Discussion of

Market Expectations in the Cross Section of Present Values

by Kelly, B., and S. Pruitt

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The problem tackled by this paper

- Market return

$$r_{mt+1} = \mu_t + \eta_{t+1}$$

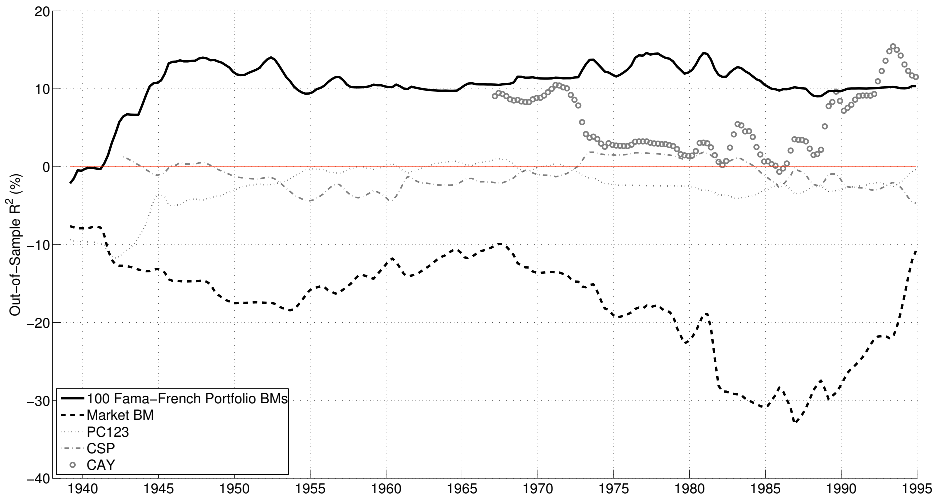
- Present-value decomposition of market M/B ratio

$$x_{mt} \approx a - b_\mu \mu_t + b_g g_t$$

  with VAR(1) dynamics of book ROE, g_t, and expected return, μ_t
- Presence of g_t obscures "signal" μ_t
- This paper: Cross-sectional information helps filter out μ_t to improve predictive regressions
- Much of my discussion focuses on Kelly and Pruitt (2012, "3PRF", WP), the paper that supplies the methodology

The results: U.S.

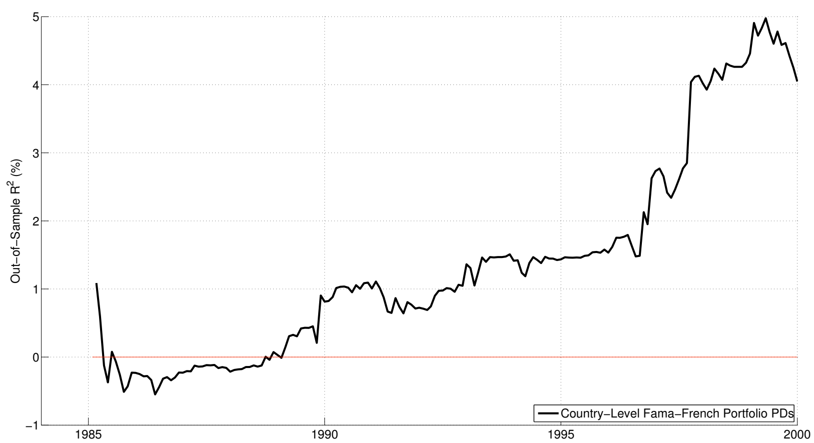
Figure 1: Out-of-Sample R^2 by Sample Split Date, One Year Returns



Notes: Out-of-sample R^2 across sample split dates. Forecasts are based on a single PLS factor from 100 book-to-market ratios of size and value-sorted portfolios, the aggregate book-to-market ratio, the cross-section premium of Polk et al. (2006), the consumption-wealth ratio of Lettau and Ludvigson (2001), and the first three principal components of the 100 book-to-market ratio cross section.

The results: International

Figure 3: Out-of-Sample R^2 by Sample Split Date, One Month International Returns



Notes: Out-of-sample percentage R^2 across sample split dates for forecasts of one month international stock returns using a single PLS factor from 42 price-dividend ratios of high value and low value portfolios across 21 countries (Fama and French (1998)). See Section III.A.5 for list of countries.

How does it work? Example

- Define: $g_t \equiv$ ROE component orthogonal to expected returns
- Assume: Three-asset cross-section where M/B ratios load on μ_t and g_t with cross-sectionally **uncorrelated** loadings

$$
\begin{aligned}
x_{1t} &= 2\mu_t - 1.5g_t & \tilde{x}_{1t} &= \mu_t - 0.5g_t \\
x_{2t} &= \mu_t & \tilde{x}_{2t} &= g_t \\
x_{3t} &= -1.5g_t & \tilde{x}_{3t} &= -\mu_t - 0.5g_t
\end{aligned}
$$

- First-stage t.s. regressions of x_{it} on r_{mt+1} (large T): slopes proportional to

$$
\begin{aligned}
\phi_1 &= 2 & \tilde{\phi}_1 &= 1 \\
\phi_2 &= 1 & \tilde{\phi}_2 &= 0 \\
\phi_3 &= 0 & \tilde{\phi}_3 &= -1
\end{aligned}
$$

- Second-stage c.s. regressions of $\tilde{x}_{it}$ on $\tilde{\phi}_i$ each t: slopes

$$F_t = \text{const.} \times [(\mu_t - 0.5g_t) + 0 - (-\mu_t - 0.5g_t)] = \text{const.} \times \mu_t$$

How does it work? Crucial assumption

- Assumption that loadings on μ_t and g_t are c.s. **uncorrelated** is important: Second stage regression slopes F_t are then...
  - "Long" in assets with positive M/B loadings on μ_t
  - "Short" in assets with negative M/B loadings on μ_t
  - "Long" and "short" in assets with M/B that load similarly on g_t: g_t exposure cancels out
  - As $N \to \infty$, sample c.s. correlation closer to zero population c.s. correlation: μ_t consistently estimated
- But what if loadings on μ_t and g_t are c.s. **correlated**?
  - Lucky: For stock market application in JF paper, correlation seems to be close to zero
  - But this may not be true in other applications
  - Remedy: Use proxies for g_t in addition to μ_t proxy
  - But that means we have to take a stand on **all** of the systematic factors driving M/B ratios

Concern: Large N, small T

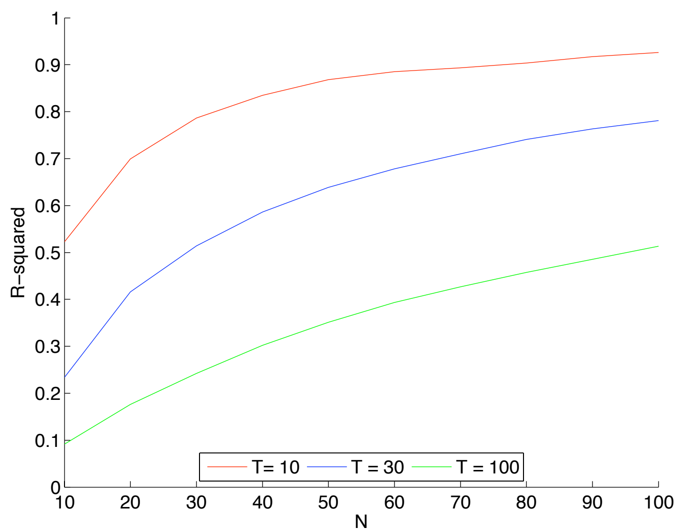
- What are the properties of the estimator under the null hypothesis of no predictability?
- Simulation: No-predictability & pure-noise M/B null

$$r_{mt+1} = \eta_{t+1}$$
$$x_t = \epsilon_t$$

where

$$\begin{pmatrix} \eta_{t+1} \\ \epsilon_t \end{pmatrix} \sim \mathcal{N}(0, I_{N+1})$$

Large N, small T: Spurious fit as N grows



Mean third-stage R-squared under no-predictability null (100 simulations for each (N, T) pair)

Large N, small T: Example with $T = 3$ and $N = 9$

Pure noise returns

Pure noise M/B

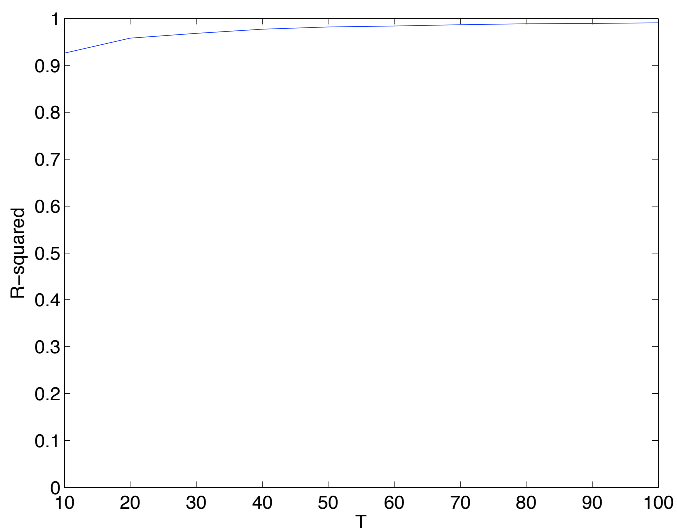
First stage (sorted)

Second stage

Third stage (fitted)

r_{mt+1}			
-1.35	0.91	-1.49	
x_{it}			$\hat{\phi}_i$
-0.52	-1.26	0.39	-5.56
-0.82	-0.44	-0.22	-1.73
-0.20	1.07	2.51	-0.49
-0.93	-2.62	0.34	-0.20
0.40	-0.06	-1.14	-0.02
1.94	0.43	2.08	1.18
-1.89	-0.96	-1.83	1.83
0.44	-0.82	1.51	10.88
-0.68	-0.37	-0.07	18.28
$\hat{F}_t$			
0.0044	0.0100	0.0040	
$\hat{y}_{t+1}$			
-0.98	0.80	-1.75	
R^2			
0.94			

N and T grow simultaneously: Spurious fit with $N = T^2$



Mean third-stage R-squared under no-predictability null with $N = T^2$
(100 simulations for each (N, T) pair)

Spurious predictability

- Thus, when N is not small relative to T, there is a bias that overstates predictability
- It seems that this is not just a small-sample bias: also asymptotic bias if N grows sufficiently fast relative to T
  - Theorem 1 in Kelly and Pruitt (2012, "3PRF") seems to need additional assumption: N cannot grow too fast relative to T
- Not a concern for the empirical results in the JF paper, as out-of-sample tests are not affected by this bias
- But concern underscores importance of out-of-sample testing
- Calls for further study of small-sample and asymptotic properties of this estimator

Summing up

- Nice idea
- Impressive empirical results: Strong out-of-sample predictability of stock market returns
- Some further work necessary on the properties of the 3PRF estimator
  - Correlated factor loadings
  - Small-T/large-N behavior
  - Asymptotic behavior when N grows fast relative to T
- These concerns do not affect out-of-sample tests: results in the JF paper are robust