1^*) = \arg\max_{C,L,h} \left\{ \hat{U}(C, L) : C \leq f(W, Y, h, \theta) - F_1(h), \right. \\
\left. L = \bar{H} - F_2(h) - h \right\}.
\] (28)

To evaluate these quantities, note that we can substitute in the time constraint and use \(O = C + F_1(h) \implies C = O - F_1(h)\) to write Eq. (28) in an equivalent form as

\[
(O_1^*, h_1^*) = \arg\max_{O,h} \left\{ \hat{U}(O) : O \leq f(W, Y, h, \theta) \right\}.
\] (29)

which can be further rewritten as

\[
(O_1^*, h_1^*) = \arg\max_{O,h} \left\{ \hat{U}(O) : O \leq f(W, Y, h, \theta) \right\}.
\] (30)

Because \(\hat{U}(a, b) = \overline{U}(a, b)\), comparing Eq. (30) to Eq. (19), it is clear that \(O_1^* = C_0^*\) and \(h_1^* = h_0^*\). Letting \(u_1 = U(C_1^*, L_1^*)\), again using the duality between the utility maximization problem and the expenditure minimization problem, we have the value of the expenditure function evaluated at \(u_1\),

\[
e(W, \theta, u_1) = Y
\] (31)

Now, let

\[
(\tilde{C}_1, \tilde{L}_1, \tilde{h}_1) = \arg\min_{C,L,h} \{C + F_1(h) - WH : \hat{U}(C, L) \geq u_1, L = \bar{H} - F_2(h) - h\}.
\] (32)

Then, by the definition of the expenditure function, we have

\[
e(W, \theta_0, u_1) = \tilde{C}_1 + F_1(\tilde{h}_1) - \tilde{W}\tilde{h}_1.
\] (33)

Note, however, that because

\[
u_1 = \hat{U}(C_1^*, L_1^*)
\]

\[
= \hat{U}(O_1^* - F_1(h_1^*), \bar{H} - F_2(h_1^*) - h_1^*)
\]

\[
= \hat{U}(O_1^*, h_1^*)
\]

\[
= \overline{U}(C_0^*, h_0^*) = u_0,
\] (34)
and using $C = O - F_1(h)$, Eq. (32) may be rewritten as

$$(\tilde{O}_1, \tilde{h}_1) = \arg \min_{\tilde{O}, \tilde{h}} \{ O - Wh : \hat{U}(O - F_1(h), \tilde{H} - F_2(h) - h) \geq u_0 \},$$

which, by the definition of $\hat{U}(O, h)$, becomes

$$(\tilde{O}_1, \tilde{h}_1) = \arg \min_{\tilde{O}, \tilde{h}} \{ O - Wh : \hat{U}(O, h) \geq u_0 \}.$$  (35)

Because $\overline{U}(a,b) = \hat{U}(a,b)$, comparing Eq. (36) to Eq. (23), it is clear that $\tilde{O}_1 = \tilde{C}_0$ and $\tilde{h}_1 = \tilde{h}_0$. Using $\tilde{C}_1 = \tilde{O}_1 - F_1(\tilde{h}_1)$, these equalities imply that Eq. (33) is equal to

$$e(W, \theta_0, u_1) = \tilde{C}_0 - W\tilde{h}_0.$$  (37)

Finally, the tax revenue is

$$R_1 = Y + Wh_{1}^*- C_{1}^* - F_{1}(h_{1}^*)$$
$$= Y + Wh_{1}^* - O_{1}^*,$$

which, because $O_{1}^* = C_{0}^*$ and $h_{1}^* = h_{0}^*$ as noted above, implies

$$R_1 = Y + Wh_{0}^* - C_{0}^*.$$  (39)

In this case,

$$DW L_1 = e(W, \theta, u_1) - e(W, \theta_0, u_1) - R_1.$$  (40)

Substitution of Eqs. (31), (37) and (39) into Eq. (40), and comparing with Eq. (27) yields the result. □

References


